ELASTO-PLASTIC ANALYSIS OF BURIED PIPELINES
SUBJECTED TO GROUND DEFORMATION

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ABSTRACT
This study presents a quasi-static approach using a matrix method based on the linear programming (LP) for the elasto-plastic analysis of a buried pipeline subjected to ground deformation. The actual buried pipeline is initially modeled as a discrete pipeline-ground system, in which pipe and ground spring elements are supposed to have elastic-perfectly plastic behaviors, respectively. The analysis procedure involves a performance of a series of incremental deformation analyses as LP problem. Herein, the objective is to maximize ground deformation increment during the successive yielding of ground spring or pipe bar element, while simultaneously satisfying the conditions of compatibility, equilibrium, yield and plastic flow as constraints. Three examples are analyzed to illustrate the features and scope of application of the approach.

INTRODUCTION
According to the records of past earthquake damage, most of seismic damage of buried pipelines is mainly caused by the permanent ground deformation such as relative ground movement, fault motion and travelling seismic waves. Therefore, as a first step to simulate the seismic behavior of buried pipelines, it is necessary to develop the elasto-plastic analysis method of the buried pipelines subjected to ground deformations. As for the work of this area, several methods (1-4) based on the seismic deformation method have been proposed in a variety of ways. However, these methods (1-4) can not exactly investigate the elasto-plastic behavior of buried pipeline, since they can not treat the range beyond the elastic limit of pipe. On the other hand, there is a tendency to utilize the ductility of steel pipe beyond the elastic limit, due to the recent rapid progress of pipe material and the accumulation of experimental data. In fact, a new design method (5) has been proposed to permit the 1% strain of pipe as the design criterion for the high pressure gas pipeline in Japan. This indicates a new and revolutionary direction for the earthquake resistant design of buried pipeline. In the analytical field, however, the elasto-plastic analysis method has not yet been established for the case including the plastic strain of pipe.
To overcome these difficulties mentioned above, this paper presents a new seismic deformation method using linear programming (LP) for the incremental analysis of buried pipeline subjected to the ground deformations. The approach essentially extends the elastic-plastic analysis of above ground structures (6) and the beam on the foundation (7) using LP. Here, the analysis process involves the maximization of ground displacement increment as an objective criterion, while simultaneously satisfying the conditions of compatibility, equilibrium, yield and plastic flow as constraints. Upon succeeding iterations, the final solutions are achieved when the ground displacement factor increment becomes zero.

In numerical examples, the buried pipelines with or without joints subjected to axial ground deformation are initially presented to illustrate the features of the approach. Then, the axial and flexural behaviors beyond elastic limit of pipe are investigated for the various ground deformations.

**ELASTO-PLASTIC ANALYSIS OF BURIED PIPELINES BY LINEAR PROGRAMMING**

Herein, the basic equations will be explained for the axial behavior of a buried pipeline and they will be also applied for the flexural behavior in the numerical example. In addition to the usual assumptions of the seismic deformation method and simple plastic theory, it is assumed that an actual pipeline is modeled as the discrete structure as shown in Fig. 1: the pipeline is characterized by a series of "bar" elements, and the ground is characterized by a series of "spring" elements. The elastic-perfectly plastic behavior characteristics are assumed for both elements as shown in Fig. 2(a), (b).

Using the external ground displacement instead of the external load in the formulation of beam on the foundation (7), the basic conditions for the elasto-plastic analysis of the buried pipeline are obtained as the following incremental forms, i.e., Eqs. (1b)-(12). As these relations are established for any ground displacement increment $\Delta \alpha (\leq \Delta \alpha_p$ : the ground displacement factor increment from the yielding of one element to the another), they can pursue the successive yielding of elements by maximizing $\Delta \alpha$. Consequently, the elasto-plastic incremental analysis of a buried pipeline subjected to the ground displacement may be formulated as the following LP problem:

