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FORCE, MIXED AND SWITCH DEGREE-OF-FREEDOM CONTROL IN HYBRID SIMULATION

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ABSTRACT

In this paper we present a review of the previous research on force-only and mixed control methods used to implement hybrid simulation tests. We delineate between methods that require calculation of the tangent stiffness matrix and the tangent-free methods and discuss benefits and limitation of each. Similarly, we present a review of the approaches for switched control at a degree of freedom. Then, we explore the feasibility of using subspace methods, such as Krylov subspace, to implement force and mixed control in a hybrid model featuring several multi-degree-of-freedom physical substructures. We also investigate using static equilibrium at the interface nodes for mixed control .We demonstrate the implementation of such methods using the OpenFresco software framework.

Introduction

Hybrid simulation is an experimental method for investigating the response of a structure to dynamic excitation using a hybrid model. A hybrid model is a consistently scaled assembly of computational and physical substructures. Continuity of the hybrid model is maintained at its nodes, where the degrees of freedom (DOF) shared by different substructures of the model are made equal. In a slow hybrid simulation test, where inertia and rate effects on the physical portion of the hybrid model can be neglected, each DOF is uniquely defined by one of the two dependent variables: force or displacement (moment or rotation). Thus, excitation of the physical portion of the structure and measurement of its response, achieved using servo-hydraulic control techniques can be by controlling either force or displacement.

Focusing on a single DOF, such control can be constant in either force or displacement, or switched between force and displacement. Actuation of a DOF of a specimen that is stiff in one direction and soft in another often necessitates such switched control. Alternatively, considering a number of DOFs in the hybrid model, control may be uniform, with all DOFs controlled in force or displacement, or mixed, with some DOFs controlled by force and others by displacement. Finally, mixed control can also be switched.

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Previous Research in Force, Mixed and Switch Control

There have been several attempts to perform force control, mixed control and switch control in recent years. One of the earliest attempts at force control was made by Thewalt and Mahin (Thewalt 1987). He proposed the effective force testing (EFT) method as a way to perform rapid/real-time hybrid testing. The discretized forcing function in Eq. 1 is $\mathbf{f}_n = -\mathbf{M}\ddot{\mathbf{u}}_g(t_n)$ at time step n, which can be known before the start of a simulation once the mass matrix, $\mathbf{M}$, is formulated.

$$\mathbf{M}\ddot{\mathbf{u}}_n + \mathbf{C}\dot{\mathbf{u}}_n + \mathbf{r}_n = \mathbf{f}_n \tag{1}$$

$\mathbf{f}_n$ become the target forces $\breve{\mathbf{f}}_n$ sent to the actuators. Once the forces are applied, the restoring forces $\mathbf{r}_n$, displacements $\mathbf{u}_n$, velocities $\dot{\mathbf{u}}_n$ and accelerations $\ddot{\mathbf{u}}_n$ are measured at each actuator DOF. The algorithm is then advanced to the next time step, $n + 1$.

The obvious advantage of the EFT method is that the target forces are easily computed, $\breve{\mathbf{f}}_n = -\mathbf{M}\ddot{\mathbf{u}}_g(t_n)$. It also tries to take into account the inertial and viscous effects during rapid or real-time testing. These factors make the EFT method a good alternative to shaking table tests. However, the computed effective forces are in the global coordinate system at the discretized nodes. It is unknown how much of these global nodal forces should be applied to the physical specimen in the case where the structural model is sub-structured.

Pan, et al. performed a mixed and switch control hybrid test on High Damping Rubber Bearing (HDRB) base isolators (Pan 2005). They used an eight-story two-span steel moment frame model isolated by HDBRs. The computed portion of the structure was modeled with two DOFs per floor, one horizontal and one vertical. The experimental part of the structure employed one horizontal actuator and one vertical actuator. The horizontal actuator was controlled in position control as the vertical actuator switched between position and force control. Both DOFs were considered to be uncoupled. Hence the horizontal target displacement was computed independently of the vertical target displacement and force.

