Polynomial Response Surface-Based Seismic Fragility
Assessment of Concrete Gravity Dams

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ABSTRACT
The consequences of a dam failure could be significant in terms of casualties and/or economic and environmental damage. Therefore, dam safety is given high priority. As knowledge in the seismicity field has increased, it has been observed that several dams fail to meet the revised safety criteria. In recent years, probabilistic methods, such as fragility analysis, have emerged as reliable tools for the seismic assessment of dam-type structures. To improve the fragility assessment of such structures by considering the influence of modelling parameters and by reducing the computational burden, this study proposes the development of a polynomial response surface meta-model. The proposed model will be used to predict the continuous maximum relative base sliding of the dam to build fragility curves and show the effect of the modelling parameters uncertainty. To do so, an accurate estimate of the seismic demand at the dam site was performed, and finite element samplings of the possible system configuration were generated with Latin Hypercube Sampling (LHS). The produced training cases are then used to fit polynomial response surface meta-models within a stepwise regression framework. The predictive capabilities of the meta-model are evaluated considering local and global goodness-of-fit estimates from 5-fold cross-validation. The best performing meta-model is then used to generate analytical seismic fragility curves, which propagate the uncertainty in the modelling parameters by considering a range of usable values. Finally, fragility regions are developed that represent the fragility curves generated with values corresponding to a 95% confidence interval of the usable range of the modelling parameters values; this is done in order to identify the parameter that introduces the most uncertainty in the fragility analysis. The proposed methodology is applied to a dam located in north-eastern Canada. It is observed that the variability of the concrete–rock cohesion modelling parameter introduces the most uncertainty in the fragility analysis, followed by the drain efficiency and, to a lesser extent, the concrete–rock angle of friction.

Keywords: gravity dams, fragility curves, meta-models, seismic hazard, modeling parameters.

INTRODUCTION
Methods for seismic analysis of dams have improved extensively in the last few decades, and growth in the processing power of computers has expedited this improvement. Advanced numerical models have become more feasible and manageable and constitute the basis of more adequate procedures of designing and assessing. A probabilistic framework is required to manage the various sources of uncertainty that may impact the system performance and the related decisions [1]. Fragility analysis, which depicts the conditional probability that a system reaches a structural limit state, is a central tool in this probabilistic framework. However, current vulnerability assessment methods develop fragility functions by using a single parameter to relate the level of shaking to the expected damage, which cannot properly represent the complex loading associated with a seismic event [2]. Furthermore, the effect of the variation of the material properties in the seismic fragility analysis is frequently overlooked owing to the costly and time-consuming re-evaluation of the numerical model. Seismic response and vulnerability assessment of key infrastructure elements often requires several non-linear dynamic analyses of complex finite element models (FEM). The substantial computational time may be reduced by using machine learning techniques to develop a surrogate or meta-model, which is an engineering method used when an outcome of interest cannot be easily measured directly; thus, a model of the outcome is used instead [3]. Therefore, the main goal of this study is to generate polynomial response surface meta-models for the seismic assessment of gravity dams and to perform fragility analysis through the use of the best performing model. The secondary goal is to explicitly account for the effect of the model parameters variation in the seismic fragility analysis and to properly depict the seismic scenario likely to occur at a specific site to enhance the accuracy of the fragility estimates. The proposed methodology is applied to a gravity dam located in north-eastern Canada.
SEISMIC ANALYSIS OF THE DAM

Finite element model and the dam considered in this study

To generate the virtual experimental data needed to develop probabilistic demand models, a nonlinear finite element model of the structure was developed to analyse the dynamic seismic response under vertical and horizontal ground motions. As mentioned above, the present study focuses on a particular dam located in north-eastern Canada. It comprises 19 unkeyed monoliths, a maximum crest height of 78 m, and a crest length of 300 m. The width is 4.6 m at the top and 62 m at the base of the largest monolith (Figure 1(a)). The dam rests on a foundation consisting mainly of anorthosite gabbro and granitic gneiss [4], which corresponds to hard rock (VS30 > 1500 m/s). The dam was chosen for its simple and almost symmetric geometry, as well as for its well-documented dynamic behaviour. In addition, forced vibration test results are available for calibration of the dynamic properties of the numerical dam model [4].

