



A PROCEDURE FOR EVALUATING SEISMIC RELIABILITY AND IMPLEMENTING PERFORMANCE-BASED DESIGN

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ABSTRACT

This paper presents an assessment of what is required to carry out reliability calculations and Performance-Based Design (PBD) in earthquake engineering. Seismic design involves many uncertainties that arise from the earthquake's ground motion, structural geometry, material properties, and analytical models. Performance-Based Design implies taking into account all the major uncertainties and determining a set of optimal design parameters, so as to satisfy several performance requirements, each with an associated minimum reliability level and possibly at a minimum total cost. Thus, PBD is an optimization process that requires, at each step, the calculation of the achieved reliabilities for a given set of design parameters. This paper presents a conceptually straightforward method for the estimation of reliability in the context of earthquake engineering. The method is based on the deterministic calculation, for different combination of the intervening variables, of the demand responses required in the reliability formulation. These responses, in turn, are represented by a Neural Network, resulting in a very efficient approach to the reliability estimate by simulation and the optimization in PBD.

Introduction

Performance-Based (PBD) design of a structure subjected to earthquake ground motions (Bertero and Bertero, 2002) requires consideration of all the major uncertainties in the problem, for the satisfaction of multiple performance criteria with associated minimum levels of reliability over its service life. Design parameters must then be obtained by optimization, possibly also achieving a minimum total cost. In this context, the basic components of PBD are: 1) a time-stepping, nonlinear, dynamic structural analysis; 2) formulation of performance requirements in terms of damage levels, or their corresponding deformation limits (total maximum displacements, inter-story drifts, etc.); 3) for each performance requirement or limit state, assignment of an associated target minimum reliability level; 4) determination of a set of ground motions (historical or simulated) representative of those likely to occur at the site; 5) estimation of the reliability achieved with a given set of design parameters (e.g., column dimensions, steel ratios); 6) in the optimization process, modification of the design parameters with the objective of minimizing the difference between the achieved reliabilities and the corresponding targets, or minimizing the total cost under those reliability constraints.

Although our state of knowledge is quite advanced as to the formulation of nonlinear, time-stepping dynamic analyses, one must be careful to ensure that the structural model reflects the actual behaviour in a realistic manner, particularly in relation to the modelling of restoring forces and hysteretic damping. The

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latter is not a material property, but a function of the particular demand history and the ensuing structural degradation in stiffness and strength. Thus, this behaviour should be captured or calculated from basic material properties or constitutive relations, and not entered as a pre-determined shape. Parameters controlling the shape of the hysteresis loops are usually determined by matching an experimentally obtained response for a given protocol cyclic history. However, these results cannot be theoretically extrapolated for use with any other demand history nor, in particular, to all other earthquakes. The consequences of these extrapolating assumptions for the reliability achieved in the design, or for the determination of optimal design parameters, have been explored (Foschi and Zhang, 2004), but remain a subject of further study. Calculating the hysteretic response is not simple, however, as the constitutive equations to be used must reflect the yielding of the material, the unloading behaviour and the formation of gaps or of cracks. In some instances, for example in the earthquake response of pile foundations, the hysteretic behaviour is properly calculated for each ground motion using the elasto-plastic properties of the pile, the formation of soil/pile gaps, and the nonlinearity of the forces between the soil and the pile (e.g. Foschi and Zhang, 2004). A similar technique can be used to estimate the hysteretic response of mechanical fasteners like bolts or nails (Foschi, 2000). A further important aspect of the analysis is the consideration of interaction between the structure and the soil foundation. An assessment of whether such interactions need to be taken into account has been recently presented (Ghalibafian et al., 2006).

The characteristics of the earthquake ground motion constitute one of the primary uncertainties, and hazard analysis should be conducted to delineate site-specific seismic characteristics, based on which a set of historical earthquake records can be selected, or a suite of artificial earthquake ground motions may be synthesized.