The predictor displacements for both actuators were calculated using the operator-splitting (OS) predictor-corrector time integration scheme. The predictor horizontal displacement was imposed on the specimen. The measured horizontal force was fed back to the scheme to correct the predictor displacement. The vertical force was computed as a product of the predictor vertical displacement and the vertical elastic stiffness of the bearings. The switching criterion was based on the sign of the computed vertical force, whether it was positive, in tension or negative, in compression. When in tension, the predictor vertical displacement was imposed while the force was applied in compression. They used both the horizontal and vertical components of the 1995 Hyogoken-Nanbu (Kobe) recorded ground motion simultaneously.

Elkhoraibi and Mosalam performed switch control hybrid simulation (Elkhoraibi 2007). They used the secant stiffness estimation with the α-OS time integration scheme to compute the target forces. The DOFs were considered to be uncoupled so the calculation of the secant stiffness was rather straightforward. The secant stiffness was averaged over 100 measured values during an iteration to minimize the errors from the resolution and noise of the measuring devices.

Elkhoraibi and Mosalam used two criteria for switching between displacement and force control, the secant stiffness k and the restoring force r. The thresholds for switching were defined beforehand. The measured stiffness k_i and force r_i were compared against the thresholds for switching. k_d and k_f were the stiffness thresholds for the displacement and the force respectively, where $k_d < k_f$. The restoring thresholds, r_d and r_f, were setup the same way. A buffer zone prevented the back and forth switching caused by small fluctuations in k.

The recent research in this area provided some novel ways of performing hybrid simulation in force, mixed and switch control. One issued that still is not addressed is how to calculate the target forces in multi-degree-of-freedom setups, where the DOFs are coupled. There also remains more work to be done in the area of control.

Force Control Hybrid Simulation Using OpenFresco

One of the advantages of hybrid simulation is that it makes use of existing finite element analysis (FEA) software programs. OpenFresco[4] already supports a wide variety of FEA software packages such as OpenSees, LSDyna, and Abaqus. Most FEA software in structural engineering use displacement-base time integration schemes to solve the equation of motion at each time step. This proves to be quite a challenge when acquiring target forces from a FEA software. One possible solution is to develop a new force-based time integration algorithm and somehow coalesce it into an existing FEA software. However, not all prepackaged FEA software programs allow the addition of custom time integration schemes.

A more viable solution is to use the target displacements from the FEA software to calculate the target forces using OpenFresco. Most time integrators used for hybrid simulation send out the target displacements and receive the measured forces. OpenFresco can receive the target displacements from the FEA software, perform the necessary transformation, convert it to the target forces and send it to the control system. Once the target forces are imposed on the specimen, OpenFresco can receive the measure displacements and forces from the control system. The measured displacements can then be converted to the corresponding forces by OpenFresco and fed back to the FEA software.

The displacement-force converter can be implemented in OpenFresco using the *ExperimentalSignalFilter* abstract class as seen in Figure 1. The *ExperimentalSignalFilter* class modifies the signal before sending it to the control system. It also modifies the signal received from the control system. Two new concrete classes *ExperimentalTangForceConverter* and *ExperimentalKrylovForceConverter* inherit from the parent class *ExperimentalSignalFilter*. These classes convert the target displacements to the target forces. Once this class is instantiated, it is aggregated in to the Experimental Control object.

There are, at least, two types of methods for computing the target force. The first method,

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[4] The Open-source Framework for Experimental Setup and Control (OpenFresco) is an object-oriented software framework for hybrid simulation of structural systems (Schellenberg 2008).