![Figure 1. (a) Cross section and (b) finite element model of the tallest monolith](image)

The tallest monolith of the dam was considered to be representative of the structure. It was modelled using the computer software LS-Dyna [5], as shown in Figure 1(b), following the recommendations of the United States Bureau of Reclamation (USBR) [6], and it was analysed using the software’s explicit solver. The proposed model considers the diverse interactions between the structure, the reservoir and the foundation. The reservoir is modelled with compressible fluid elements, while the concrete body of the dam and the rock foundation were modelled with linear elastic materials. By modifying the properties of the dam and the foundation materials, the fundamental period of the dam-reservoir-foundation system turned was 0.25 s, which matches the fundamental period from a previous model of the same monolith calibrated from in situ forced vibration tests [4]. Further details on the modelling assumptions and the validation of the numerical model can be found in Bernier et al. [7].

Ground motion records selection

To proceed with the evaluation of the vulnerability of the dam considered in this study through the development of fragility functions, a representative set of ground motion time series (GMs), which properly accounts for the aleatory uncertainty, is necessary [8-9]. A probabilistic seismic hazard analysis (PSHA) was performed at the dam site using the computer software OpenQuake [10] to characterize the target earthquake scenarios at various intensity levels. The hazard levels were defined in terms of spectral acceleration at the fundamental period of the structure (Sa(T)) to conveniently cover the range of values corresponding to return periods between 700 to 30000 years. To proceed with the selection of a representative set of ground motion time series (GMTS), the generalized conditional intensity measure (GCIM) approach [11] was adopted. The purpose of using the GCIM approach is to include the most influencing seismic intensity measures (IM) with respect to the structural response. For the case of gravity dam-type structures, PGV was found to be one of the best-performing structure-independent ground-motion scalar IMs to correlate with damage [12]. Similarly, the vertical spectral acceleration SaV is also expected to be relevant in heavy structures of this type. As a result, the set of considered IMs in the GCIM is {Sa(T); Sav(T); PGV}, where Sa(T) and Sav(T) are computed at 20 vibration periods in the range of T = [0, 2T1 − 2T1] as proposed by Baker [13], leading to a total of 41 IMs to be considered in addition to the conditioning IM, Sa(T1). The GCIM distribution computed using the abovementioned IMs was then used to simulate and select 250 ground motions. The records were selected from the PEER NGA-West2 database [14] owing to the limited availability of strong ground motion records in the PEER NGA-East database [15]. Further details on the PSHA and the record selection procedure can be found in Segura et al. [16].
Design of experiments

To minimize the associated cost of running a dynamic non-linear FEM, while analysing an adequate number of loading conditions and structural system configurations, an appropriate experimental design method should be used. Structural and material properties likely to affect the seismic response of the structure should be considered, and their associated ranges should be based on experimental data or values found in the literature. The procedure used in this study to generate the meta-model can be summarized as follows: (i) generate an experimental design matrix for the finite element simulations to generate the sample points representing the different configurations of the system, (ii) conduct the finite element simulations for each row of the experimental matrix and (iii) fit regression meta-models to the training points. Latin Hypercube sampling (LHS) is adopted to generate the sample points representing the different configurations of the system under study. This sampling technique was selected because of its ability of dividing the desired range of values for each parameter into n equiprobable intervals and then selecting a sample once from each interval, to ensure that the set of samples reflects the entire range of the parameters, as demonstrated in past earthquake engineering applications [17-18].

Table 1 presents the parameters that were considered as random variables in the numerical analysis of the dam response and for which the uncertainty was formally included through their probability distribution function (PDF). All the remaining input parameters were kept constant and represented by their best estimate values. For the studied dam, owing to the limited availability of material investigations, the probability distributions were defined using the empirical data of similar dams. The uniform distribution was used for most parameters except for damping, for which a log-normal distribution was adopted as proposed by Ghanaat et al. [19].

