



## PROBABILISTIC ASSESSMENT OF COINCIDENCE OF EARTHQUAKE DAMAGE TO COLLOCATED LIFELINES

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### ABSTRACT

Earthquake damages to collocated lifeline facilities cause damage spread as well as conflicts in repair works. In this study, a probabilistic model has been developed to assess coincidence of damage to collocated facilities. First, occurrence rate of coincidence of damage to two different systems is derived assuming uniform rate of damage occurrence. Next, the basic model is extended to deal with non-uniform damage rate which spatially varies depending on seismic intensities and vulnerability of facilities. A case study is carried out for water delivery systems and sewer systems in Chiba prefecture, Japan.

### Introduction

Lifeline systems collectively support people's daily lives and socioeconomic activities in modern cities. In spite of recent development of earthquake countermeasures, lifeline systems are still prone to suffer damage in an earthquake, resulting in subsequent loss of urban functions that causes significant societal impacts. One of the critical issues in lifeline earthquake disaster is the problem of system interactions. Compound, cascade and/or feedback effects among lifeline damage exacerbate the extent and significance of loss of functions and also interrupt the recovery process. Such problems were clearly demonstrated in an unprecedented scale in The Hanshin-Awaji (Kobe) earthquake disaster, 1995, Japan (Nojima and Kameda, 1996). Rinaldi et al. (2001) categorized critical infrastructure interdependencies into physical, cyber, geographic and logical ones. Chang et al. (2009) categorized societal impacts of infrastructure failure interdependencies (IFIs) into economic, environmental, health, safety and social ones.

Among various aspects of lifeline system interactions, this study focuses on problems arising from geographical proximity of underground pipelines. Since large part of pipelines of water delivery, city gas supply and sewer systems are mainly buried under roads or streets, those facilities are collocated (i.e., closely located from each other). When elements of multiple infrastructures are located in close spatial proximity, a geographic interdependency occurs (Rinaldi et al., 2001). For example, leak of water can wash out a road, gas leak can cause gas explosion, resulting in damage to neighboring facilities; leak of wastewater can contaminate

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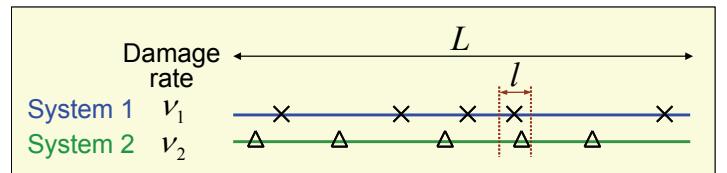
water (Edwards, 2008). When pipe damages coincidentally occur to the collocated facilities, conflicts among multiple lifeline service providers in recovery work are serious concerns as well. Organizational coordination across those involved providers may be required so as to avoid such conflicts, making recovery plans less effective than unrestricted ones. However, little attention has been paid to quantitative assessment of such situations.

In this view of the problem, a probabilistic model is developed in this study for assessment of coincidence of damage to collocated facilities which may cause system interactions in earthquake disasters. The proposed model combines individual damage rate for involved systems in a probabilistic manner. The occurrence rate of coincidence of damage to two different systems is first derived assuming uniform rate of damage occurrence, and next assuming non-uniform damage rate. A numerical example for Chiba prefecture, Japan is shown to demonstrate the applicability of the proposed model.

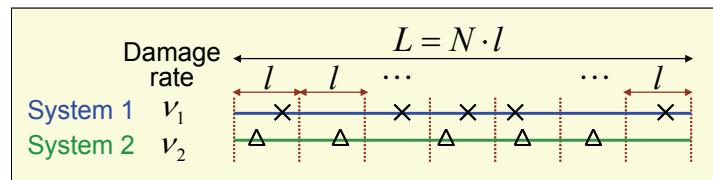
### Occurrence Rate of Coincident Damage to Two Systems with Uniform Damage Rates

#### Modeling and Formulation

Consider two lifeline systems (System 1 and 2) within a certain region. In reality, these systems are geographically distributed networks. In this study, for simplification, these systems are modeled as one-dimensional linear facilities closely collocated as shown in Fig. 1. Assume that occurrence of damages is mutually independent and random over the entire length of  $L$  with uniform rates of damage occurrence (= expected number of damage in a unit length) of  $v_1$  and  $v_2$ , respectively. Occurrence of damage to the two systems is also mutually independent.



