Ninth Canadian Conference on
Earthquake Engineering
Ottawa, Ontario, Canada
26-29 June 2007

ROBUST ACTIVE CONTROL OF STRUCTURES UNDER SEISMIC EXCITATIONS

S. Hayati[1], A. H. Davaie-Markazi[1] and H. Rokni Damavandi Taher[3]

ABSTRACT

In this paper, an H_∞ optimal control scheme is employed to reduce the vibrations of a structure under seismic excitations and in the presence of *floors and columns mass uncertainty*. It is known that in an occasionally-populated building, e.g., a theatre, in which considerable amount of mass uncertainty exists; a non-robust active vibration control system may lead to closed-loop instability, while the uncontrolled structure is stable by nature. It is also known that, when the relative mass of columns, compared to the mass of floors, is high, in particular in the lower floors of the building, the structural mass matrix turns out to have off-diagonal terms. Such additional terms are modeled as a full block uncertainty matrix acting on the first two floors of a 5-story building, and an H_∞ active control system is designed. Next, by employing the μ-analysis technique, the robustness of the feedback control system under mass variations is studied, and the tolerable bound of mass uncertainty is further increased to 10%, using the μ-synthesis technique. Simulation studies depict the effectiveness of the designed control system.

Introduction

With the increased number of highly populated buildings, the safety of such structures under seismic excitations is an important issue. Active and semi-active control systems have provided new possibilities and opportunities for making buildings a safer place to be in. Recent researches and achievements on magneto-rheological (MR) dampers have improved the effectiveness of such devices for constructing better semi-active control systems. In order to reduce the response of an excited structure with parametric uncertainty on the stiffness and damping, Calise (1998) proposed an H_∞ control approach. For the situation that the available peak control force is limited, Yang and Jabbari (2004) used the H_∞ control scheme, without taking the plant uncertainty into account. In Majia (2005), the design of a robust controller using QFT and sliding mode control was investigated, and a six story building with a semi active control strategy was presented. In Wang (2004), a robust active control with uncertainties on the control input and disturbance matrices was proposed.

The present paper concentrates on the building mass uncertainty, which is the most obvious kind of uncertainty in highly populated buildings, like theatres and schools. A possible mathematical structure of mass uncertainty will be studied in this paper. In particular, the mass matrix for an ordinary lumped model of a building turns out to be diagonal, while for lower floors where the mass of columns may be comparable to the mass of floors, some off-diagonal terms may appear in the mass matrix, which can be

[1]Former MSc student, Dept. of Mechanical Engineering, Iran University of Science and Engineering (IUST), Tehran, Iran.
[2]Associate professor, Dept. of Mechanical Engineering, IUST, Tehran, Iran, email: markazi@iust.ac.ir
[3]Former BSc student, Dept. of Mechanical Engineering, IUST, Tehran, Iran.

modeled as an uncertainty model in a 2-by-2 full block form. The main objective of this paper is to add such a mass uncertainty to the otherwise diagonal mass matrix of the lumped model, and further design a robust closed-loop control system which not only retains the closed loop stability, but also provides a good performance in terms of vibration reduction under seismic excitations. For the purpose of illustration, we have used a simple model of a 5-story building, and employed the H_∞ method to design an active vibration control system for it. By using the μ-analysis technique, the level of tolerable mass uncertainty is studied and by employing the μ-synthesis approach, the level of the robustness of the closed-loop control system under structured mass uncertainty is increased. Simulation studies are provided to assess the effectiveness of the designed control system.

Model of structure

The specification of the 5-story building considered in this paper, is given in Fig.1 and Table 1.

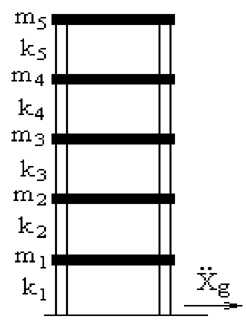

Figure 1. General view of the structure.