![Table 1. Adopted PDF for the modelling parameters](image)

<table>
<thead>
<tr>
<th>Parameters</th>
<th>PDF</th>
<th>PDF Parameters</th>
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</thead>
<tbody>
<tr>
<td>Concrete–rock tensile strength (MPa), CRT</td>
<td>Uniform</td>
<td>L = 0.2</td>
</tr>
<tr>
<td>Concrete–concrete tensile strength (MPa), CCT</td>
<td>Uniform</td>
<td>L = 0.3</td>
</tr>
<tr>
<td>Concrete–rock cohesion (MPa), CRC</td>
<td>Uniform</td>
<td>L = 0.3</td>
</tr>
<tr>
<td>Concrete–concrete cohesion (MPa), CCC</td>
<td>Uniform</td>
<td>L = 0.9</td>
</tr>
<tr>
<td>Concrete–concrete angle of friction (°), CRF</td>
<td>Uniform</td>
<td>L = 42</td>
</tr>
<tr>
<td>Concrete–concrete angle of friction (°), CCF</td>
<td>Uniform</td>
<td>L = 42</td>
</tr>
<tr>
<td>Concrete damping (%) , CD</td>
<td>Log-normal</td>
<td>λ = -2.99</td>
</tr>
<tr>
<td>Drain efficiency, DR</td>
<td>Uniform</td>
<td>L = 0.0</td>
</tr>
</tbody>
</table>

**PROBABILISTIC SEISMIC DEMAND MODEL**

A surrogate, or meta-model, is an engineering method based on techniques that use data to identify the relationships existing between the quantities of interest, and it is used when an outcome of interest cannot be easily measured directly, making it necessary to use a model of the outcome instead. The basic idea is for the surrogate to act as a "curve fit" to the available data so that results may be predicted without recourse to the use of the expensive simulation code. The meta-model considered herein is within an adaptive scheme, i.e., the functions in the meta-model can change according to the input data to reduce the burden of manual selection of several parameters in the meta-model.

A polynomial response surface (PRS) is a regression technique where an n-dimensional surface that predicts desired responses is developed using a computationally efficient closed-form polynomial function developed from a set number of input variables [20]. The sparse polynomial response surface can be represented as:

\[ y_i = \mathbf{\theta}^T g(\mathbf{X}) + \nu \]  

(1)

In Eq. 1, \( y_i \) is the engineering demand parameter as a result of the finite element model simulation, \( g \) is a column vector that includes explanatory functions that are powers and cross products of powers of the predictors in \( \mathbf{X} \) up to a predefined degree, \( \mathbf{\theta}^T \) is the row vector of model parameters, which are unknown constant coefficients, and \( \nu \) is the model error due to the lack of fit of the surrogate model. In this study, in addition to the original parameters, transformations on the response and predictors are also evaluated, including logarithm and natural logarithm transformations. Therefore, for each model, \( \mathbf{X} \) includes the original predictors as well as their transformed forms.

**Polynomial response surface meta-model**

The response surface methodology in this study is employed to predict the dam’s maximum relative base sliding under seismic loads. By selecting various configurations of the model parameters in Table 1, 250 samples of the FEM were generated with LHS and paired with the selected ground motions. The maximum relative sliding at the base was computed from non-linear simulations, and polynomial response surface meta-models of order 1 to 4 were fitted to the structural response. The final models developed for the responses do not include all the explanatory functions in the initial training set and to select the best
explanatory functions, the stepwise regression algorithm in MATLAB [21-22] is used. This algorithm starts with a constant term to predict the response. In the next step, one explanatory function is added to the model, and the performance of the model is evaluated based on the Bayesian information criterion (BIC) [21-22]. If the model performance improves, the added term is retained; otherwise, it is removed, and this process is repeated until all explanatory function candidates are tested. As a result of this process, the final set of explanatory functions in the final model is a subset of the initial set. The prediction accuracy of the developed models is evaluated with the 5-fold cross-validation method [3], in which the available data set is divided into five subsets, called folds, and the model is trained on a set comprised of four folds, and the left-out fold is used as the test set. Using a new fold as the test set each time, this procedure is repeated five times, and the average root mean square error (RMSE), relative maximum absolute error (RMAE) and coefficient of determination (R^2) value of the five repetitions is used as the model goodness-of-fit. In addition to the 5-fold cross-validation method, which shows the overall accuracy of the model, p-values associated with the explanatory functions are controlled to be smaller than 0.05, assuring that the final terms included in the model are not selected by chance. Table 2 lists the comparison of the predictive capabilities of the PRS meta-model of different orders in terms of goodness-of-fit estimates calculated with the entire training set and from 5-fold cross-validation. As can be seen, an overall good performance of the three meta-models is observed, with the PRS of order 4 being the one that performs best.