Given: $c_N, c_S, l_N, l_S, k, k^{-1}, s, v, \bar{w}, \bar{V}$  
Find: $\Delta N, \Delta S, \Delta u, \Delta N, \Delta S, \phi_N, \phi_S$  
Subject to:  
\[ D c_N^T \Delta N + c_S^T \Delta S = 0 \]
\[ L_N \Delta N + k_N^1 \Delta N - C_N^1 \Delta u = 0 \] (1 c)
\[ L_S \Delta S + k_S^1 \Delta S - C_S^1 (\Delta u - \Delta u_S) = 0 \] (1 d)
\[ L_N \Delta N + (-\Phi_N^1) = L_N^T N - \Phi_P^1 , -\Phi_N^1 \geq 0 \] (1 e)
\[ L_S \Delta S + (-\Phi_S^1) = L_S^T S - \Phi_P^1 , -\Phi_S^1 \geq 0 \] (1 f)
\[ (L_N^T N - \Phi_P^1)^T \Delta N^e = 0 , (L_S^T S - \Phi_P^1)^T \Delta S^e = 0 \] (1 g), (1 h)
\[ (L_N^T N - \Phi_P^1)^T \Delta N = 0 , (L_S^T S - \Phi_P^1)^T \Delta S = 0 \] (1 i), (1 j)
\[ \Delta N \geq 0 , \Delta S \geq 0 \] (1 k), (1 l)

in which Eq. (1a) means the activation of a new yield mode. Eq. (1b) expresses the equilibrium condition, Eqs. (1c), (1d) define the compatibility equations for the pipe and the spring elements, respectively. Eqs. (1e), (1f) are the yield conditions for the pipe and the spring elements, respectively, and Eqs. (1g), (1i) and Eqs. (1h), (1j) are the plastic flow conditions of the pipe and the spring elements, respectively. It is noted that Eqs. (1i), (1j) consider the unloading phenomenon, i.e., they ensure that plastic flow can not occur ( \( \Delta \lambda_S = 0 \) ) if the stress increment decreases ( \( \Delta \lambda_S < 0 \) ). Abbreviations \( \Phi_N^1, \Phi_S^1 \) mean the conditions of equilibrium, compatibility, yield and plastic flow, respectively.

In Eq. (1), the increments of stress and deformation are simultaneously found by only a LP application and, as such, they are referred as a modified forced method. Having the increments \( \Delta N, \Delta S, \Delta u, \Delta \lambda_N, \Delta \lambda_S \) from Eq. (1), the total quantities in the present stage are found by adding the increments to the values in the previous stage. The specially devised computer program determines the stresses and deformations at each stage as well as the sequence of the yielding elements. That is, the computer program initially involves the determination of the first yield element with the ground displacement factor \( \alpha_1 \) in which the maximum stress has just reached the elastic limit. Then, the second yield element with \( \alpha_2 \) is found by using Eq. (1) with the previous data. Upon succeeding iterations, if the analysis finds \( \Delta N = 0 \), the operation is completed.

EXAMPLES

EXAMPLE 1: In order to illustrate the features of the present method, the axial behavior is firstly investigated for a straight steel pipeline (\( \phi=100 \) m) with or without expansion joints. The pipeline is discretized into the 20 pipe elements as shown in Fig. 1 and properties of pipe and ground soil are summarized in Table 1 and Table 2. Note that both end elements (1), (2) in Fig. 1 are selected as dummy elements in order to express the joints elements or the actual fixed conditions. For example, adopting the small rigidity at both end elements, say, \( 1/1000 \) of other pipe elements, this model is transformed into the pipeline with
expansion joints. The two types of sinusoidal waves are chosen as the input axial ground displacements \( u_s \), i.e., the one wave and the half wave length as shown in Fig. 3.

Performing incremental analysis in case of pipeline with expansion joint, the historical collapse modes with \( \alpha_{\text{max}} \) for the two types of ground displacements are found to be shown in Fig. 3. Note that the numerals of yielded elements in Fig. 3 indicate the sequence of yielding. It should be noted from Fig. 3 that the final collapse modes in both cases are completed by only yielding of ground spring elements. The maximum ground displacement factor \( \alpha_{\text{max}} \) and the maximum strain of the pipeline \( \varepsilon_{\text{max}} \) are summarized in Table 3 for the various combinations of the ground displacement mode, the existence of joint and the limit relative displacement \( e_y \).

Figures 4 and 5 show the relationship between the ground displacement factor \( \alpha \) and the maximum strain of pipe \( \varepsilon_{\text{max}} \) in both cases of ground displacement of one wave length and half wave length, respectively. It is found from Fig. 4 that the maximum strain \( \varepsilon_{\text{max}} \) with joints becomes larger than the one without joints. On the other hand, it should be noted from Fig. 5 that \( \varepsilon_{\text{max}} \) with joints decreases about 1/2 in comparison with the one without joints.