ExperimentalTangForceConverter deploys the tangent stiffness matrix calculated from the *ExperimentalTangentStiff* class in combination with the target displacements to formulate the target forces. Researchers have been looking at alternatives to computing the Jacobian matrix at every iteration of the Newton method because this is sometimes a costly operation. This section explains in detail how some alternatives for computing the Jacobian could be used in hybrid simulation to find the target forces. The second method, *ExperimentalKrylovForceConverter* does not directly use the tangent stiffness matrix. The implementation of this tangent-free method is explored in this section as well.

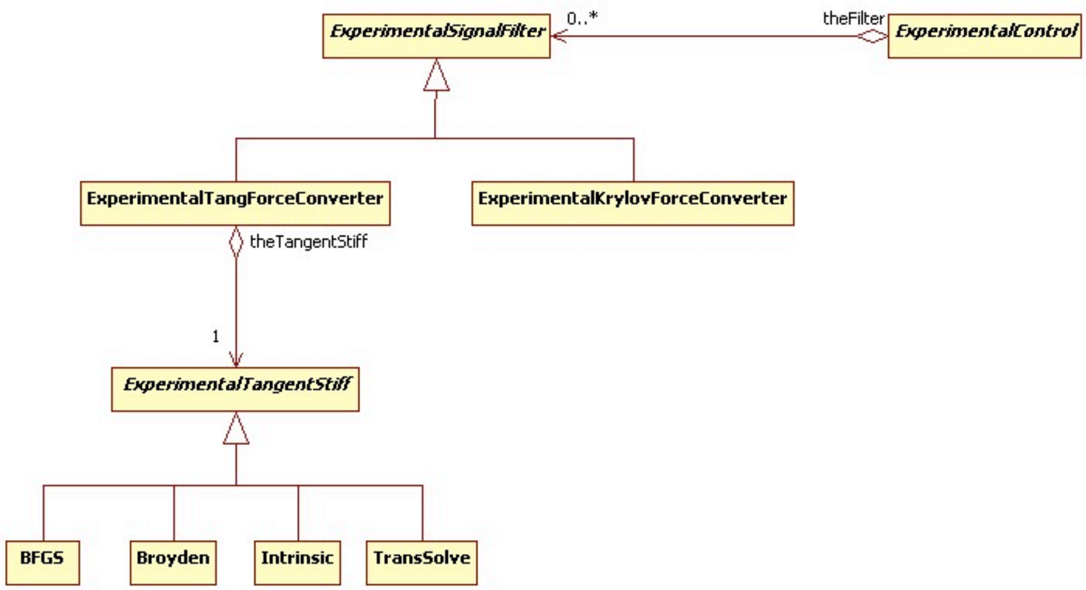

Figure 1. New proposed software architecture for OpenFresco for force control.

Calculating Target Force using Tangent Stiffness Matrix

In the global coordinate system, the resisting force of a structure, **r**, is computed as the product of the structural stiffness matrix, **K**, and the deformation, **u**, **r**(**u**) = **Ku**, in linear structural analysis. In non-linear analysis, the Newton method is commonly used to find the deformation given the input force and the tangent stiffness matrix. The tangent stiffness matrix, $\mathbf{K}_t$, is the Jacobian of the resisting force function, **r**(**u**). In the actuator coordinate system, the forces **f** can be calculated in the same way as the product of the tangent stiffness matrix $\mathbf{K}_t^{Act}$ and the displacements **x,** Eq. 3.

$$\mathbf{f} = \mathbf{K}_t^{Act}\mathbf{x} \tag{3}$$

Once OpenFresco receives the target displacements in the global coordinate system $\breve{\mathbf{u}}$ from the FEA software, it transforms them into actuator target displacements $\breve{\mathbf{x}}$. (The target response quantities are denoted with a breve [□] and the measured response quantities are marked with a hat [ˆ].) The target forces $\breve{\mathbf{f}}$ obtained from Eq. 3 is then applied to the specimen. The control system sends the measured displacements and forces to OpenFresco. The measured displacements $\hat{\mathbf{x}}$ are converted to the corresponding measure forces $\hat{\mathbf{f}}$ via Eq. 3. OpenFresco makes the necessary transformations and feeds it back to the FEA software. $\mathbf{K}_t^{Act}$ is updated at the beginning of each time step from the previous measured displacements and forces before the new target displacements are converted to the target forces. This subsection discusses methods for formulating $\mathbf{K}_t^{Act}$.