<table>
<thead>
<tr>
<th>Table 2. PRS meta-model comparison</th>
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<tr>
<td>PRS order</td>
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<tr>
<td>-----------</td>
</tr>
<tr>
<td>O1</td>
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<tr>
<td>O2</td>
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<tr>
<td>O3</td>
</tr>
<tr>
<td>O4</td>
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Best performing meta-model for maximum relative base sliding prediction

From the above comparison, the selected surrogate is a polynomial response surface of order 4 (PRS-O4) as a function of three model parameters, three seismic intensity measures and their transformations and pairwise products.

\[
y = g(CRC, CRF, DR, PGV, l_a, PGA_v) + v
\]

\[
v \sim \mathcal{N}(0, \sigma^2)
\]

where \(CRC, CRF, DR\) are the model parameters corresponding to the concrete-rock cohesion, the concrete-rock angle of friction and drain efficiency, respectively; \(PGV\) is the peak ground velocity, \(PGA_v\) is the peak ground acceleration in the vertical direction and \(l_a\) is Arias intensity. A normally distributed model error term \(v\), with zero mean and standard deviation equal to the RMSE is added to the selected surrogate model to contemplate the lack of fit. Figure 2 (a) shows that the predicted values with the selected meta-model are in agreement with the simulated dataset, while in Figure 2 (b) it can be seen that the residual normal distribution error hypothesis is respected.

SEISMIC FRAGILITY ANALYSIS

The first step in a seismic fragility analysis is the identification of the limit states that are relevant to the system performance. When subjected to strong ground motion, gravity dams may be damaged in different ways. In recent years, typical damage modes that could lead to the potential collapse of dams after a seismic event have been identified, and seismic damage levels have been established. Preliminary analyses have identified sliding as the critical failure mode for the case study of this dam [7], and other failure modes would only occur after sliding has already been observed. As a result, the limit state considered in this study is sliding at the concrete-to-rock interface at the base of the dam, which is characterized by the sliding damage states presented in Table 3.

<table>
<thead>
<tr>
<th>Table 3. Considered limit states for the case study dam</th>
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<tbody>
<tr>
<td>Sliding damage state</td>
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<tr>
<td>LS0 - Slight</td>
</tr>
<tr>
<td>LS1 - Moderate</td>
</tr>
<tr>
<td>LS2 - Extensive</td>
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<tr>
<td>LS3 - Complete</td>
</tr>
</tbody>
</table>
Dam sample generation and fragility functions

Regarding the generation of the samples where the meta-model will be evaluated to predict the dam’s response, independence between all the model parameters was considered to generate $5 \times 10^5$ samples with LHS. The fragility curves were generated as a function of PGV, because they came up as the structure-independent scalar seismic intensity measure that correlates the most with the analysed damage states [10]. To consider a range of values corresponding to return periods from 700–30000 years at the dam site, the possible values of PGV were bounded as $0.8 \leq \text{PGV} \leq 20 \text{ cm/s}$.

To perform the fragility analysis and obtain the fragility point estimates from the meta-models, the multiple stripes analysis (MSA) methodology [23] was used. The range of considered values of PGV was divided in 100 intervals and the fragility point estimates were generated for each limit state. It should be noted that for a specific site, the seismic IMs are correlated. For the dam in this study, and considering the 250 GMTS used to train the meta-models, a linear correlation is to be expected as it can be seen from Figure 3. Based on this, the samples of these parameters are taken from a jointly log-normal distribution with their respective correlation coefficients. Regarding the modelling parameters CRC, CRF and DR, their uncertainty was propagated in the analysis by considering a range of usable values listed in Table 1.