(a) Coincidence of damage within an infinitesimal length of  $l$



(b) Coincidence of damage within a segment of length of  $l$

Figure 1. Models for coincident damage to two lifeline systems with the entire length  $L$

#### Coincidence of Damage within an Infinitesimal Length $l$

First, consider coincident damage (= a situation where damages occur to both System 1 and 2) within an infinitesimal length of  $l$  as shown in Fig. 1(a). The occurrence rate of coincident damage is obtained as follows under the condition that  $v_1 l$  and  $v_2 l$  are small enough.

$$\nu_1(1 - e^{-\nu_2 l}) \approx \nu_1 \nu_2 l \quad \text{or} \quad \nu_2(1 - e^{-\nu_1 l}) \approx \nu_1 \nu_2 l \quad (1)$$

Since the occurrence rate of coincident damage is given by the product of the damage rates of the two systems and the prescribed length  $l$ , it can be said that there is a multiplier effect to coincident damage. Especially, in catastrophic disaster, lifeline interactions such as damage spread and recovery interruption may be exacerbated, because both  $\nu_1$  and  $\nu_2$  take on high value.

From the discussion above, the probability distribution of the number,  $k$ , of coincident damage to two systems over the entire length  $L$  follows Poisson distribution with parameter  $\nu_1 \nu_2 l L$ . The probability mass function is represented by:

$$P(k) = \frac{(\nu_1 \nu_2 l L)^k e^{-\nu_1 \nu_2 l L}}{k!} \quad (2)$$

The expected number of coincident damage and its standard deviation are given by  $\mu_k = \nu_1 \nu_2 l L$  and  $\sigma_k = \sqrt{\nu_1 \nu_2 l L}$ , respectively.

### **Coincidence of Damage within a Segment of Length of $l$**

In the previous subsection, the prescribed length  $l$  is assumed to be infinitesimal so that  $\nu_1 l$  and  $\nu_2 l$  are small enough. In actual situations, however, system interactions can be induced even though two damages occur within a certain distance. For this reason, a segmented system composed of  $N$  segments with length of  $l$  ( $=L/N$ ) are considered as shown in Fig. 1(b). Assuming that occurrence of damages is random and mutually independent with uniform rates of  $\nu_1$  and  $\nu_2$  over the entire length, the probability of occurrence of coincident damage to each segment is identically represented by the product of damage probability of System 1 and that of System 2. As shown in Eq. 3, the multiplier effect which is similar to Eq. 1 is observed herein.

$$p_s = (1 - e^{-\nu_1 l})(1 - e^{-\nu_2 l}) \quad (3)$$

From the assumption, the sequence of occurrence and non-occurrence of coincident damage in each segment constitutes Bernoulli sequence of  $N$  trials with the probability represented by Eq. 4. Therefore, the probability distribution of the number,  $N_s$ , of segments where coincident damage occurs follows binomial distribution with parameter  $p_s$ . The probability mass function is represented by:

$$P(N_s) = \binom{N}{N_s} p_s^{N_s} (1 - p_s)^{N - N_s} \quad (4)$$

The expected number of segments where coincident damage occurs and its standard deviation are given by  $\mu_{N_s} = N \cdot p_s$  and  $\sigma_{N_s} = \sqrt{N(1 - p_s)p_s}$ , respectively.

## Numerical Examples

Numerical examples of probability distributions derived in the previous section are shown for two systems with the entire length  $L=100$ . Table 1 compiles the occurrence rates of damage to System 1 and 2 for four cases (a)-(d) studied herein. Case (a) is the base case with  $v_1=v_2=0.1$ . The damage rates are one fifth in Case (b) and fivefold in Case (c). Different values of damage rates for System 1 and 2 are set in Case (d), keeping their product to be 0.01.

