Residual Strength and Post-Liquefaction Deformations

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ABSTRACT

Post-liquefaction stability and displacements of earth structures are controlled primarily by the residual strengths of the liquefied soils. Determination of the appropriate residual strength to use in post-liquefaction analysis is a controversial matter. This paper reviews current research on the parameters controlling residual strength. Estimation of residual strength by laboratory tests and by interpretation of case histories is reviewed. Validation of a procedure for estimating post-liquefaction displacements is presented, based on blind predictions of many failures in flood protection dikes in Hokkaido, Japan, during the 1993 and 1994 offshore earthquakes in the magnitude range $M = 7.8-8.0$.

INTRODUCTION

In the context of this review, liquefaction is synonymous with strain softening of relatively loose sands in undrained shear as illustrated by curve 1 in Fig. 1. When the sand is strained beyond the point of peak strength, the undrained strength drops to a value that is maintained constant over a large range in strain. This is conventionally called the undrained steady state or residual strength. If the strength increases after passing through a minimum value, the phenomenon is called limited or quasi-liquefaction and is illustrated by curve 2 in Fig. 1. Limited liquefaction may result in significant deformations because of the strains necessary to develop the strength to restore stability.

![Fig. 1. Types of contractive deformation (after Vaid et al., 1989).](image)

A major challenge facing engineers is the selection of the appropriate residual strength of liquefied materials for use in analyses to assess the post-liquefaction stability of embankments. The residual strength has a major impact on the cost of remediation. Laboratory research has done much to clarify the factors that control residual strength and provides some basis for selecting residual strength for design. However, the appropriate residual strength for design and analysis is still controversial.

Residual Strength Based on Case Histories

Seed (1987), back-analyzed a number of embankments in which significant displacements had occurred during earthquakes as a result of liquefaction. He published a chart linking the residual strength to the $(N_1)_{60-c}$, the normalized penetration resistance for clean sands (Seed, 1987). A revised version of this chart by Seed and Harder (1990) is shown in Fig. 2. Use of this chart for estimating residual strength is widespread in engineering practice.

Concerns have been expressed that in some case histories in Fig. 2, displacements may not have been sufficient to have mobilized residual strength. In others, because of lack of direct data, either assumptions or data from adjacent locations had to be used to generate the appropriate $(N_{1})_{60-c}$. There seem to be very few case histories for clean sand in the correlation. The correction for fines content has also been queried. Recently, the National Science Foundation held a workshop on residual strength at which these questions were discussed (NSF, 1997). A committee has been appointed to re-examine these case histories and to report in due course. This report should be of great interest and may result in proposed changes to the correlation. It is hoped that the review will also provide the data to facilitate independent interpretation of the case histories.

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SPT data and residual strength parameters measured
O SPT data and residual strength parameters estimated
O Construction-induced liquefaction and sliding case histories

Fig. 2. Relationship between corrected "clean sand" blowcount \( (N_1)_{50-ss} \) and undrained residual strength, \( s_u \), from case studies (after Seed and Harder, 1990).

Another limitation of the analysis of field data is the assumption that sliding takes place on a single slip surface as in conventional slope stability analysis. This assumption is not valid when a significant liquefied zone is deforming. Energy is dissipated throughout the deforming liquefied zone. The only way to capture this energy dissipation is by a finite element analysis that incorporates the proper distribution of residual strength. Neglecting the volumetric dissipation of energy is analogous to ignoring the zones of radial shear AGC and BCD in the classical Prandtl solution for undrained bearing capacity of a clay (Fig. 3) and assuming shear is mobilized only on the bounding sliding surfaces such as ACDE and FGCB. If the zones of radial shear are neglected, the estimated bearing capacity is 4.36\( c_u \), which is 85% of the Prandtl value of 5.14\( c_u \), where \( c_u \) is the undrained shear strength.