Table 1. Properties of the structure

5	4	3	2	1	Floor
593.5 Usage: Office	593.5 Usage: Office	593.5 Usage: Office	593.5 Usage: Trainig Center	593.5 Usage: Theatre	floor mass (ton) (850 m^2 plan area)
5641	1988	1991	1912	2245	Elastic stiffness between floors ($10^2\,kN/m$)
Damping ratio for all modes is 5% or less.					Damping ratio

Stiffness matrix of this structure is in the form of

$$K=\begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2+k_3 & -k_3 & 0 & 0 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & -k_5 \\ 0 & 0 & 0 & -k_5 & k_5 \end{bmatrix} \quad (1)$$

The mass matrix (M) is a diagonal matrix whose diagonal entries are the masses of the floors. We can write the lumped equation of motion for this structure as

(2) $$M\ddot{X}(t)+C\dot{X}(t)+KX(t)=Du(t)+Ef(t),$$

where, $X(t)$ is the vector of floors displacements, $u(t)$ is the control force and $f(t)$ is the earthquake excitation force. Pre-multiplication of this equation with M^{-1} gives

(3) $$\ddot{X}(t)=-M^{-1}C\dot{X}(t)-M^{-1}KX(t)+M^{-1}Ef(t)+M^{-1}Du(t).$$

By using $X(t)$ and $\dot{X}(t)$ we define the new column vector $q(t)=\left[x(t),\dot{x}(t)\right]^T$.

Now, equation (3) can be re-arranged as

(4) $$\begin{Bmatrix}\dot{X}\\ \ddot{X}\end{Bmatrix}=\begin{bmatrix}O & I_n\\ -M^{-1}K & -M^{-1}C\end{bmatrix}\begin{Bmatrix}X\\ \dot{X}\end{Bmatrix}+\begin{bmatrix}0\\ M^{-1}D\end{bmatrix}\begin{Bmatrix}u(t)\\ 0\\ 0\\ 0\\ u_n(t)\end{Bmatrix}+\begin{bmatrix}0\\ M^{-1}E\end{bmatrix}\begin{Bmatrix}f(t)\\ 0\\ 0\\ 0\\ f_n(t)\end{Bmatrix}$$

The matrices B and H, defined below, depict the positions of the control and external forces in the new coordinate system, respectively, i.e.,

(5) $$B=\begin{bmatrix}0\\ M^{-1}D\end{bmatrix},\ H=\begin{bmatrix}0\\ M^{-1}E\end{bmatrix}$$

Controller Design with No Mass Uncertainty

In order to design a controller, the generalized plant is constructed first and an H_∞ controller is designed afterwards (K. Zhou 1998). By measuring the accelerations of the floors, a very good performance in terms of vibration reduction is achieved, yet with a relatively large control effort, particularly at lower floors. By imposing a heavier penalty on the control input, through selection of appropriate weighting functions, the peak control forces are reduced to acceptable levels. Comparison of required forces, with different measuring instruments, i.e. position, velocity and acceleration, at the 1st floor is shown in Fig. 2. The larger forces correspond to the acceleration feedback case, depicting a more influencing control strategy, compared to the two other cases.

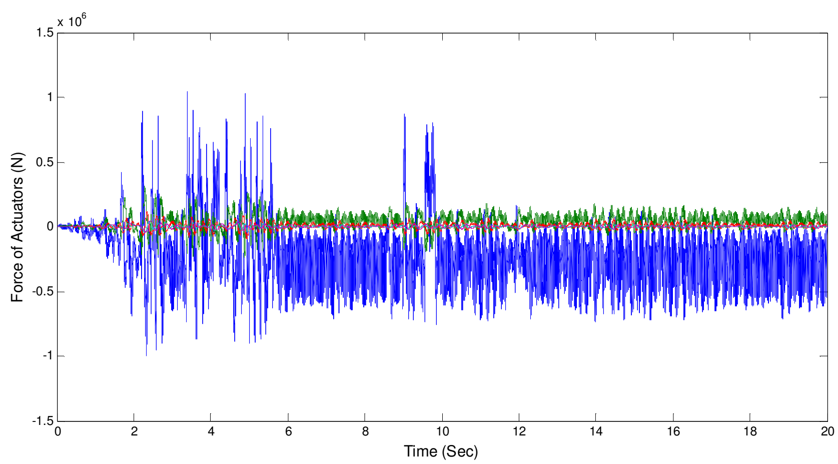

Figure 2. Actuator forces applied to the 1st floor with the optimal controller strategy under three kinds of feedbacks (position, velocity and acceleration). The larger force corresponds to the case with acceleration feedback.

Based on the above analysis, the acceleration feedback was selected and an optimal controller of 20th order was designed. For the sake of brevity the details are omitted here.