Table 1. Setting of damage occurrence rates of two systems.

Case	$v_1$	$v_2$	Expected value $v_1v_2lL$	Standard deviation $\sqrt{v_1v_2lL}$
(a)	0.1	0.1	1	1
(b)	0.02	0.02	0.04	0.2
(c)	0.5	0.5	25	5
(d)	0.5	0.02	1	1

## Poisson Distribution

Fig. 2 compares Poisson distributions represented by Eq. 2 for the number of coincident damage  $k$ . In this case study, infinitesimal length is set as  $l=1$ . The expected number and the standard deviation are shown in Table 1. Case (a) resulted in zero to four coincident damages. In Cases (b) and (c), multiplier effects are clearly observed where less and more coincident damages than in Case (a) occur, respectively. Case (d) gives exactly identical result with the base Case(a) because of the same vales of  $v_1v_2$ .

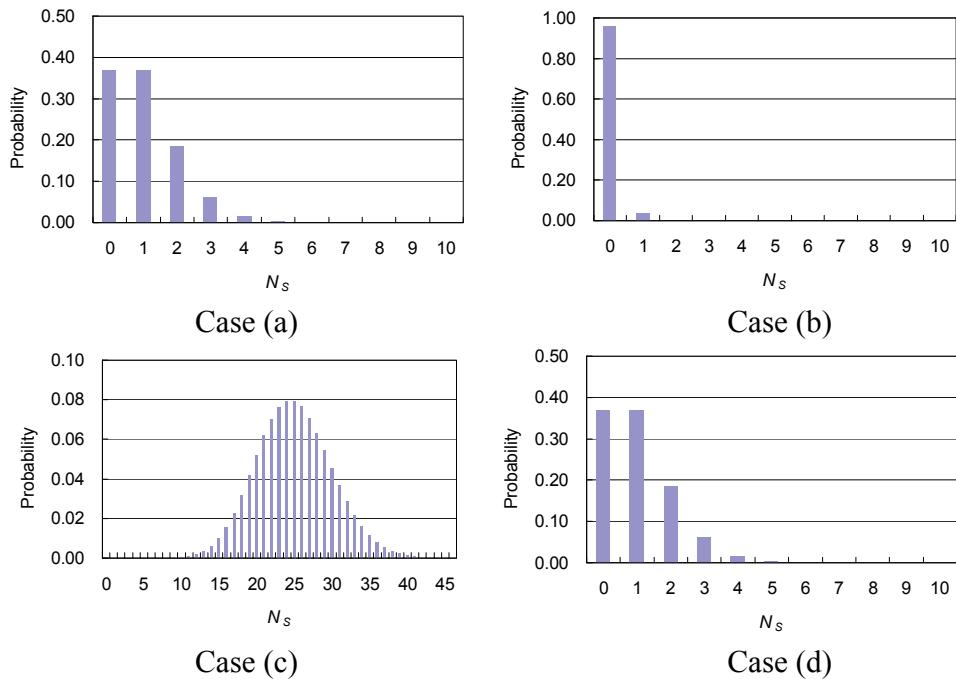


Figure 2. Poisson distributions for the number of coincident damage. (Values sums up to unity.)

## Binomial Distribution

Fig. 3 compares binomial distributions represented by Eq. 4 for the number of segments  $N_S$  where coincident damage occurs. The vertical axis represents  $N_S$ , and the horizontal axis represents the number of segments  $N (=L/l)$  ranging from 1 to 50. Since the entire length  $L=100$ , the segment length  $l (=L/N)$  ranges from 100 to 2).