**RECENT DEVELOPMENTS IN ESTIMATING RESIDUAL STRENGTH**

**Factors Controlling Residual Strength**

Extensive research has been conducted in the laboratory on the factors controlling the residual strength. Originally, it was thought to be a function only of the void ratio for a given sand (Castro, 1969; Poulos et al., 1985). Research studies, since 1988, some of which will be described below, suggest that the residual strength measured within the strain capacity of laboratory equipment is a function of:

- sample preparation technique
- stress path followed during loading
- effective confining pressure

**Effect of Sample Preparation**

Many laboratory studies in liquefaction use samples prepared by moist tamping because it is the easiest way to form relatively loose samples. However, it frequently results in void ratios which are not accessible to the same sand under deposition conditions in the field. Other methods in use are air pluviation and pluviation under water. Vaid et al. (1998a) has demonstrated that the residual strength measured on samples prepared in different ways are quite different (Fig. 4). Vaid et al. (1998b) tested frozen samples of two different sands to determine the residual strength. He then reconstituted the same samples to the same void ratio using pluviation in water. The reconstituted samples gave residual strengths very similar to the frozen samples. These results are a strong argument for using pluviation under water to form representative samples of soils which were originally deposited under water or were placed by hydraulic fill construction. The moist tamping method would seem to be more appropriate for forming representative samples of unsaturated compacted soils.
Stress Path

Uthayakumar and Vaid (1998) and Yoshimine et al. (1998) have explored the effects of stress path on residual strength over a wide range of stress paths defined by \( \alpha \), the inclination of the principle stress to the vertical axis of the sample, and the parameter \( r = (\sigma_2 - \sigma_3) / (\sigma_1 - \sigma_3) \) which is a measure of the intermediate principle stress. The samples were tested using the hollow cylinder torsional shear device. Typical results of this kind of data (Yoshimine et al., 1988) are shown in Fig. 5 which suggest that different residual strengths should be assigned to different parts of a liquefied zone in an embankment depending on the predominant stress conditions. This selective use of shear strength for design is not new. Bearing capacity under offshore structures in the North Sea is evaluated using compression, simple shear and extension strength data to suit stress conditions at different locations along potential sliding surfaces.

Residual Strength as a Function of Effective Confining Pressure

The practice of expressing the residual strength as a fraction of the effective confining pressure has been used in practice on several water-retaining and tailings dams since it was used on Sardis Dam in 1989 (Finn, 1990). For the most part the ratio selected has been between 0.06 and 0.1. A value of \( S_r / p' = 0.23 \) was used for Duncan Dam, based on extensive testing of frozen samples (Byrne et al., 1994).

Vaid and Thomas (1994) using extension tests with frictionless end platens, determined the residual strength of Fraser River sand over a range of void ratios and confining pressures. The results clearly demonstrated the dependence of residual strength of overburden pressure. Their results are replotted in Fig. 6; normalized with respect to the effective confining stress. Despite some scatter, the variation of \( S_r / p' \) is well represented by a straight line. For \( S_r / p' \) to be a constant for a given soil, it is necessary for the steady-state line to be parallel to the isotropic consolidation line. This is not generally the case, although data from Yoshimine et al. (1998) suggest that it is approximately true for Toyoura sand. Similar results were obtained for simple shear tests (Vaid and Sivathayalan, 1996).

Fig. 4. The effect of sample preparation on undrained simple shear response of Syncrude sand (Vaid et al., 1998a).

Fig. 5. Effect of stress path on undrained behaviour of Toyoura sand (Yoshimine et al., 1998).

Fig. 6. Relationship between the residual strength normalized by the effective confining stress and void ratio in extension tests on Fraser River (after Vaid and Thomas, 1994).
POST-LIQUEFACTION DISPLACEMENT ANALYSES

The use of large displacement analysis in evaluating post-liquefaction response and assessing the adequacy of proposed remediation measures has now become part of engineering practice. It was first applied to Sardis Dam in 1989 (Finn, 1990). Post-liquefaction analysis and its application in design of remediation measures have been the subject of several reviews (Finn, 1993; Finn and Ledbetter, 1993).

Between 1994 and 1998, displacement analyses were conducted on many flood protection dikes in Hokkaido, Japan, which had been damaged during the Kushiro-oki and Nansei-oki earthquakes in 1993 and 1994, respectively. The objective was to develop a criterion based on potential post-liquefaction crest settlement to prioritize the remediation of dikes against future earthquakes. A 2-step strategy was adopted for the studies. First, failures of dikes in eastern Hokkaido would be simulated. If these simulations were satisfactory, then the analyses would be used to predict crest settlements in a number of dikes in western Hokkaido which had significant post-liquefaction displacements. The displacement analyses were conducted in Vancouver, and the predictions were verified by engineers from the Advanced Construction Technology Center (ACTEC), Tokyo, and the Hokkaido Development Bureau on the basis of field data known only to them.