There is a subclass of Newton method called the secant or quasi-Newton method. They are called such because no true Newton equation is ever formed throughout all iterations (Klie 2006). In other words, the Jacobian is not directly formulated at all. A secant approximation of the Jacobian is used instead. This approximation is executed through successive rank one updates to the current Jacobian matrix. One of the most well known of these is the Broyden method (Broyden 1965). Igarashi, et al. used the Broyden-Fletcher-Goldfarb-Shanno (BFGS) method to update the tangent stiffness matrix during a hybrid simulation of a 5-story full-scale building (Igarashi 1993). They utilized the stiffness matrix for scaling the actuator movement. The algorithm uses the measured incremental displacement/force pairs at each DOF. BFGS is a variation on the Broyden method. The stiffness matrix is assumed to be symmetric and positive-definite. The algorithm follows the minimal update approach. The incremental stiffness with a smaller Forbenius norm updates the stiffness matrix.

In recent years, researchers have formulated non-secant based methods for estimating the tangent stiffness matrix. Ahmadizadeh and Mosqueda developed such a method to improve the performance of the OS time integration scheme (Ahmadizadeh 2008). They update the stiffness matrix by using the "intrinsic" coordinate system. $\mathbf{K}_n^{Act}$ results from the transformation of the stiffness matrix in the intrinsic coordination system $\mathbf{P}_n$ via the transform matrix $\mathbf{T}_p$. $\mathbf{T}_p$ is used to transform the displacements and forces from the actuator coordinate system to the intrinsic coordinate system. These transformed displacements and forces are used to formulate $\mathbf{P}_n$. The important result of this transformation is that $\mathbf{P}_n$ is now diagonal and the DOFs in the intrinsic coordinate system are decoupled. The intrinsic coordinate system needs to be determined for each experimental setup and specimen. Then, $\mathbf{T}_p$ and $\mathbf{P}_n$ are formulated from the intrinsic coordinate system.

Another non-secant base method was proposed by Hung and El-Tawil (Hung 2008). Their intent was the same as Ahmadizadeh and Mosqueda to improve the OS scheme. They based their algorithm on the assumption that the tangent stiffness matrix does not change drastically between time steps. The matrices, $\Delta\hat{\mathbf{F}}$ and $\Delta\hat{\mathbf{X}}$ are formed from m incremental force vectors $\Delta\hat{\mathbf{f}}$ and displacement vectors $\Delta\hat{\mathbf{x}}$ respectively from time step $n + 1$ to $n + 2 - m$. Now Eq. 4 is formed using $\mathbf{K}_{n+1}^{Act}$ at time step $n + 1$.

$$\Delta\hat{\mathbf{F}} = \mathbf{K}_{n+1}^{Act}\Delta\hat{\mathbf{X}} \tag{4}$$

Both sides of Eq. 7 are transposed. m must be greater than or equal to the number of DOFs. When m equals the number of DOFs, $\mathbf{k}_{n+1}^{i}$, the i^{th} row of $\mathbf{K}_{n+1}^{Act}$, are obtained by Gaussian elimination using $\Delta\mathbf{f}^{i}$, the i^{th} column of $\Delta\hat{\mathbf{F}}$. When m is greater than the number of DOFs, the rows of $\mathbf{K}_{n+1}^{Act}$ are found via the least-squares method.

These four methods were not originally formulated to be used for force control hybrid simulation. However, they can be used for such a purpose because they all try to estimate the tangent stiffness matrix of the specimen. The study of the performance of these methods in force control is one of the topics left for future work.