**EXAMPLE 2**: The pipeline subjected to the forced axial displacement at both ends as shown in Fig. 6(a) is analyzed to investigate the elasto-plastic behavior of pipeline in the case occurring plastic strain of pipe. From incremental analysis, the historical collapse mode and the displacement factor-strain curves are found to be shown in Figs. 6(b) and 7, respectively.

(a) It is found from Fig. 6(b) that the final collapse mode of pipeline-ground system is achieved by only yielding of pipe elements.

(b) It should be noted, however, that unloading phenomenon occurs for the ground spring elements in the yielding process even when the applied ground displacement increases. Consequently, the frictional resistant forces of these elements are equal to zero.

(c) It is recognized from Fig. 7 that the ground displacement factor becomes \( \alpha = 22 \) corresponding to the compressive yield strain \( \varepsilon_y = 0.69\% \), which is specified in the "Recommended Practice for Earthquake Resistant Design of High Pressure Gas Pipeline" (5).

**EXAMPLE 3**: This approach is applied to the flexural behavior of the buried pipeline subjected to the ground displacement perpendicular to pipeline axis. It is assumed that the ground reaction coefficient in perpendicular direction to the pipeline axis is constant \( k_{sy} = 76.6 \text{ kg/cm}^2 \) and the ground spring force is also constant \( R_p = k_{sy} \cdot a_0 \cdot e_y = 38.3 \text{ ton} \) throughout. Figures 8(a), (b), (c) show the input ground displacement mode, the output historical collapse mode and the deflection curves of pipe, respectively. It is found from Figs. 8(b) and (c) that the final collapse mode is formed with only yielding of pipe and, as such, the earthquake resistant capacity for the flexural deformation is remarkably larger than that for the axial deformation.
CONCLUSION

The following conclusions may be drawn from this study.

(1) An efficient analysis method for the elasto-plastic behavior of buried pipeline has been developed by extending the technique for the above ground structure.

(2) This approach can exactly evaluate the strain and the deflection of pipeline beyond the elastic limit of pipe.

(3) It is found from examples that there are two types of collapse modes, i.e., the one is due to only yielding of ground springs as shown in Fig. 3, and the other is due to only yielding of pipe elements as shown in Figs. 6 and 8.

(4) It should be noted that the maximum strain with joints is larger than that without joint for the ground deformation of one wave length. Therefore, the position of joint should be carefully arranged in the actual pipeline system.

(5) It is recognized that local unloading of a yield element occurs for the increasing ground deformation after a certain amount of plastic deformation has taken place. Such a phenomenon can only be detected by a complete incremental analysis.

(6) The present approach will be able to give us a useful information on the earthquake resistant capacity, if the properties of pipe and ground soil are known for the existing buried pipeline in a local area.

REFERENCES


NOTATION

\( a_0 \) : length of pipe element
\( C_N, C_S \) : compatibility matrix relating to pipe element, ground spring element
\( e = e_e + e_p \) : relative displacement between pipe and ground
\( e_e \) : elastic part of \( e \)
\( e_p \) : plastic part of \( e \)
\( e_y \) : limit relative displacement
\( k_N^{-1}, k_S^{-1} \) : assembly flexibility matrix relating to pipe element, ground spring element
\( L_N, L_S \) : matrix whose rows are the outward unit normal vectors to the yield planes relating to \( N, S \)
\( N, S \) : vector of axial force, frictional resistant force
\( \Delta N, \Delta S \) : increment vector of \( N, S \)
\( N_p, S_p \) : limit value of \( N, S \)
\( u \) : vector of nodal displacement of pipe element
\( \Delta u \) : increment vector of \( u \)
\( u_N \) : vector of ground displacement
\( \alpha \) : ground displacement factor
\( \alpha_{\text{max}} \) : maximum ground displacement factor
\( \Delta \alpha \) : increment of \( \alpha \)
\( \delta = \delta_e + \delta_p \) : expansion or contradiction of pipe element
\( \delta_e \) : elastic part of \( \delta \)
\( \delta_p \) : plastic part of \( \delta \)
\( \delta_y \) : yielding value of \( \delta \)
\( \Phi_N, \Phi_S \) : vector of yield function of pipe and spring elements
\( \Delta A_N, \Delta A_S \) : vector of plastic multipliers increment of pipe and spring elements
### Table 1 Dimensions of pipe

