



## DESIGN CONFINEMENT REINFORCEMENT FOR REINFORCED CONCRETE COLUMNS

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### ABSTRACT

This paper presents the development of new design equations in the CSA standard A23.3-04, for determination of confinement reinforcement for ductile and moderately ductile earthquake-resistant rectangular and circular columns and walls. These equations are based on performance measured in terms of curvature demand, and are developed from parametric studies relating confinement reinforcement to the level of sectional ductility and account for concrete strength, transverse reinforcement yield strength, axial load level, and transverse confinement reinforcement spatial distribution.

### Introduction

Transverse reinforcement specified in design codes for beams and columns has three main functions: (i) prevent buckling of longitudinal bars, (ii) avoid shear failure, and (iii) confine the concrete core to provide sufficient deformability or ductility. These three actions are typically considered separately in design codes (ACI-318 2005, CSA-A23.3 2004, NZS-3101 1995). This paper addresses only the confinement requirements. The confinement reinforcement in the current ACI Code is based on the work of Richart et al. (1929) and was developed so that the compressive strength of the confined core of a column after spalling should be equal to the strength of the gross section of the column before spalling. The resulting confinement reinforcement requirements are only a function of the ratio of gross area of column section to area of concrete core and the ratio of the specified compressive strength of concrete and the specified yield strength of transverse reinforcement. In the CSA Standard (1994), columns in ductile frame had to comply with the same confinement requirements as specified in the ACI Code (2005). These requirements have been shown to be inadequate (Paultre et al. 2006) and new equations were developed for the 2004 CSA-A23.3 Standard. The objective of this article is to describe how these equations were developed.

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## Predicting Ductility of Columns

Paultre et al. 2006 showed that the main parameters influencing the curvature ductility capacity,  $\mu_\phi$  of a column are the axial load level  $k_p = P/P_0$  and the effective confinement index  $I'_e = f'_{le}/f'_c$ , where  $P$  is the applied axial force on the column,  $P_0$  is the axial capacity of the column ( $P_0 = 0.85A_g f'_c + A_s f_y$ ),  $f'_c$  is the concrete strength,  $f'_{le}$  is the effective confinement pressure at peak stress which is a measure of the restraint applied by the stirrups to the expansion of the confined concrete core under compression. For a circular column,  $f'_{le}$  is given by:

$$f'_{le} = \frac{1}{2} K_e \rho_s f'_h \quad (1)$$

while for rectangular columns in the  $y$  direction (see Fig. 1),  $f'_{le}$  is given by:

$$f'_{le} = \frac{A_{shy} f'_h}{c_y s} \quad (2)$$

where  $K_e$  is the geometric confinement effectiveness coefficient, which measures the effectiveness of the confinement reinforcement to confine concrete and varies from 1 for a continuous tube to 0 when ties are spaced more than half the core cross section minimum dimension,  $A_{shy}$  is the total section of confinement reinforcement for the set of ties in direction  $y$ ;  $c_y$  is the cross-section dimension in direction  $y$ , measured center-to-center of peripheral ties or spiral;  $s$  is the center-to-center spacing between ties;  $\rho_s$  is the ratio of volume of spiral reinforcement to total volume of core, measured center-to-center of spiral and  $f'_h$  is the stress in the confinement reinforcement at peak stress. Details for the determination of  $f'_h$  can be found in Légeron and Paultre (2003). It has been demonstrated by Cusson and Paultre (1994) and Paultre et al. (2001) that the confinement reinforcement does not always reach yield at peak concrete stress. Azizinamini et al. (1994) have demonstrated that high-yield strength steel in certain cases do not provide any significant ductility improvements for columns under constant axial load and reversed flexure.

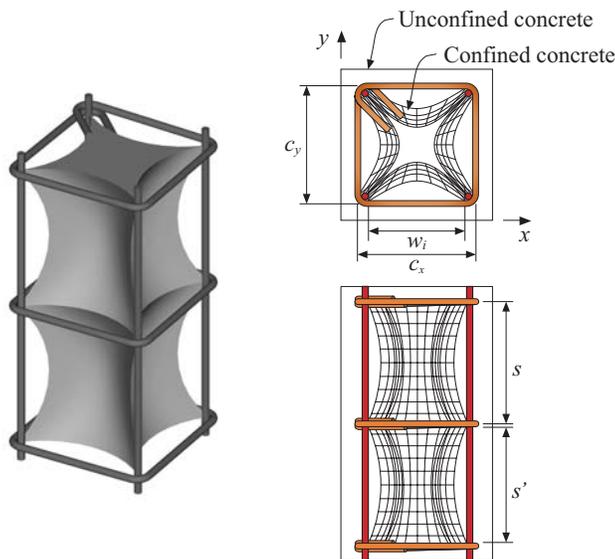


Figure 1. Arching effect.