Reliability analysis must then be applied to evaluate the probability of non-performance in each of the limit states. These can be related to either final collapse or to different levels of intermediate damage, or to situations of continuing service. While the structural analysis provides information on displacements, forces and moments, there is a need to relate these quantities to underlying levels of damage (Bozorgnia and Bertero, 2004). This is largely an experimental task, which must be carried out in combination with subjective evaluation of damage levels and of associated repair costs.

Finally, the probability estimations could be carried out with Monte Carlo simulations using variance reduction techniques. However, as the probability of non-performance in a well-designed structure is generally small, the simulation may entail a great number of performance function evaluations, each requiring the execution of the nonlinear dynamic analysis, a task that could be very computationally intensive. Other techniques like FORM, or First Order Reliability Method, may require fewer calls to the dynamic analysis, but have the drawback that the estimation of the non-performance probability is influenced by the non-linearity of the performance function, a difficulty not present in simulation techniques. In order to reduce the computational effort and to improve efficiency, this paper introduces an approach based on the Design of Computer Experiments for the construction of response databases and subsequent representation by Neural Networks (Hurtado, 2004). This means that the dynamic analysis is run for a chosen arrangement of combinations of the intervening random variables (structural dimensions, structural properties, ground motion, peak intensity, etc.), including the design parameters. In the design of this computer experiment, the combinations must be chosen so that the range of each variable is appropriately covered. The response database so generated will include all those responses relevant to the formulation of the performance criteria. Thus, the database contains a set of discrete responses, for example, the maximum inter-story drift corresponding to a particular combination of dimensions, material properties and ground motion. This process may also be computationally intensive, but the generated database is very useful, particularly because it would not require modifications for different statistical descriptions of the variables, and would be applicable to different structures within the type for which it was developed. For further analysis, this discrete database must be represented by a mathematical form, an objective which may be achieved by different response surfaces or, most efficiently, by Artificial Neural Networks. These are fitted, or trained, to the calculated database, and act as an interpolating tool within the boundaries of the database. They will then act as a substitute for further dynamic analyses, for variable combinations not in the database, allowing a very straightforward implementation of reliability

calculations by simulation or optimization by gradient-free algorithms. The following section describes briefly the experimental computer design and the neural network architecture. Two case studies are then presented to demonstrate the applicability and efficiency of the proposed approach: a bridge bent with or without seismic isolation and a steel pipe pile in a sandy soil.

Performance Functions and Performance-Based Design Formulation

The calculation of the probability of non-performance in a given limit state requires the formulation of an associated performance or limit state function $G(x)$, in the form

$$G(x) = C(x_c, d_c) - D(x_d, d_d) \quad (1)$$

containing the vectors of random variables x_c and d_c , associated with the capacity C , and x_d and d_d associated with the demand D . The variable vectors d_c and d_d are the design parameters, or the objective of the performance-based design. In earthquake engineering, the capacity C can be given in terms of a limiting damage or deformation, while the demand D requires the calculation (using the nonlinear dynamic analysis) of the damage or deformation associated with the random variables x_d and design parameters d_d . The simulation then calculates the probability P_f of the performance function $G(x)$ being negative, or the probability of $D > C$. This probability can be expressed in terms of an associated reliability index β , uniquely related to the probability P_f through the Normal cumulative distribution Φ :

$$P_f = \Phi(-\beta) \quad (2)$$

Performance-based design is an optimization problem that is formulated as follows: to find the optimal design parameter vector d (that is, d_c and/or d_d) by minimizing the objective function Ψ ,

$$\Psi = F(d) \rightarrow \text{Minimum} \quad (3)$$

subject to reliability constraints $\beta_j(d) \geq \beta_j^T$ ($j = 1, \text{ND}$)

and geometric constraints $d_i^l \leq d_i \leq d_i^u$;

or , in a more restrictive problem,

$$\Psi = \sum_{j=1}^{\text{ND}} (\beta_j^T - \beta_j(d))^2 \rightarrow \text{Minimum} \quad (4)$$

subject to geometric constraints $d_i^l \leq d_i \leq d_i^u$.