Uncertainty model

Let's assume that the building under consideration is comprised of a theatre hall in the first floor and a populated training center in other floors, i.e., there is an occasional relatively large mass uncertainty during day times. As shown in Table 1, the specific mass of each floor is around 700 kg/m^2. Considering the potential usage of a typical theatre or training center, one can assume a typical value of 10% mass uncertainty. On the other hand, due to the usually larger mass of the columns in the lower floors, a mass uncertainty matrix of 2-by-2 is selected, which adds some off-diagonal terms to the otherwise diagonal mass matrix. Here, we assume that such a full block uncertainty exists in the first two floors of the building only, and the rest of the mass uncertainty appears as diagonal additive terms with a bounded uncertainty of 10% on other floors. In other words, we assume that the uncertain mass matrix (M_{unc}) can be written, with respect to the nominal mass matrix ($\bar{M}$), as $M_{unc} = \bar{M} + (0.1 \times \bar{M} \times \Delta_m)$, where the structure of Δ_m is as

$$\Delta_m = \begin{bmatrix} \delta_m I_{3\times3} & \\ & \Delta_{2\times2} \end{bmatrix} \tag{6}$$

where $\|\delta_m\|_\infty < 1$ and $\bar{\sigma}(\Delta_{2\times2}) < 1$, i.e., the largest singular value of $\Delta_{2\times2}$ is less than one.

It is customary to show such uncertainties as a linear fractional transformation, i.e.,

$$M = F_L\left(\begin{bmatrix} \bar{M} & 0.1\bar{M} \\ I & 0 \end{bmatrix}, \Delta_m\right) \tag{7}$$

where F_L is a linear fractional transformation in a lower feedback configuration (K. Zhou, 1998), i.e.,

$$M^{-1} = (\bar{M} + (0.1\bar{M}\Delta_m))^{-1} = \bar{M}^{-1}(I + 0.1\Delta_m)^{-1} = \bar{M}^{-1}\left(I - 0.1\Delta_m(I + 0.1\Delta_m)^{-1}\right) \tag{8}$$

Bring in the following upper linear fractional representation, F_u:

$$M^{-1} = F_u\left(M_{unc}^{-1}, \Delta_m\right) \tag{9}$$

where,

$$M_{unc}^{-1} = \begin{bmatrix} -0.1I & \bar{M}^{-1} \\ -0.1I & \bar{M}^{-1} \end{bmatrix} \tag{10}$$

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Fig. 3 depicts a graphical representation of this transformation.

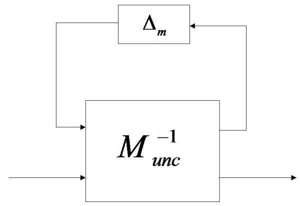

Figure 3. Graphical representation of (10)

The overall dynamical equations of the system together with the mass uncertainty model are shown in Fig.4.

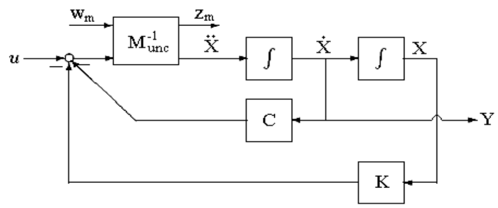

Figure 4. Dynamic model of structure including the mass uncertainty.

Here, W_m and Z_m are the disturbance input signal (earthquake) and the output signal (structural vibrations), respectively, and u is the input to the plant, which includes the actuators force.

Robust Stability Analysis

Considering the structure shown in Figure 4, our objective is to analyze the robust performance of the closed-loop control system, and further improve the design for achieving a robust performance under a wider range of mass uncertainty.

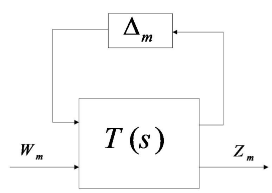

Figure 5. Upper LFT form of Fig. 4.

An upper LFT form of Figure 4 with some loop shaping weighting functions included in T(s) is shown in Fig. 5; see (K.Zhou 1998) .

For notational purposes, we drop "s" form *T(s),* and partition it according to the dimensions of the input and out vector, i.e.,

$$T = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix}.$$

It is a well-known fact (K. Zhou 1998) that the system in Fig. 5, is robustly stable for any stable $\Delta_m(s)$ satisfying

$$\bar{\sigma}(\Delta_m(j\omega)) < \frac{1}{\bar{\sigma}(T_{11}(j\omega))} \quad \text{for all } \omega \in R \ .$$

Similar tests exist for the robust performance of the closed-loop system, in which a fictitious full block uncertainty term Δ_F of size ($n_i \times n_o$) is added to $\Delta_m(s)$, where n_i and n_o are the number of inputs and outputs of T respectively, and $\|\Delta_F\|_\infty < 1$. In other words, the new uncertainty block matrix is defined as

$$\Delta = \begin{bmatrix} \Delta_m & \\ & \Delta_F \end{bmatrix},$$

and the robust performance condition would be as

$$\bar{\sigma}(\Delta(j\omega)) < \frac{1}{\bar{\sigma}(T_{11}(j\omega))} \quad \text{for all } \omega \in R$$

The nominal performance condition norm (with $\Delta = 0$) for our studied case is shown in Fig. 6. It can be seen that the value of $\bar{\sigma}(\Delta(j\omega))\bar{\sigma}(T_{11}(j\omega))$ is less than one everywhere, i.e., the nominal performance is achieved.