In the base Case (a), almost all the segments suffer coincident damages when the number of segments  $N$  is small, because the segment length  $l$  is long enough to be almost certainly damaged. Although  $N_S$  increases with increasing  $N$ , the variance increases as well. As a result,  $\mu_{N_S} = 4$  peaks at  $N=8$ . In Case (b), the probability distribution shifts downward, and  $\mu_{N_S} = 0.8$  peaks at  $N=2$ . In Case (c), on the contrary, the probability distribution shifts upward, and  $\mu_{N_S} = 20$  peaks at  $N=40$ . The probability distribution shifts downward also in Case (d), and  $\mu_{N_S} = 1.8$  peaks at  $N=12$ . In binomial distributions, unlike in Poisson distributions which are dependent on the product of the two damage rates, results of Cases (a) and (d) are not identical; the difference of two damage rates also affects the chance of coincident damage. However, binomial distributions asymptotically approach to Poisson distributions with large  $N$  (=short  $l$ ), and  $\mu_{N_S}$  approaches  $\mu_k$ .

$$\mu_{N_S} \approx N \cdot \nu_1 l \cdot \nu_2 l = \nu_1 \nu_2 l L = \mu_k \quad (5)$$

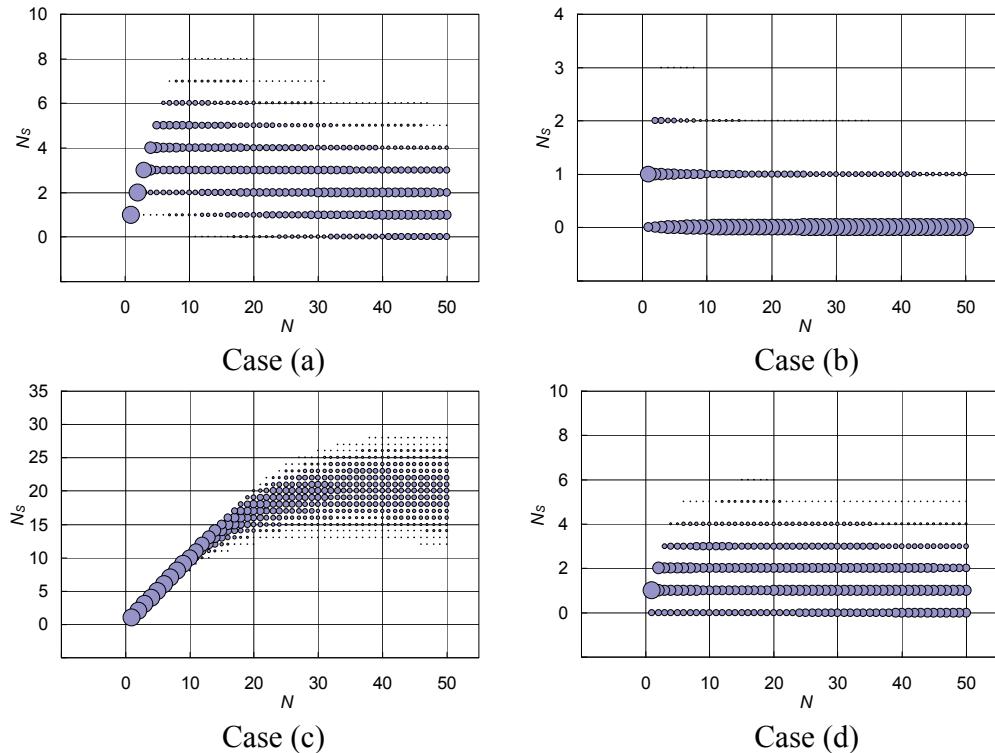


Figure 3. Binomial distributions for the number of segments where coincident damage occurs. (Circle symbols represent the values of probability mass whose summation in the vertical direction sum up to unity.)

## Occurrence Rate of Coincident Damage to Two Systems with Non-uniform Damage Rates

In the previous chapter, damage rates are assumed to be uniform over the entire system. However, in reality, damage rates of lifelines vary location to location across the service area covered by the extended networks according to spatial distributions of seismic intensity, soil (or ground) condition and seismic vulnerability of facilities. Let  $I(x)$ ,  $S(x)$  and  $V(x)$  denote the three main factors (*Intensity*, *Soil* and *Vulnerability*) to the non-uniform damage rates, respectively, as functions of location  $x$ . Widely-used formula for estimation of the number of damage to pipelines are in the form of the following expression.