In these formulations,

$F(d)$ is a function expressing the total expected cost of the structural system;

ND is the number of performance criteria or limit states;

β_j^T is the target reliability index for the performance criterion j ;

$\beta_j(d)$ is the calculated reliability index for performance criterion j , given the design vector d ;

d_i^l is the lower bound for the design parameter d_i ;

d_i^u is the upper bound for the design parameter d_i .

Computer Experimental Design and Neural Network Representation

In general, the functions C or D cannot be given an explicit form and only discrete values, deterministically obtained, are available for specific combinations of the intervening variables. To implement simulation

procedures for reliability estimation, those discrete values need to be represented by a mathematical form.

The first step towards this representation is to choose an adequate set of variable combinations to develop the discrete database. This implies an experimental design (Sacks et al., 1989) and several techniques are available: a random selection may be used within given bounds for each variable, or a deterministic grid approach may be utilized. A combination of the two would use a random selection within a grid of cells for each variable. For example, Fig.1 shows combinations of two variables, each ranging between 0 and 1, with five cells per variable and one combination per cell, randomly selected within the cell. Regardless of the selection policy, the variable bounds for database development must be chosen sufficiently wide, to accommodate the range of values implied by the statistics of each variable.

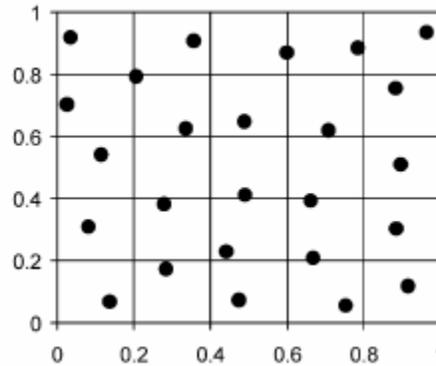


Figure 1. Combinations arrangement, 2 variables example.

The relevant outputs are obtained for each variable combination in the database and, depending on the number of combinations, this task can also be computationally intensive. Thus, the number of combinations must be chosen with care. However, having accomplished this task, subsequent reliability estimation or performance-based design optimization are made very efficient through the implementation of a neural network representation.

A neural network architecture is shown in Fig. 2. Each input variable occupies a “neuron” in the input layer, with the outputs occupying similar neurons in the output layer. The transition between input and output requires intermediate or hidden layers with a number of neurons. The information proceeds along paths, connecting the neurons, and each path contains a calibration parameter or weight. Furthermore, the incoming information into a neuron in the hidden layer is processed internally before being sent forward.

The weights associated with each path must be calibrated (or the network “trained”) so that the difference between the calculated outputs and those known for the input combinations is minimized in a least squares sense. The number of neurons in the hidden layer is also optimized to achieve a best result. Of all the combinations in the dataset, only 80% of them are used in the training, leaving the remaining 20% for validation of the network performance. Details on neural networks and training can be found in the appropriate literature (Patterson, 1996).

Neural Network Representation Using Grouped Variables

Considering the demand D in Eq. 1, let the variables x_d be separated into two sets: x_{du} and x_{dg} . For fixed values of x_{du} and d_d , the demand D can be calculated for different combinations of the variables grouped under x_{dg} . This allows the calculation of response statistics over the sub-set x_{dg} , for example, the mean response $\bar{D}(x_{du}, d_d)$ and the corresponding standard deviation $\sigma(x_{du}, d_d)$.