Tangent-Free Method for Calculating Target Force

Using the tangent stiffness matrix to acquire the target force is promising, but these methods do have an inherent difficulty of overcoming spurious updates caused by the noises from the measuring instruments. There are also some questions as to how accurate these estimates are. A more ideal way would be to not use the tangent stiffness matrix to find the target forces. This section describes a subspace method for acquiring the target forces directly from $\Delta\hat{\mathbf{x}}$ and $\Delta\hat{\mathbf{f}}$.

Scott and Fenves used a Krylov subspace to accelerate the Newton method. In a similar way, a Krylov subspace can become the basis for the target forces (Scott 2006). Starting with Eq. 5 where the flexibility matrix in the actuator coordinate system $\mathbf{F}^{Act}$ is the inverse of the stiffness matrix $\mathbf{K}^{Act}$

$$\mathbf{0} = \Delta\breve{\mathbf{x}}_{n+1} - \mathbf{F}^{Act}\Delta\breve{\mathbf{f}}_{n+1} \tag{5}$$

where $\Delta\breve{\mathbf{x}}_{n+1}$ is computed by the FEA software. $\Delta\breve{\mathbf{f}}_{n+1}$ can be seen as an addition of two vectors $\mathbf{p}_{n+1}$ and $\mathbf{q}_{n+1}$.

$$\Delta\breve{\mathbf{f}}_{n+1} = \mathbf{p}_{n+1} + \mathbf{q}_{n+1} \tag{6}$$

Now Eq. 5 becomes

$$\mathbf{0} = \Delta\breve{\mathbf{x}}_{n+1} - \mathbf{F}^{Act}\Delta\mathbf{p}_{n+1} - \mathbf{F}^{Act}\Delta\mathbf{q}_{n+1} \tag{7}$$

The key to this algorithm is to minimize the 2-norm of the difference of $\Delta\breve{\mathbf{x}}_{n+1}$ and $\mathbf{F}^{Act}\mathbf{p}_{n+1}$.

$$\min\left\|\Delta\mathbf{x}_{n+1} - \mathbf{F}^{Act}\mathbf{p}_{n+1}\right\|_2 \tag{8}$$

$\mathbf{p}_{n+1}$ is assumed to be a linear combination of the previous force increment vectors, $\Delta\breve{\mathbf{f}}_1, \Delta\breve{\mathbf{f}}_2, \ldots, \Delta\breve{\mathbf{f}}_n$.

$$\mathbf{p}_{n+1} = c_1\Delta\breve{\mathbf{f}}_1 + c_2\Delta\breve{\mathbf{f}}_2 + \ldots + c_n\Delta\breve{\mathbf{f}}_n \tag{9}$$

Now then, the vector $\mathbf{F}^{Act}\mathbf{p}_{n+1}$ is

$$\mathbf{F}^{Act}\mathbf{p}_{n+1} = c_1\mathbf{F}^{Act}\Delta\breve{\mathbf{f}}_1 + c_2\mathbf{F}^{Act}\Delta\breve{\mathbf{f}}_2 + \cdots + c_n\mathbf{F}^{Act}\Delta\breve{\mathbf{f}}_n \tag{10}$$

From Eq. 5, $\mathbf{F}^{Act}\Delta\breve{\mathbf{f}}_1, \mathbf{F}^{Act}\Delta\breve{\mathbf{f}}_2, \cdots, \mathbf{F}^{Act}\Delta\breve{\mathbf{f}}_n$ are estimated by the previous measured incremental target displacements $\Delta\hat{\mathbf{x}}_1, \Delta\hat{\mathbf{x}}_2, \cdots, \Delta\hat{\mathbf{x}}_n$. Solving for the constants $c_1, c_2, \cdots, c_n$ using Eq. 8 becomes a least squares problem. Using $c_1, c_2, \cdots, c_n$, $\mathbf{F}^{Act}\mathbf{p}_{n+1}$ is calculated through Eq. 10. The least square residual $\mathbf{v}_{n+1}$ is then

$$\mathbf{v}_{n+1} = \Delta\breve{\mathbf{x}}_{n+1} - \mathbf{F}^{Act}\mathbf{p}_{n+1} \tag{11}$$