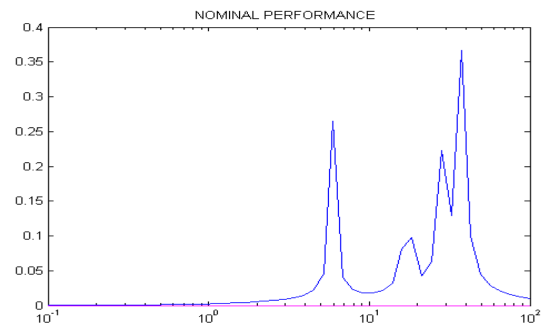

Figure 6. Nominal performance test.

The robust performance condition test with 10% mass uncertainty is shown in Fig. 7. It can be seen that, in this case, the value of $\bar{\sigma}(\Delta(j\omega))\bar{\sigma}(T_{11}(j\omega))$ is larger than one in some parts of the frequency range, i.e., the robust performance condition is not satisfied.

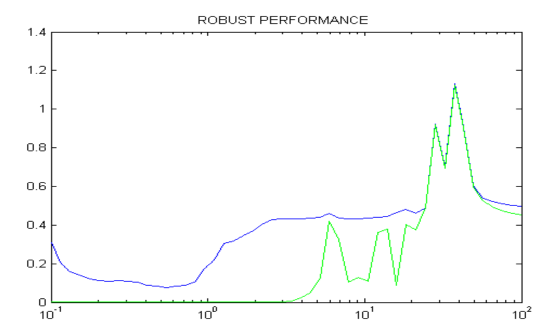

Figure 7. Robust performance with 10% mass uncertainty

We may apply the worst case performance test and show that the system is robust under 4% mass uncertainty, or so (Fig. 8). The problem is analyzed once more by assuming 4% uncertainty. Results of the new test show that this controller may still be robust under somewhat higher level of uncertainties. Some iteration provide the largest value of the mass uncertainty, 7.5%, under which the robust performance is preserved, as depicted in the Fig. 9.

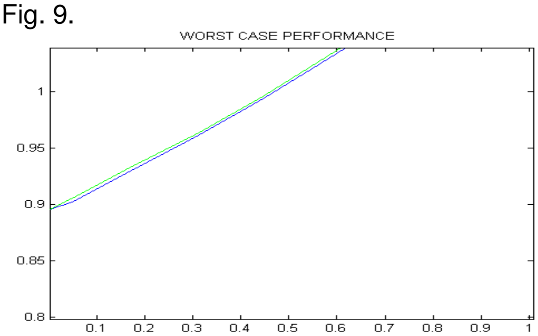

Figure 8. Worst case test with 10% mass uncertainty

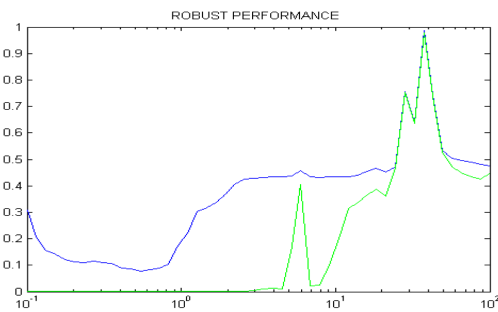

Figure 9. Robust performance with 7.5% mass uncertainty.

In order to achieve an even larger of tolerable mass uncertainty, the μ-synthesis technique is employed. In particular, the so called D-K iteration strategy is employed (see (K. Zhou 1998); details are omitted here).

The new controller turns out to be of order 34, and the robust performance test is shown in Fig. 10, which proves the robust performance under 10% mass uncertainty.

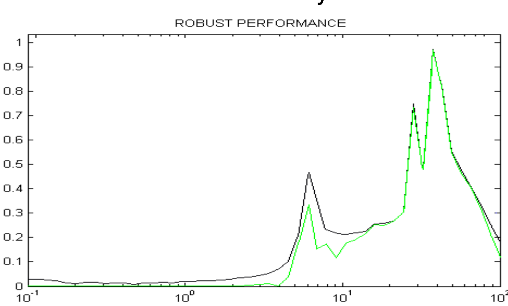

Figure 10. Robust performance with 10% mass uncertainty (D-K iteration).