Simulation of Dike Failure at Section 9K850 in Eastern Hokkaido

The failure mode at a location on the left bank of the Kushiro river is shown in Fig. 7. The height of the dike before the earthquake was about 7 m and the crest width was about 8 m. As a result of earthquake shaking, the crest of the dike settled about 2 m and movement of the slope of the order of 3 m took place towards the river. The ground was frozen to a depth of about 0.7 m at the time of the earthquake. The brittle nature of the frozen layer is probably responsible for the sharp step feature in the crest near the upstream slope. This frozen layer was taken into account during the simulation.

Appropriate input motions for seismic analysis were specified by Jishin Kogaku Kenkyusho, Tokyo. Relevant soil properties were provided by ACTEC. Dynamic analysis was conducted in the effective stress nonlinear mode using the program TARA-3 (Finn et al., 1986). The large strain post-liquefaction deformations were calculated using the program TARA-3FL (Finn and Yogendrakumar, 1989). This program allows the liquefied region to deform at constant volume and uses a Lagrangian updating scheme to handle large strains.

The computed deformed shape of the dike is shown in Fig. 8. The sharp break in the surface shows the effect of the frozen ground. The deformed shape and the magnitudes of displacements agree fairly well with the displacements measured after the earthquake. The computed maximum settlement and horizontal displacement are 2.3 m and 2.7 m, respectively, compared to measured displacements of 2 m and 3 m.

The simulation studies were considered satisfactory and a major parametric study was approved to investigate the effects of some of the important cross-sectional parameters that control the consequences of liquefaction, such as the thickness of a non-liquefiable layer overlying the liquefied layer, the thickness of the liquefied layer itself, and the height and side slopes of the dikes. The effects of these parameters were characterized by the settlements of the crests of the dikes after liquefaction.
Estimation of Crest Settlements

The crest settlements were estimated first for dikes with side slopes 1:2.5 as shown in Fig. 9. The thicknesses of the liquefied and nonliquefied layers were varied and the resulting displacements after liquefaction are plotted in nondimensional form in Fig. 10.

![Typical cross-section of dike for analysis](image)

**Fig. 9. Typical cross-section of dike for analysis.**

The nondimensional computed crest settlements, \( S/H_D \), are shown by the curve in Fig. 10. The equation of the curve is given by,

\[
\frac{S}{H_D} = 0.01 \exp \left( 0.922 - \frac{H_D}{H_{NL}} \right)
\]

where \( S \) is the crest settlement, \( H_D \) is the height of the dike; \( H_L \) and \( H_{NL} \) are the thicknesses of the liquefiable and non-liquefiable layers, respectively. This curve was adopted for predicting crest settlement.

Engineers from ACTEC and the Hokkaido Development Bureau compared the measured crest settlements from a wide variety of dikes in western Hokkaido which underwent noticeable displacements during the Nansei-oki earthquake in 1994 with those predicted by Eqn. 1. The data points and the prediction curve are plotted in Fig. 10. The black points represent the real cases corresponding to some of the idealized analyses done to develop the curve; the open points represent other dikes. The agreement was very good for dikes with slopes of 1:2.5, but the field data showed that the side slopes had an important affect on the crest settlement and that a separate criteria would be necessary for two other predominant slopes; uniform side slopes of 1:5, and unequal slopes, 1:5 and 1:10. Parametric studies were conducted for different values of \( S/\sigma'_{vo} \). Dikes with \( S/\sigma'_{vo} \geq 0.5 \) showed only tolerable displacements.

**CONCLUSIONS**

It can be seen that the criterion based on crest settlement can be useful for deciding which dikes should be remediated first. The larger the predicted settlement, the more urgent the need for remediation. The Hokkaido dikes study is an important case history because it is the only instance in which post-liquefaction displacement analysis has been validated independently in blind tests in a large number of earth structures undergoing different levels of post-liquefaction displacements.

**REFERENCES**


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