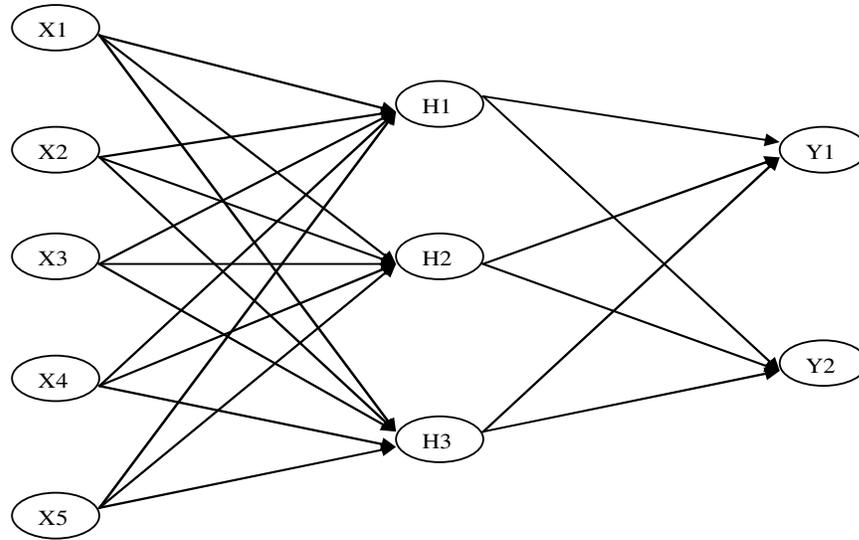


Figure 2. Neural network architecture.

Each of these statistics, in turn, can be represented by a Neural Network of lesser input dimension (Foschi, 2005). Finally, the demand $D(x_d, d_d)$ can be written using these two neural networks and the introduction of a lognormal distribution (if appropriate):

$$D = \frac{\bar{D}}{\sqrt{1 + (\sigma / \bar{D})^2}} \exp\left(R_N \sqrt{\ln[1.0 + (\sigma / \bar{D})^2]}\right) \quad (5)$$

in which R_N is a Standard Normal variable. The same approach may be used for the capacity C . A similar approach can be used if an Extreme type I distribution is used in Eq. 5 instead of a lognormal.

In an earthquake engineering problem, the grouped variables in the demand D could be chosen as those associated with the characteristics of the ground motion: the duration of its strong phase and the characteristics of the record (frequency content). Thus, neural networks are constructed both for the mean and for the standard deviation of the response demands over a set of earthquake records likely to occur at a site, (Zhang and Foschi, 2004). The peak ground acceleration is a random variable which could either be left within the basic set x_{du} or could also be made part of the grouped set, with its influence then reflected in the statistics represented by the corresponding neural networks.

In the simulation, Eq. 5 is used to calculate the demand D as follows: a random value R_N is chosen and, for specific values of (x_{du}, d_d) , taken from their probability distributions, the neural networks are executed to obtain the mean $\bar{D}(x_{du}, d_d)$ and the standard deviation $\sigma(x_{du}, d_d)$. The process is repeated over the number of simulations.

Example 1: Bridge Bent With or Without Seismic Isolators

This example considers a reinforced concrete bridge bent, with or without seismic isolation, subjected to earthquake ground motion. Fig. 3 shows a schematic diagram for the bent with isolation. The earthquake records are simulated following Shinozuka's approach (Shinozuka, 1967) and using a power spectral density function (Clough and Penzien, 1975), Fig. 5, with a predominant ground frequency, ω_g , and a

modulation function (Amin and Ang, 1968) with a strong motion duration T_s , as shown in Fig. 4. Five random variables are considered for the case without isolators: namely, peak ground acceleration PGA, predominant ground frequency, strong motion duration, diameter of the column D , and total vertical load or deck weight P . For this example, 20 artificial ground motion records were generated using 20 different sequences of random phase angles. In the case with isolators, the width of the isolator B is added as the sixth variable, with the height of the isolator fixed as 0.40 of its width. The maximum absolute values of the horizontal displacement of the bent cap beam, column moment and column plastic rotation are chosen as the output variables.