Using a numerical parametric study, Paultre et al. 2006 showed that the curvature ductility and axial load level can be related directly to effective confinement index as:

$$I'_e = 0.0111k_p\mu_\phi \quad (3)$$

It is interesting to note that within the limit of the variables considered in the parametric study, the concrete strength, the volumetric ratio of longitudinal reinforcement, the yield strength of reinforcement and the size of the column were found to have only limited influences on curvature ductility and that the most important parameters controlling ductility are the effective confinement index and the relative level of axial load. This is consistent with experimental evidence (Légeron and Paultre 2000).

### Design Equations for Confinement Reinforcement

Equation (3) can be used for displacement based or performance based seismic design. Typical structures are designed with the elastic seismic forces divided by a force reduction factor that accounts for the ductility and overstrength of the structure. Local ductility contributes to overall displacement ductility, and is ensured for columns primarily by a sufficient amount and arrangement of confinement reinforcement at potential plastic hinge locations. The objective of this section is to develop simplified design equations providing directly  $A_{sh}$  or  $\rho_s$  for two different levels of ductility, which in this research are targeted as (i) moderate ductility level corresponding to a ductility-related force modification factor of 2.5 and curvature ductility factor  $\mu_\phi$  of at least 10 and (ii) ductile level corresponding to a ductility-related force modification factor of 4 and curvature ductility factor  $\mu_\phi$  of at least 16. These ductility levels are consistent with the National Building Code of Canada (2005). Replacing these values of  $\mu_\phi$  in Eq. (3), the required effective confinement index for a ductile structure is:

$$I'_e = 0.178k_p \quad (4)$$

and, for structures with moderate ductility

$$I'_e = 0.111k_p \quad (5)$$

To obtain simplified design equations relating to  $\rho_s$  or  $A_{sh}$  from Eqs. (1), (2), (4) and (5),  $K_e$  and  $f'_h$  must be expressed in simple forms. The two following sections will aim for this purpose.

### Geometric Coefficient of Confinement Effectiveness, $K_e$

The geometric coefficient of confinement effectiveness used here was first proposed by Sheikh and Uzumeri (1982), and latter modified by Mander et al. (1984). For rectangular columns its expression is:

$$K_e = \frac{\left(1 - \frac{\sum w_i^2}{6c_x c_y}\right) \left(1 - \frac{s'}{2c_x}\right) \left(1 - \frac{s'}{2c_y}\right)}{1 - \rho_{cc}} \geq 0 \quad (6)$$

where all the all the dimensions are explained in Fig. 1. For circular columns confined with spiral, Mander et al. (1984) proposed a similar expression. It is convenient to divide the geometric coefficient of confinement effectiveness into two parts: (i) an horizontal arching coefficient,  $K_h$ , and (ii) a vertical arching coefficient,  $K_v$ , such that:

$$K_e = K_h K_v \quad (7)$$

For members with rectangular hoops, the horizontal arching coefficient can be written:

$$K_h = 1 - \frac{\sum w_i^2}{6c_x c_y} \quad (8)$$

and for members with circular hoops or spirals,  $K_h = 1$ . It is possible to simplify equation Eq. (8) for square columns with longitudinal reinforcement distributed regularly as

$$K_h = 1 - \frac{2}{n_l} \quad (9)$$

At the design stage, many parameters are not yet known and the approximation given by Eq. (9) is sufficient for rectangular columns.