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Outer diameter</td>
<td>40.64 cm</td>
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<tr>
<td>Wall thickness</td>
<td>0.79 cm</td>
</tr>
<tr>
<td>Cross-sectional area</td>
<td>98.9 cm²</td>
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<tr>
<td>Young's modulus</td>
<td>2.1x10⁶ kg/cm²</td>
</tr>
<tr>
<td>Moment of inertia</td>
<td>19640 cm⁴</td>
</tr>
<tr>
<td>Ultimate strength of pipe</td>
<td>4100 kg/cm²</td>
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<tr>
<td>Yield stress of pipe</td>
<td>2300 kg/cm²</td>
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<tr>
<td>Axial plastic capacity</td>
<td>227470 kg</td>
</tr>
<tr>
<td>Plastic moment</td>
<td>2222x10³ kg.cm</td>
</tr>
<tr>
<td>Length of model pipeline</td>
<td>100 m</td>
</tr>
<tr>
<td>Length of pipe element</td>
<td>300 cm</td>
</tr>
<tr>
<td>Axial stiffness of pipe element</td>
<td>415380 kg/cm</td>
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</table>

### Table 2 Properties of ground spring element

<table>
<thead>
<tr>
<th>Direction</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial direction</td>
<td>Limit frictional spring force</td>
<td>5745 (1915) kg</td>
</tr>
<tr>
<td></td>
<td>Ground spring constant</td>
<td>19150 kg/cm</td>
</tr>
<tr>
<td></td>
<td>Ground spring constant per unit length</td>
<td>76.6 kg/cm²</td>
</tr>
<tr>
<td></td>
<td>Limit relative displacement</td>
<td>0.3 (0.1) cm</td>
</tr>
<tr>
<td>Perpendicular direction</td>
<td>Limit ground reaction force</td>
<td>38300 kg</td>
</tr>
<tr>
<td></td>
<td>Ground spring constant</td>
<td>38300 kg/cm</td>
</tr>
<tr>
<td></td>
<td>Ground spring constant per unit length</td>
<td>76.6 kg/cm²</td>
</tr>
<tr>
<td></td>
<td>Limit relative displacement</td>
<td>1.0 cm</td>
</tr>
</tbody>
</table>
Table 3 Maximum ground displacement factor ($\alpha_{\text{max}}$) and maximum strain ($\varepsilon_{\text{max}}$) (Example 1)

<table>
<thead>
<tr>
<th>Ground displacement mode</th>
<th>Joint at both ends</th>
<th>Limit relative displacement ($e_r$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.1 cm</td>
</tr>
<tr>
<td>without</td>
<td>$\alpha_{\text{max}}$</td>
<td>0.458</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{\text{max}}$</td>
<td>82</td>
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<tr>
<td>with</td>
<td>$\alpha_{\text{max}}$</td>
<td>1.519</td>
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<tr>
<td></td>
<td>$\varepsilon_{\text{max}}$</td>
<td>156</td>
</tr>
<tr>
<td>without</td>
<td>$\alpha_{\text{max}}$</td>
<td>1.198</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{\text{max}}$</td>
<td>174</td>
</tr>
<tr>
<td>with</td>
<td>$\alpha_{\text{max}}$</td>
<td>1.329</td>
</tr>
<tr>
<td></td>
<td>$\varepsilon_{\text{max}}$</td>
<td>84</td>
</tr>
</tbody>
</table>

($\varepsilon_{\text{max}} : x 10^5$)

Fig. 1 Discretized pipeline model
Fig. 2 Elastic-plastic behaviors of (a) pipe element and (b) ground spring.

Fig. 3 Example 1: Behavior of pipeline with expansion joints at both ends (e=0.3 cm) for the ground displacements of (a) one-wave length and (b) half-wave length.

Fig. 4 Example 1: Ground displacement factor-maximum strain curves with or without joint (one wave length).

Fig. 5 Example 1: Ground displacement factor-maximum strain curves with or without joint (half wave length).
Fig. 7 Example 2: Ground displacement factor-strain curves of pipe element

Fig. 6 Example 2: Behavior of pipeline subjected to forced displacement at both ends (ey=0.3cm)

Fig. 8 Example 3: Flexural behavior of pipeline