A total of 200 combinations of the input random variables were generated using the aforementioned experimental design method. For each combination, the dynamic structural analysis program CANNY (Li, 1996) was run for each of the 20 ground motions and the corresponding responses were obtained, taking into account material nonlinearity and $P-\Delta$ effects. Subsequently, for each combination, the mean and standard deviation of each response were calculated, and a lognormal distribution was fitted to the data as shown, in Fig. 6, for a typical case of the maximum cap-beam displacement.

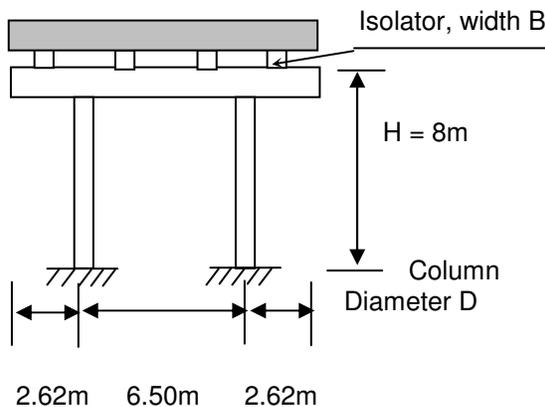


Figure 3. Bridge bent with isolation.

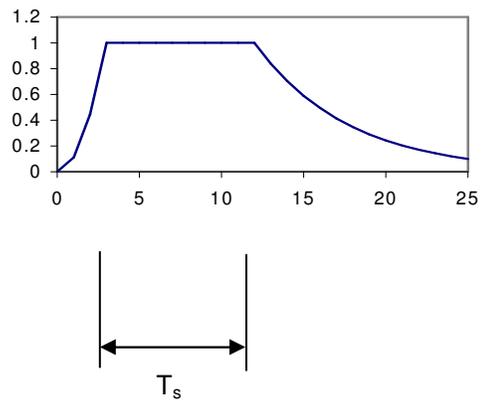


Figure 4. Modulation function.

This process is repeated for all the 200 sample combinations and, finally, two response databases are constructed: one for the mean response and the other for the standard deviation of the response over the earthquake records considered. Corresponding neural networks are then trained using the databases.

Numerical results are given here only for Δ_0 , the maximum cap-beam displacement, using two performances levels ($\Delta_0 = H/400$ for serviceability, and $\Delta_0 = H/40$ at collapse). For this example, it is assumed that the site seismicity is described by an earthquake class with a mean arrival rate of 0.20 or, on average, once every five years. For these earthquakes, the peak ground acceleration (PGA) has a lognormal distribution with a mean of 0.12g and a coefficient of variation of 0.66. This information results in a 475-year return acceleration of 0.40g (also corresponding to an exceedence probability of 0.10 in a 50-year window). Statistics for the remaining variables are shown in Table 1.

Table 2 shows the results for the reliability index β , when no isolators are present, in the case of the earthquake event and for a 75-years exposure window. Table 3, on the other hand, shows the corresponding results for isolators with a mean width $B = 750\text{mm}$ and a small coefficient of variation 0.01. It is clear that the isolators, in reducing the shear force transmitted to the bent, allow a marked increased in reliability at both performance levels. All results were obtained using Monte Carlo simulation and the neural network formulations.

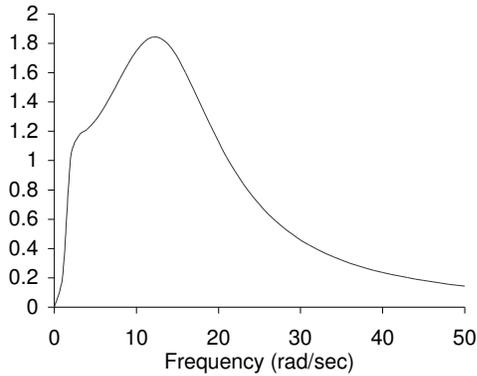


Figure 5. Clough-Penzien PSD function.