The expressions for  $K_v$  depends on the spacing of ties, hoops or pitch of spirals which is one of the desired quantity with  $A_{sh}$  for rectangular columns or part of the unknown  $\rho_s$  for circular columns. The solution would therefore require iterations which is clearly impractical. Conservative values of  $K_v$  for columns respecting all the minimum requirements in ACI and CSA codes could be used instead. A minimum for  $K_v$  should be sought as this would result in conservative amounts of confinement reinforcement. To arrive at a conservative expression for  $K_v$ , more than 500 different columns have been investigated using minimum transverse reinforcement required by ACI and CSA Codes. For rectangular and square columns, a conservative expression for  $K_v$  as a function of the ratio  $A_{ch}/A_g$  can be found as shown in Fig. 2 for fully ductile square and rectangular columns ( $\mu_\phi = 16$ ) and:

$$K_v = 1.05 \left( \frac{A_{ch}}{A_g} \right) \quad (10)$$

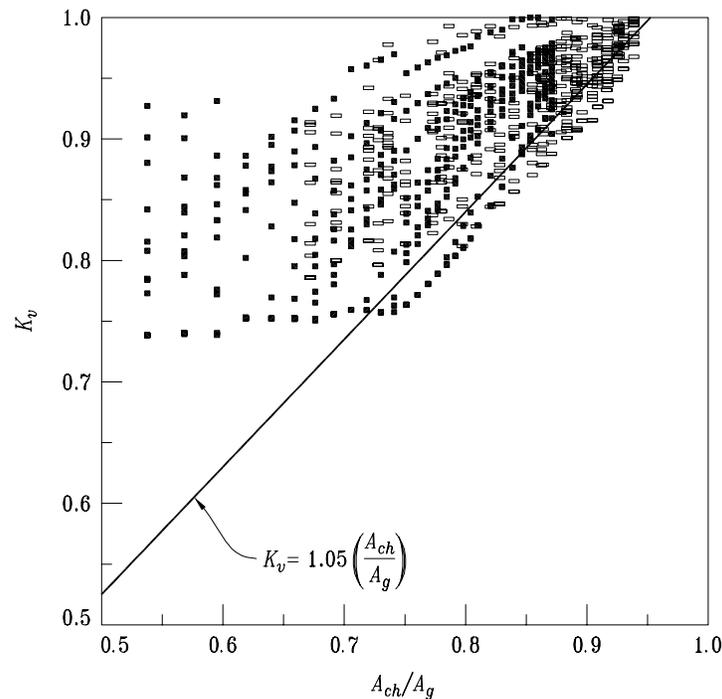


Figure 2.  $K_v$  coefficient for square and rectangular columns with  $\mu_\phi = 16$ .

For rectangular or square columns in a moment-resisting frame with moderate ductility, i.e.,  $\mu_\phi = 10$ , we can find similarly:

$$K_v = 0.95 \left( \frac{A_{ch}}{A_g} \right) \quad (11)$$

For circular columns with spiral reinforcement for ductile moment-resisting frame or moment-resisting frame with nominal ductility,  $K_v$  is always greater than 0.90 and is used hereafter.

### Effective Stress in Confinement Steel

It has been experimentally demonstrated by Paultre et al. (2001), and explained by Légeron and Paultre (2003) that the yield strength of ties are not fully used for confinement effect. Therefore, it is necessary to use an effective stress instead of the yield stress when determining confinement reinforcement. Légeron and Paultre (2003) have proposed expressions to accurately determine the effective transverse reinforcement stress,  $f'_h$ , at peak strength. Using this expression,  $f'_h$  is calculated for a number of columns reinforced with only non confinement minimum transverse reinforcement for columns required by ACI and CSA Codes. It is found (Paultre et al. 2006) that  $f'_h/f_{yh}$  for circular columns and that for rectangular columns  $f'_h = 0.83f_{yh}$  when  $\mu_\phi = 16$  and  $f'_h = 0.68f_{yh}$  when  $\mu_\phi = 10$ , assuming  $f_{yh} = 400$  MPa .

### Implementation in CSA Standard for Circular Columns

From Eqs. (4) and using simplifications described in the previous two subsections, we obtain the following equations for  $\rho_s$  for circular columns with  $\mu_\phi = 16$  :

$$\rho_s = \frac{0.356k_p f'_c}{0.9f_{yh}} = 0.40k_p \frac{f'_c}{f_{yh}} \quad (12)$$

This equation has been adopted directly in the new edition of the Canadian Standard (CSA A23.3 2004) for circular columns in ductile moment resisting frames. It is noted that the concrete core area is measured to the outside diameter of the spiral which provides some additional conservatism.

For circular columns part of moment resisting frame with moderate ductility ( $\mu_\phi = 10$ ) we have:

$$\rho_s = 0.3k_p \frac{f'_c}{f_{yh}} \quad (13)$$

where the coefficient 0.3 has been rounded from 0.25.