![Figure 2. PRS-O4 (a) predicted vs. simulated values and (b) residuals histograms](image1)

![Figure 3. Linear correlation between the seismic IMs from the GMTS (a) PGV–I_a (b) PGA_V–I_a and (c) PGV–PGA_V](image2)
To compute the fragility, a Weibull cumulative distribution function (CDF) was fitted to the data points according to Eq. 4, in which $\beta > 0$ is the shape parameter and $\theta > 0$ is the scale parameter.

$$P_f(PGV) = 1 - \exp\left[-\frac{(PGV/\theta)^\beta}{\theta}\right]$$ (4)

Following the recommendations of Baker [23], when using MSA, the maximum likelihood estimation (MLE) method was employed to fit Eq. 4 to the fragility point estimates and to approximate its parameters. Figure 4 presents the obtained fragility curves.

![Figure 4. Fragility curves for the case study dam](image)

Model parameters uncertainty impact in the fragility analysis

In the fragility curves shown in the section above, the uncertainty related to the model parameters present in the meta-model is propagated in the analysis by sampling from the probability density function with LHS. In the same manner, and to explicitly account for the variability in the modelling parameters, Figure 5 shows the fragility curves for each limit state in red; the shaded areas represent the fragility curves generated with the values corresponding to the 95% confidence interval of the usable range of the modelling parameter values (Table 1). A Weibull CDF was used in all cases except for the concrete–rock cohesion, where a log-normal CDF provided better goodness-of-fit. It can be seen that the concrete–rock cohesion variability introduces the most uncertainty in the fragility analysis, followed by the drain efficiency and, to a lesser extent, the concrete–rock angle of friction.

CONCLUSIONS

The main objective of this study was to train PRS meta-models of various orders for the seismic fragility assessment of gravity dams and to select the best-performing model for the generation of fragility functions displaying the effect of the model parameter variation on the dam’s seismic response. Polynomial response surface meta-models of order 1 to 4 were fitted to the seismic response of the studied dam selecting as engineering demand parameter the maximum relative sliding at the base of the dam. The 250 ground motion time series used for the analysis was selected with the GCIM method considering the horizontal and vertical spectral acceleration and PGV. The dataset used to train the surrogate model was generated by pairing the LHS sampling of the modelling parameters with the ground motion records. Stepwise regression was used to identify the best set of covariates to train the surrogates. The performance of the proposed meta-models was evaluated based on the goodness-of-fit estimates from the 5-fold cross-validation. The PRS-O4 came up as the optimal surrogate model to predict base sliding seismic response of the studied dam, and it was used to generate general fragility curves as a function of PGV considering the correlation between the seismic IMs for a specific site and by sampling from a multivariate log-normal distribution. To explicitly account for the variability of the modelling parameters in the fragility analysis, fragility curves were generated by propagating the uncertainty of these parameters and by considering the extreme values of the usable range of values. It was observed that the modelling parameter affecting the fragility analysis the most is the concrete–rock cohesion. Finally, it should be noted that machine learning techniques are indispensable when assessing the vulnerability of structures with computationally expensive FEM. Likewise, the use of surrogate models allows the exploration of the impact of different parameters in the fragility without the costly re-evaluation of the FEM simulations. Nevertheless, a more thoroughgoing exploration of various machine learning techniques to develop surrogate or meta-models that efficiently approximate the
seismic response of the dam, besides the PRS method, should be performed to determine the most viable technique for a seismic fragility analysis.

Figure 5. Fragility curves and 95% confidence interval of the modelling parameters values for the case-study dam (a) LS0 (b) LS1 (c) LS2 and (d) LS3

ACKNOWLEDGMENTS

The authors acknowledge the financial support of the Natural Sciences and Engineering Research Council of Canada (NSERC), the Fonds de recherche du Québec – Nature et technologies (FRQNT), the Centre d’études interuniversitaire des structures sous charges extrêmes (CEISCE) and the Centre de recherche en génie parasismique et en dynamique des structures (CRGP). Computational resources for this project were provided by Compute Canada and Calcul Québec. Any opinions, findings, and conclusions or recommendations expressed in this paper are those of the authors and do not necessarily reflect the views of the sponsors.

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