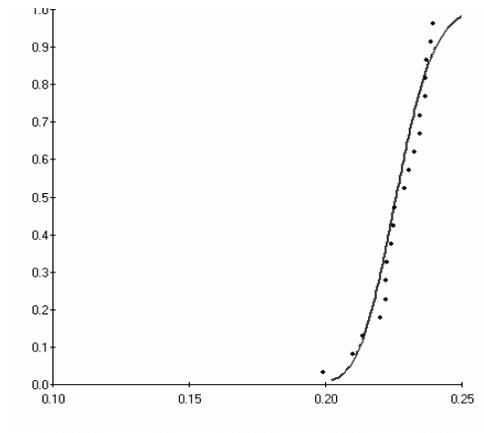


Figure 6. Variability with seismic record, lognormal.

Table 1. Example 1: Variable Statistics and Bounds for Database Generation.

Variable	Mean	Std. Deviation	Distribution	Lower Bound	Upper Bound
X(1) = PGA	0.12 g	0.0792 g	Lognormal	0.01 g	2.0 g
X(2) = ω_g	5π	π	Normal	π	10π
X(3) = T_s	20 sec	5 sec	Normal	1.0 sec	60.0 sec
X(4) = D	1800 mm	90 mm	Normal	1500 mm	2100 mm
X(5) = B	750 mm	7.5 mm	Normal	500 mm	1000 mm
X(6) = P	2400 KN	240 KN	Normal	1200 KN	3600 KN
X(7) = R_N	0.0	1.0	Normal		

Table 2: Results, No isolators.

Performance Definition	β (event)	β (75-years)
$\Delta < H/400$	1.74	0.10
$\Delta < H/40$	3.71	2.95

Table 3: Results, With Isolators, $\bar{B} = 750$ mm.

Performance Definition	β (event)	β (75-years)
$\Delta < H/400$	3.50	2.69
$\Delta < H/40$	4.63	4.04

Finally, Fig. 7 shows the variation in 75-years reliability, for both criteria, as a function of the mean width \bar{B} , always with a 0.01 coefficient of variation. The solid line corresponds to the collapse state, while the dashed line is associated with the serviceability limit state. Information such as that in Fig. 7 permits the performance-based design of the isolators, given target reliabilities for the service life. Obviously, it is unlikely that the same value of the design parameter \bar{B} will satisfy both criteria with arbitrarily requested reliability targets, and an optimal value can then be found by optimization or weighting of the criteria. If the target chosen for 75-year serviceability is $\beta = 2.0$ and the target for collapse is $\beta = 4.0$, Fig. 7 shows that \bar{B} could be chosen, in lieu of an optimization, as 800mm (the minimum between 800mm satisfying the collapse requirement and 900mm, satisfying serviceability).

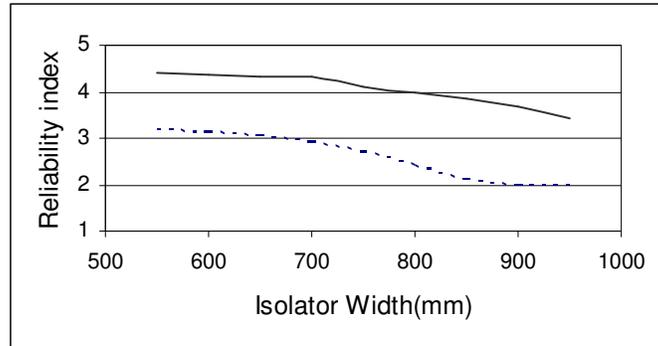


Figure 7. 75-year reliability indices β as a function of mean isolator width \bar{B} .
 ————— Collapse limit state; - - - - - Serviceability limit state