### Implementation in CSA Standard for Square and Rectangular Columns

From Eqs.(4) and using simplifications described previously, we obtain the following equations for  $A_{shy}$  for square or rectangular columns with  $\mu_\phi = 16$  :

$$A_{shy} = \frac{0.178k_p k_n c_y s f'_c}{1.05 \frac{A_{ch}}{A_g} 0.83 f_{yh}} = 0.20k_p k_n c_y s \frac{f'_c}{f_{yh}} \frac{A_g}{A_{ch}} \quad (14)$$

where  $k_n = 1/K_h = n_t/(n_t - 2)$ . Equation (14) was adopted in the Canadian Standard (CSA A23.3 2004) for rectangular columns forming part of a ductile moment resisting frame with the exception that in CSA Standard  $h_c$  replaces  $c_y$  and together with  $A_{ch}$  these dimensions are measured to outside of peripheral hoops. In addition to the amounts required by Eq. (14), the minimum for large columns was carried from previous CSA Standard as:

$$A_{shy} = 0.09 \frac{f'_c}{f_{yh}} s h_c \quad (15)$$

For columns in moment resisting frames with moderate ductility ( $\mu_\phi = 10$ ), from Eqs. (5) and previous simplifications:

$$A_{shy} = 0.15k_n k_p \frac{A_g}{A_{ch}} \frac{f'_c}{f_{yh}} s h_c \quad (16)$$

where the coefficient 0.15 was rounded down from 0.17 as it was found to give ample safety for most cases of practical interest. Therefore, for moderately ductile columns, the total effective area in each of the principal directions of the cross section within spacing  $s$  of rectangular hoop reinforcement shall not be less than the amounts required by Eq. (16) and Eq. (15).

### Comparison with Existing Codes and Experimental Data

Figure 3 compares the confinement reinforcement requirements for two 500 mm-square columns and two 1000 mm-square columns. It is seen that for small axial loads the proposed equations require less reinforcement than the current ACI and the 1994 CSA codes, but for high axial load, it is the contrary. This is consistent with experimental evidence. The proposed equations, although simpler, give comparable results to the requirements in the New Zealand Standard. For small circular columns, the minimum requirement for non seismic design controls most of the time. For large circular columns, the proposed equations result in more confinement reinforcement, which is similar to the New Zealand Standard.

In order to validate the equations previously developed, comparison of displacement ductility of 60 columns to percentage of compliance to Eqs. (12) and (14) as well as to the confinement reinforcement required by the ACI Code are presented in Figures 4 and 5. The 60 columns have been tested by Sheikh and Khoury (1993), Watson and Park (1994), Sheikh et al. (1994), Li et al. (1994), Bayrak and Sheikh (1998), Légeron and Paultre (2000), Paultre et al. (2001), Robles et al. (2003) and Saatcioglu and Baingo (1999). As a reference, the required displacement ductility for a column forming part of a ductile frame is supposed to be at least 4. The vertical dotted line in Figs. 4 and 5 corresponds to the limit between compliance and non compliance to amount of confinement reinforcement recommended in codes. Column specimens falling on the left do not comply with the quantity of transverse confinement reinforcement required by a specific code. The horizontal dotted line corresponds to the limit between ductile and non ductile behavior, the limit being set in this case is a displacement ductility of 4. Columns above this line are ductile enough to be part of a ductile moment frame. The set of two dotted lines divide the space into four quadrants. Columns should theoretically be in the upper right corner, i.e., they have enough reinforcement according to the code and behave in a ductile manner or in the lower left corner, i.e., they have insufficient confinement reinforcement according to code and behave in a non-ductile manner. As codes are conservative overall, it is not surprising to find columns in the upper left corner. However, no column should be in the lower right corner, meaning they have the required quantity of confinement reinforcement according to the code, but do not display enough ductility.

The CSA 2004 equations are a great improvement compared to those in the current ACI Code and the 1994 CSA Standard as seen in Figures 4 and 5. They provide both more economical and safer design. They are also more satisfactory since they are based on a rational method that links directly performance measured by curvature ductility to requirement for confinement and are validated on experimental results on a large number of columns.