$$\nu_1(x) = R_1(I(x), S(x)) \cdot V_1(x) \quad (6)$$

$$\nu_2(x) = R_2(I(x), S(x)) \cdot V_2(x) \quad (7)$$

where  $R(\ )$  represents standard damage rate function for specified pipe material and pipe diameter, and subscripts 1 and 2 corresponds to System 1 and 2, respectively. The two systems have their own characteristic standard damage rate function  $R(\ )$  and vulnerability factor  $V(x)$ . On the other hand, seismic intensity  $I(x)$  and soil condition  $S(x)$  are common to the two systems, because they are closely located and little difference is expected between them in terms of  $I(x)$  and  $S(x)$ . Therefore, two damage rates  $\nu_1(x)$  and  $\nu_2(x)$  are spatially correlated due to these common factors. At a particular site  $x$  with strong ground motion and/or bad ground condition, both  $\nu_1(x)$  and  $\nu_2(x)$  take on high values; therefore, both systems tend to suffer damage.

Now let  $E_1(x)$  and  $E_2(x)$  represent the events of damage occurrence to System 1 and 2 at the site  $x$ . Even though there exists spatial correlation in damage distribution of the two systems, it can be assumed that the two events,  $E_1(x)$  and  $E_2(x)$ , are statistically independent. Since the correlation between  $\nu_1(x)$  and  $\nu_2(x)$  does not stem from causal relationships but from the common factors mentioned above, damage occurrence in one system does not influence that in the other system. In probabilistic terms,  $P(E_1|E_2) = P(E_1)$  and  $P(E_2|E_1) = P(E_2)$  hold at arbitrary site, which means statistical independence between the two events. Therefore,  $P(E_1E_2) = P(E_1|E_2)P(E_2) = P(E_2|E_1)P(E_1) = P(E_1)P(E_2)$  holds from location to location.

Based on the discussion above, the results derived in the previous section can be extended so as to deal with the general case with non-uniform damage rates. As shown in Fig. 4, spatially distributed occurrence rate of coincident damage can be defined as  $\nu_1(x)\nu_2(x)/l$ , which is a general form of Eq. 1. By piecewise application of non-stationary Poisson process, the number of coincident damage can be evaluated using Eq. 2. When dealing with segmented systems, the probability of coincident damage at a site  $x$  is obtained by Eq. 8.

$$p_s(x) = \{1 - e^{-\nu_1(x)/l}\} \{1 - e^{-\nu_2(x)/l}\} \quad (8)$$

The spatial contrast in distribution of coincident damages should become more apparent than in the case of uniform damage rate, potentially bringing about significant system interactions especially in a catastrophic disaster. In the next section, numerical examples are shown using Eq. 8.

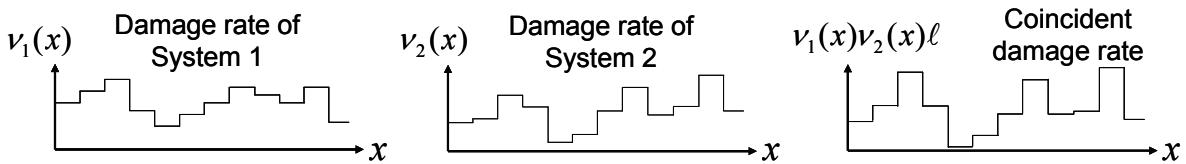


Figure 4. Spatially varying damage rates of two systems and their coincident damage rate

### Case Study of Coincident Damage to Water Delivery System and Sewer System in Chiba Prefecture, Japan

#### Outline of Data Taken from Earthquake Damage Assessment

Water delivery system and sewer system play vital roles of supply and disposal of water, being analogized to artery and vein of a human body. Disruption of any one of them may lead to malfunction of daily lives and socio economic activities. Also, there are unique and significant features of system interactions between the two systems. When sewer system fails, restrictions may be placed on use of water, even though water delivery system has not been affected or has been restored rapidly. Leak of sewage from damaged sewers can cause ground water contamination, and if pipe breaks occur to water distribution system in the neighborhood, sewage may invade into water pipelines. In this view of the problem, a case study has been carried out for water delivery system and sewer system as a practical application of the probabilistic assessment model of coincident damage.