Example 2: Performance-Based Design of A Pile Foundation

Fig. 8 shows a cylindrical steel pile of length L and outside diameter D_c , supporting a mass M , subjected to earthquake ground motion. As a response, the pile cap will displace an amount Δ . The dynamic analysis considered the displacement of the pile shaft, w , and the reaction from the soil, $p(w)$, with consideration of the gap formation between the pile and the soil. The $p(w)$ soil reaction, always in compression, was taken as a nonlinear function of w (Yan and Byrne, 1992), and the pile itself was considered elasto-perfectly plastic. Suppose that it is of interest to assess the probability that the displacement Δ will exceed given levels expressed as fractions of the pile diameter D_c . That is, the performance function G is defined as

$$G(X, d) = \lambda D_c - \Delta(a_G, \omega_g, T_s, M, D_R, r) \quad (6)$$

in which M , the applied mass, is the design parameter d , and the vector X of intervening random variables includes

- a_G = peak ground acceleration;
- ω_g = soil dominant frequency;
- T_s = duration of the strong motion;
- D_R = soil relative density;
- r = nominal variable representing the soil accelerogram record.

The parameter λ = fraction of D_c , is used to define the limiting displacement. The pile has a diameter $D_c = 0.356\text{m}$, with wall thickness $t = 0.10\text{m}$, and a length $L = 30\text{m}$. Yield strength and elastic modulus were assumed to be deterministic and to have nominal values for mild steel (respectively, 250 Mpa and 200000 Mpa). Twenty earthquake records were simulated as non-stationary processes using, as in Example 1, the Clough-Penzien power spectrum density and the same envelope modulation function. Twenty records were thus simulated with twenty different sequences of random phase angles. The dynamic analysis, based on a beam finite element representation of the pile (Foschi, 2000), allowed the calculation of the hysteretic response from the basic properties for the pile and the soil. The formation of gaps resulted in corresponding stiffness losses and pinching. Fig. 10 shows, for example, the calculated loop for the cyclic pile cap displacement shown in Fig. 9.

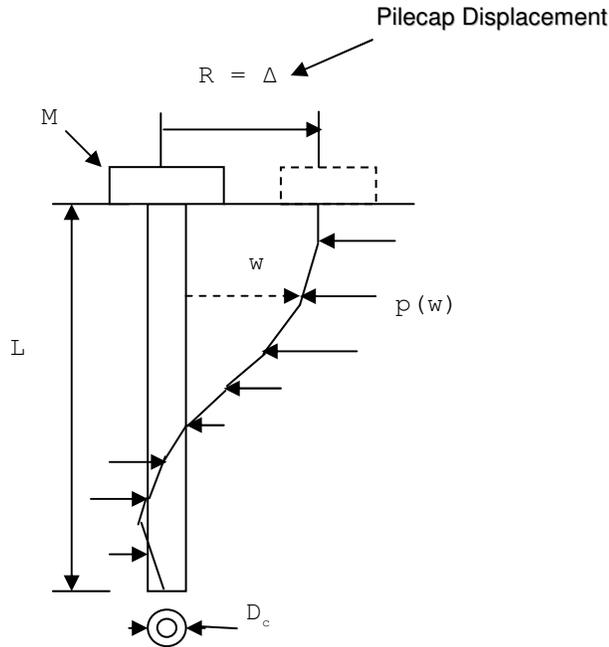


Figure 8. Pile under earthquake excitation.

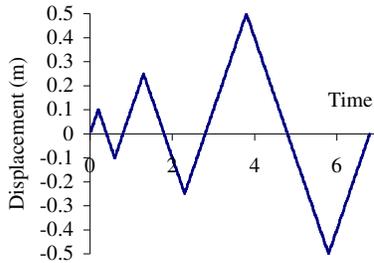


Figure 9. Cyclic cap displacement.

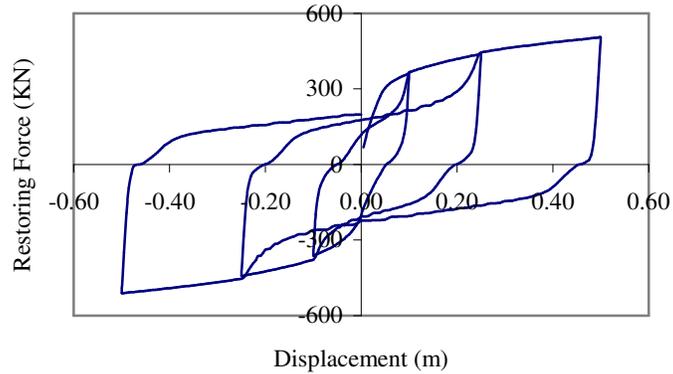


Figure 10. Calculated response to cyclic history.