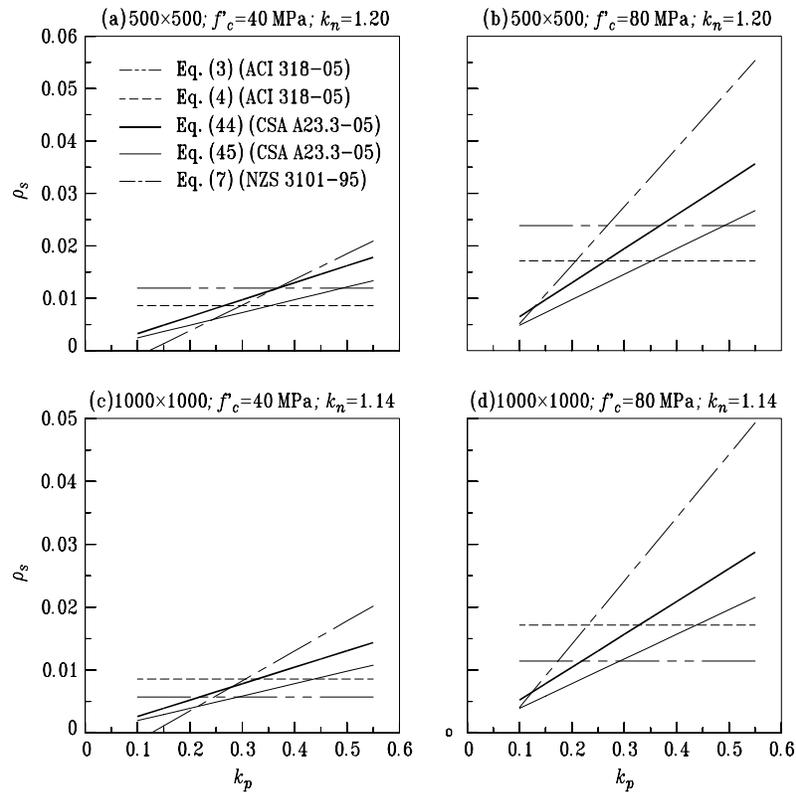


Figure 3. Comparison between confinement reinforcement ratios required by ACI Code and NZS and CSA Standards for square columns.

It is important to note here that the relation between experimental ductility and the level of compliance to confinement reinforcement requirements shall not be understood as direct. In fact the proposed equations are supposed to be safe and conservative in all cases. In some cases, they may underestimate slightly the actual displacement ductility. Simple equations cannot provide more than what they are intended for: to be conservative in most cases. As well, the approach is related only to confinement, and other factors may alter experimental behavior, namely buckling of longitudinal bars and insufficient shear strength. In addition, all authors do not measure ductility in the same way, which makes it difficult to compare results. If the engineer is interested to know exactly the level of ductility, he must use a complete approach as described in Légeron (2001), Légeron and Paultre (2003) or Dodd and Cook (1992) and not only base his design on code equations.

### Confinement Reinforcement for Concentrated Reinforcement in Walls

Concentrated reinforcing bars at the ends of walls must be tied by hoops in accordance with the requirements for conventional construction and in the region of plastic hinging the hoop spacing shall not exceed the smallest of (a) six longitudinal bar diameters, (b) 24 tie diameters, (c) one-half the least dimension of the member and (d) additional requirements if the maximum compressive strain  $\epsilon_{cu}$ , exceeds 0.0035.

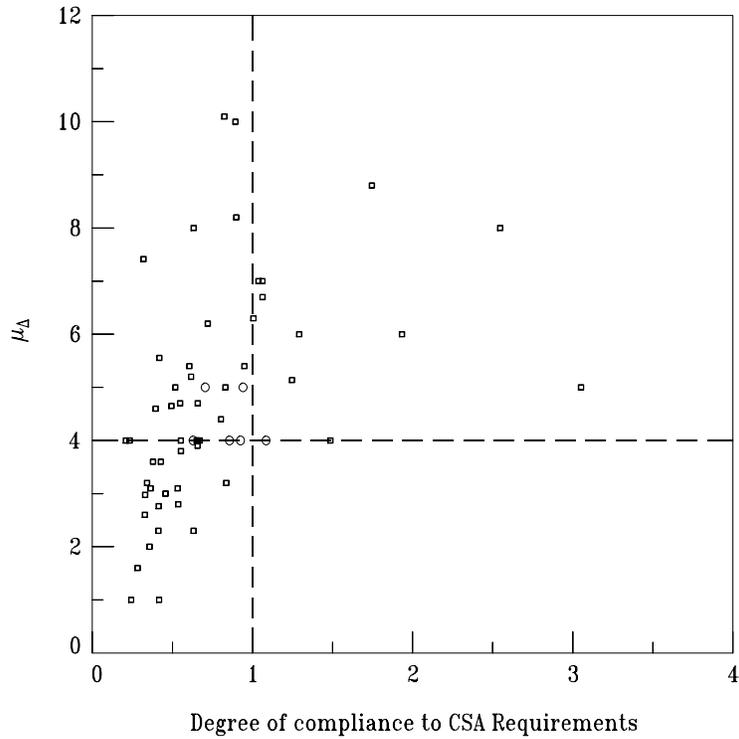


Figure 4. Comparison between degree of compliance to amount of confinement reinforcement required by new CSA Standard to achieved displacement ductility of columns.