The study area is Chiba prefecture, Japan, one of four prefectures in Tokyo metropolitan area. The hypothetical Northern Tokyo Bay earthquake ( $M=7.3$ ) was adopted for the case study. Basic data, taken from those prepared for earthquake damage assessment (Chiba Prefectural Government Offices, 2008), were compiled on grid cells of the size of longitude  $11.25''$  by latitude  $7.5''$  which is approximately 250m by 250m. The total number of grid cells (80,315) covers the entire area ( $5,157\text{km}^2$ ) of Chiba prefecture composed of 56 municipalities. Fig. 5 shows a distribution of JMA (Japan Meteorological Agency) seismic intensity and that of liquefaction potential  $P_L$ . Distributions of pipeline extension (km) for water distribution networks (23,600km, penetration ratio=93%) and sewer networks (14,400km, penetration ratio=64%) are shown in Fig. 6(a) and Fig. 7(a), respectively.

Fig. 6(b) and (c) show damage rate and the number of damages of water distribution networks. The highest damage rate is 4.94 breaks/km, the largest number of damage is 60 breaks/cell, and the estimated total number of damage is 8,135. As for sewer networks, meandering of pipeline and laxation of joints, which are uncountable, are dominant damage modes rather than countable pipe breaks and joint failures. Therefore, seismic damage is usually measured in terms of damaged pipe length (km), and damage rate (%) is defined as damaged pipe length (km) divided by pipeline extension (km). Fig. 7(b) and (c) show damage rate (%) and the damaged pipe length (km) of sewer networks. The highest damage rate is 16%, the longest damaged pipe length is 1 km/cell, and total damaged pipe length is estimated as 428.8km.

## Analysis and Evaluation of Coincident Damage

In order to apply the assessment model proposed in this study, uncountable damaged pipe length of sewer lines has been converted to equivalent number of damage that is countable so as to be incorporated in Eq. 8. On the basis of judgment by Ministry of Land, Infrastructure, Transport and Tourism (MLIT, 2005), equivalent damaged pipe length of  $L_S = 25\text{m}$  is regarded as one damage. As for the segment length  $l$  in Eq. 8, opinions of domain experts were collected. As far as individual repair work is concerned, system interaction such as a conflict in recovery effort between two systems (namely, simultaneous repair work is impossible) occurs within relatively short distance. So the value of  $l$  should be in the order of ten meters. On the contrary, if damage investigation and/or recovery works are conducted with exclusively occupying a certain area, the value of segment length  $l$  should be as long as several hundred meters. It also applies to a case where damage spread such as water contamination due to sewage leak is concerned. On this basis, those cases with  $l = 10, 25, 50, 100, 250, 500$  and  $1000\text{m}$  were parametrically examined. Since pipeline extension of water distribution lines is different from that of sewer lines, the shorter of them was adopted for the entire length  $L$  in each grid cell.

Evaluated are 41,444 grid cells where both water distribution lines and sewer lines are placed. Fig. 8(a) shows the distribution of the expected number of coincident damages to the two systems when  $l=25\text{m}$ . At this level, the problem of system interactions may not be so serious. Fig. 8 (b) and (c) show the results for  $l=100$  and  $250\text{m}$ , respectively. Relatively many coincident damages occur especially in highly populated regions exposed to strong ground shaking and/or high liquefaction potential. The expected total numbers of coincident damage in Chiba prefecture are 280, 983 and 1926 for  $l = 25, 100$  and  $250\text{m}$ , respectively.

Obviously the number of coincident damages strongly depends on the parameters  $l$  and  $L_S$ . In addition, compared with the numbers of damage to individual systems, those of coincident damages are relatively small and spread over wide regions. Although it is hard to specify the locations where system interactions may occur, the result of the total number coincident damage and its spatial distribution are useful to outline the potential system interactions.