For fixed combinations of X and d , the responses Δ were obtained for the set of 20 records r , calculating the mean value $\bar{\Delta}(X, d)$ and the standard deviation $\sigma_{\Delta}(X, d)$ over all records. Each of these statistics was then represented by a corresponding Neural Network, following the methodology previously described, and having as input the variables X and the mass M for d . As in Example 1, R_N being a Standard normal variable, the pile-cap displacement can be written as a lognormal distribution over the records,

$$\Delta(X, d) = \frac{\bar{\Delta}}{\sqrt{1 + (\sigma_{\Delta} / \bar{\Delta})^2}} \exp\left(R_N \sqrt{\ln\{1 + (\sigma_{\Delta} / \bar{\Delta})^2\}}\right) \quad (7)$$

Reliability assessments were carried out for different values of λ or different performance levels. Table 4 shows the statistical data used for the intervening variables, and Table 5 the reliability results.

Table 4. Statistical data for the intervening variables.

Variable	Distribution	Mean	Standard Deviation
a_G (m/sec ²)	Lognormal	1.0	0.6
ω_g (rad/sec)	Normal	4π	π
T_s (sec)	Normal	12	1.2
M (kN.sec ² /m)	Normal	150	15
D_R	Normal	0.5	0.1

The statistics for the peak ground acceleration a_G correspond to the event, and are consistent with a design acceleration (475 years return period) of 0.31g, assuming that earthquakes have a Poisson arrival rate of 0.2 (on average, one every five years).

Table 5. Reliability results.

Limit definition, factor λ	Reliability index β
0.1	-0.14
0.2	1.10
0.4	2.51
0.6	3.20
0.8	3.73
1.0	4.24

Finally, for performance-based design, two performance criteria were chosen: a displacement level associated with moderate damage level, $\lambda = 0.40$, and another associated with more substantial damage, $\lambda = 1.0$. For the first criterion, the target reliability was $\beta = 2.5$, while for the second the target was $\beta = 4.5$. The objective was to determine the design parameter, the mean mass M , optimally achieving the target reliabilities, allowing a coefficient of variation in M of 0.10. The results are shown in Table 6.

Table 6. Performance-based design, optimum mean applied mass M .

Optimum Mean Mass (kN.sec ² /m)	Performing criterion λ	Target reliability β	Achieved reliability β
139.8	0.4	2.50	2.59
	1.0	4.50	4.51

Conclusions

This paper has shown an efficient and straightforward method for reliability assessment and performance-based design in earthquake engineering. The strategy involves the deterministic generation of a response database for combinations of the random variables and design parameters, for different records likely to occur at a site. Data are obtained, for each combination, to calculate the mean and the standard deviation of the responses over the set of records. These data, in turn, are represented by neural networks. These facilitate simulations for reliability estimation and optimization in performance based design. The approach is more efficient than calculating first vulnerability functions (or exceedance probabilities conditional on a hazard level) and then integrating over all hazard levels at a site. When the design parameters are included in the neural networks' input layer, the optimization for performance-based design is easily implemented. Although the approach shown here can be directly applied for design, without going through a codified procedure, it can also be used to calibrate such a procedure. Thus, factors like "force reduction" or "ductility" can be calibrated so that designs following the code will approximately achieve prescribed target reliabilities. With a few factors to calibrate, however, the targets may not be reached uniformly over all design situations or locations. This disadvantage of a Code is overcome with the direct approach shown here, as it is always possible to achieve an optimum solution for a specific site.

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