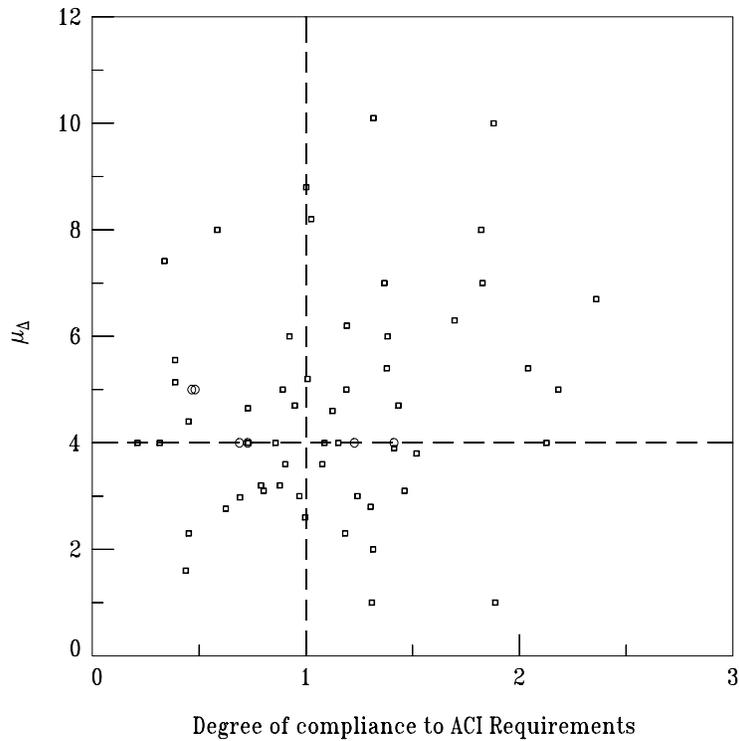


Figure 5. Comparison between degree of compliance to quantity of confinement reinforcement required by ACI Code to achieve displacement ductility of columns.

The confinement expression for columns given in Eq. (14) expresses the amount of confinement reinforcement required as a function of the level of axial load in columns. The confinement expression for concentrated reinforcement at the ends of walls uses the maximum concrete compressive strain in the wall instead of the level of axial load as the main parameter. The equivalent factor for the level of axial load,  $k_p$ , is given by  $k_p = 0.1 + 30\varepsilon_{cu}$ . When  $\varepsilon_{cu}$  equals 0.0035 no additional confinement reinforcement is required, as  $\varepsilon_{cu}$  increases from 0.0035 to the maximum concrete compressive strain permitted of 0.014, the equivalent value of  $k_p$  varies from about 0.20 (equivalent to a lightly loaded column) to about 0.50 (representative of a heavily loaded column).

Because of the significant compressive strain gradient in walls it is necessary to provide requirements for the length over which confinement is required, given by  $c(\varepsilon_{cu} - 0.0035)/\varepsilon_{cu}$ , where  $c$  is the depth of compression in the wall. When  $\varepsilon_{cu}$  equals 0.0035 this length is zero (i.e., no confinement reinforcement is required) and when  $\varepsilon_{cu}$  equals the maximum permitted value of 0.014 the required length over which confinement reinforcement is equal to  $0.75c$ .

### Conclusions

At this moment, high-strength steel is not used widely in construction. Should this change, the framework of the proposed equations will adequately take into account high-yield strength steel. By suppressing all limits on yield strength in exchange for calculation of effectiveness of confinement steel, use of very high strength steel might be possible, which might prove very useful for high axially loaded columns of high-rise buildings using high-strength concrete. The overall work presented in this paper can be used in ductility based design, as described in this paper and included in CSA standard A23.3-04, or displacement based design. By considerably simplifying design, this should favor the use of these rational methods of design for a wide range of structures.

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