Simulation studies

The displacement response of the structure under a typical seismic excitation is depicted in Figures 11 and 12. In particular, the effectiveness of the designed robust active control system, compared to the uncontrolled system, is demonstrated. Furthermore, a considerable reduction in the acceleration response is observed as in Figure 13.

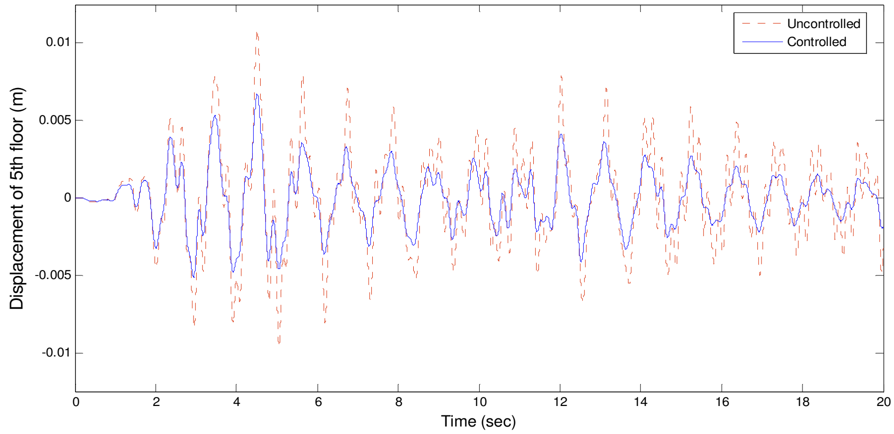

Figure 11. Displacement response for the 5th floor.

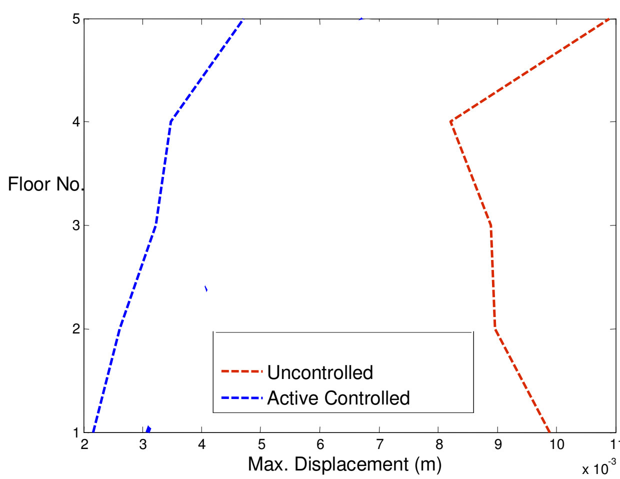

Figure 12. Maximum displacement of the excited floors with active control and with no control.

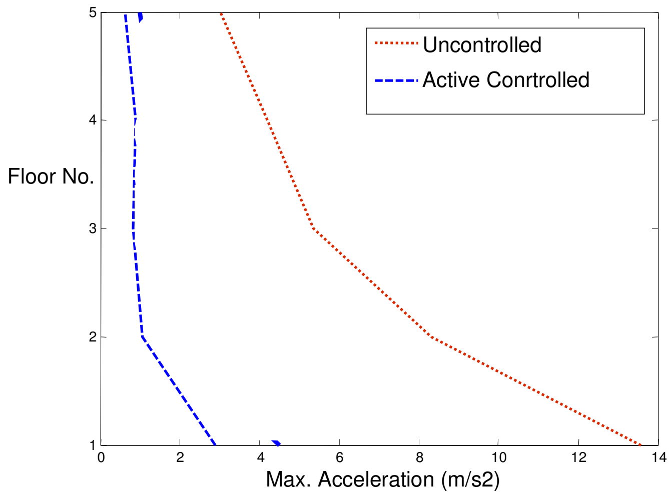

Figure 13. Maximum acceleration of the excited floors with active control and with no control.

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Conclusions

In this paper we have concentrated on the design of a robust control system, which effectively reduces the structural vibrations under seismic excitations, and in the presence of floors mass uncertainty. The nominal, as well as, robust performance of the controlled system was studied and the design was improved so that the desired level of performance under 10% mass uncertainty is achieved. The structure of the uncertainty block was selected so that the effect of the relatively high mass of the columns between 1^{st} and 2^{nd} floors is taken into account. The effectiveness of the design was demonstrated through simulation studies.

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