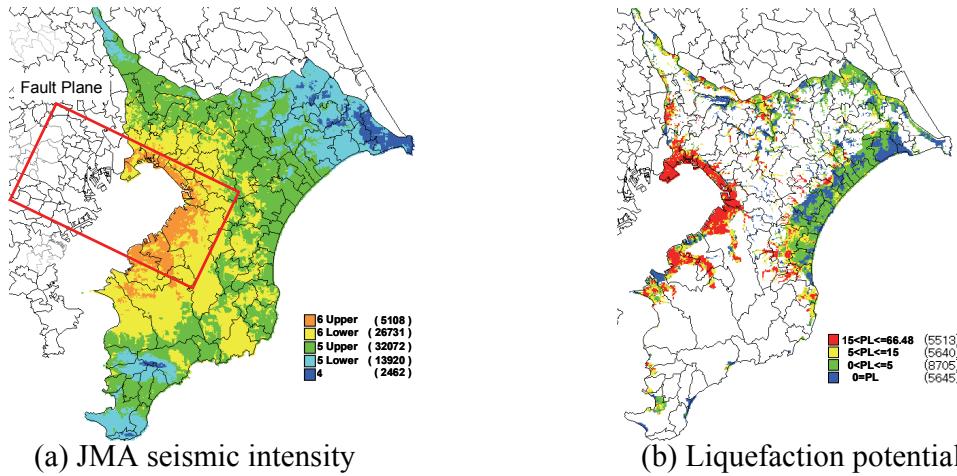


Figure 5. Estimated distributions of seismic intensity and liquefaction potential estimated in the hypothetical Northern Tokyo Bay earthquake.

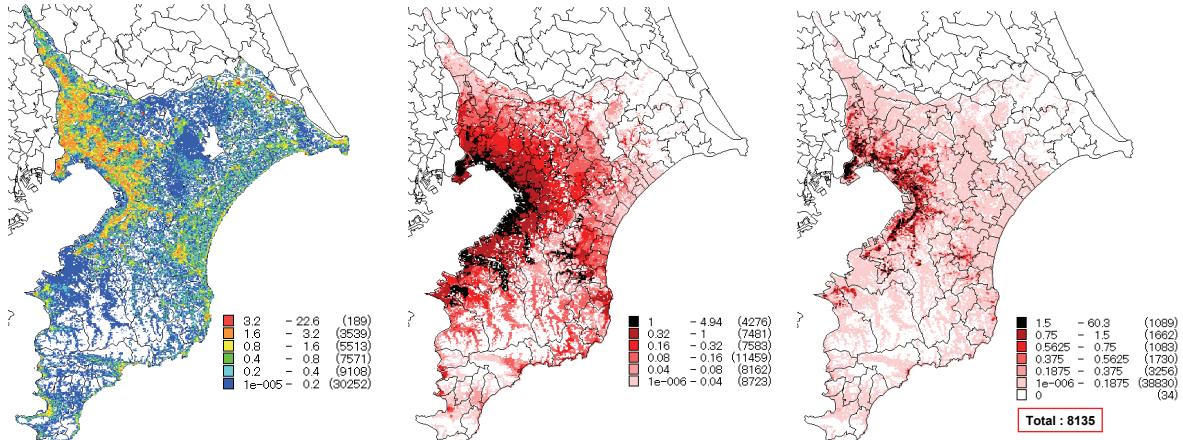


Figure 6. Damage assessment of water distribution networks.

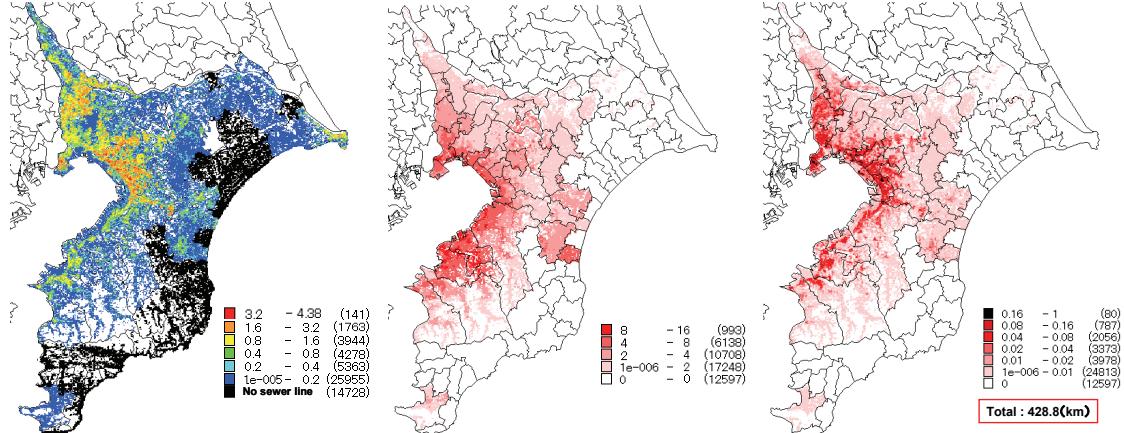


Figure 7. Damage assessment of sewer networks.

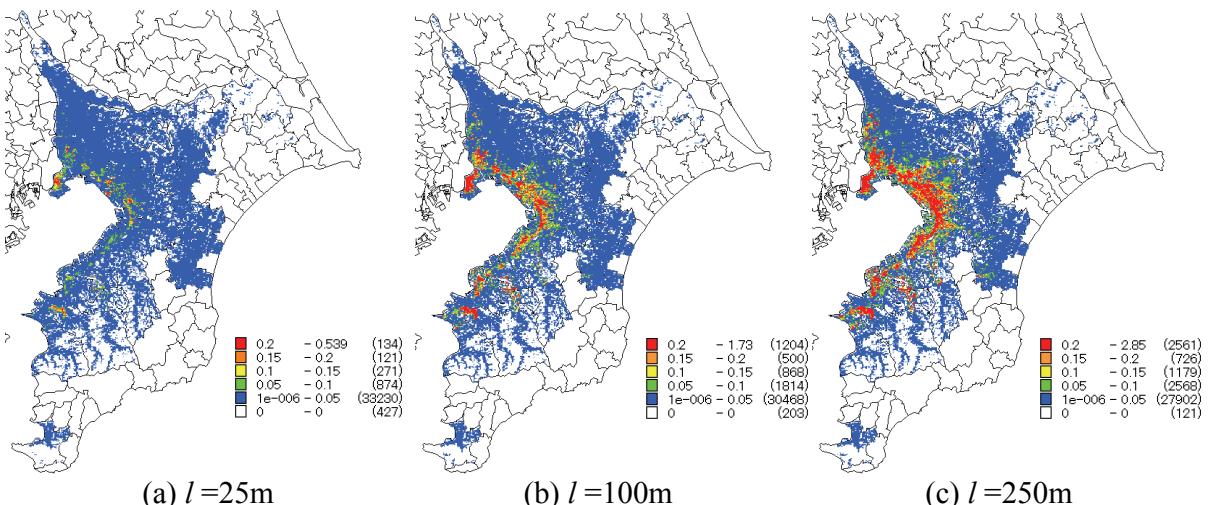


Figure 8. Estimated distributions of coincident damage to water delivery networks and sewer networks in the hypothetical Northern Tokyo Bay earthquake ( $L_S=25\text{m}$ ).

## Conclusions

- 1) A probabilistic model has been proposed for assessment of coincidence of earthquake damage to collocated lifelines. For the fundamental case with an assumption of uniform damage rate, the occurrence rate is represented by the product of the damage rates of the two systems concerned and the infinitesimal length  $l$  within which coincident damage occurs.
- 2) For practical applications, the fundamental model has been extended for non-uniform damage rate and arbitrary segment length  $l$  defining the coincident damage. Even though there exists spatial correlation between the damage rates of the two systems, the proposed model is applicable by piecewise application of non-stationary Poisson process.
- 3) A case study for water delivery systems and sewer systems in Chiba prefecture, Japan, has been carried out. The expected total number of coincident damage to the two systems is in the order of hundred to thousand depending on the prescribed length  $l$ . The results are useful for disaster response planning for avoiding conflicts in recovery efforts.

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