Eq. 5 is rewritten in terms of $\mathbf{v}_{n+1}$ as

$$\mathbf{0} = \mathbf{v}_{n+1} - \mathbf{F}^{Act}\mathbf{q}_{n+1} \tag{12}$$

$\mathbf{q}_{n+1}$ is solved using the stiffness matrix $\mathbf{K}$.

$$\mathbf{q}_{n+1} = \mathbf{K}^{Act}\mathbf{v}_{n+1} \tag{13}$$

Finally, the incremental target force is found via Eq. 6.

The only time $\mathbf{F}^{Act}$ or $\mathbf{K}^{Act}$ are used is in Eq. 13. Either the initial stiffness matrix $\mathbf{K}_0^{Act}$ or the tangent stiffness matrix $\mathbf{K}_t^{Act}$ can be used in Eq. 13. The previous section discusses methods for estimating the tangent matrix. There is also the issue of how large the subspace needs to be. In other words, how many incremental measure displacements vectors are required to use to form an adequate subspace. This is reserved for future investigation.

Mixed Control Hybrid Simulation Using OpenFresco

Methods for mixed control hybrid simulation are closely related to the methods for force control. Instead of all DOFs being in force control, some actuator DOFs are in force control while the rest in displacement control, hence the term mixed. The displacement-controlled actuators receive the unconverted target displacements to impose on the specimen. At the same time, the force-controlled actuators impose the converted target forces from OpenFresco. The resisting forces and the displacements are measured at all DOFs and sent back to OpenFresco. The measure displacements in the force DOFs are converted to the corresponding measured forces in OpenFresco and fed back to the FEA software. The measured forces from the displacement DOFs are unmodified and fed back to the FEA software. Mixed control algorithms can be implemented in OpenFresco using the *ExperimentalTangForceConverter*, *ExperimentalKrylovForceConverter* and *ExperimentalControl* classes. This section discusses this implementation. One type uses the tangent stiffness matrix directly while the other does not.

Mixed Control using Tangent Stiffness Matrix

Few modifications are needed to the force control tangent stiffness matrix method to implement the mixed control method. The force control tangent stiffness method is utilized to find the target forces for all the actuator DOFs. The *ExperimentalControl* object ensures that the target forces are imposed only in the force-controlled actuator DOFs and the target displacements calculated from the FEA software are imposed in the displacement-controlled actuator DOFs. The same tangent stiffness matrix is deployed once again to convert the measured displacements from the force DOFs to corresponding measured forces to feed back to the FEA software while the measure forces from the displacement DOFs are fed back without any modification. The measured displacements and forces at all DOFs from the previous time step are used to update $\mathbf{K}_t^{Act}$ at the beginning of the current time step before the target displacements are converted.

Tangent-Free Mixed Control

The Krylov subspace method for force control can be implemented for mixed control as well. It is used to calculate the target forces for all the actuator DOFs. However, only the force-controlled actuators impose the computed target forces while the displacement-controlled actuators impose the unconverted target displacements on the specimen. The feedback process works the same way as the mixed control tangent stiffness method.

Another implementation is to not have the force controlled DOFs in the finite element model at all. A specimen is often loaded with force in the direction where it is very stiff. Therefore, the deformation in the force DOFs are comparatively small to the deformation in the displacement DOFs. For example, an axially loaded beam-columns are analyzed without axial deformations in the model shown Figure 2 (left). The red beam column is simulated experimentally while the rest in black are simulated numerically. The axial load is the target force to be imposed on the specimen. The axial load is calculated using static equilibrium. The target displacements $\breve{\mathbf{u}}_{n+1}$at time step n + 1 are imposed on the FEA model as depicted in Figure 2 (right).

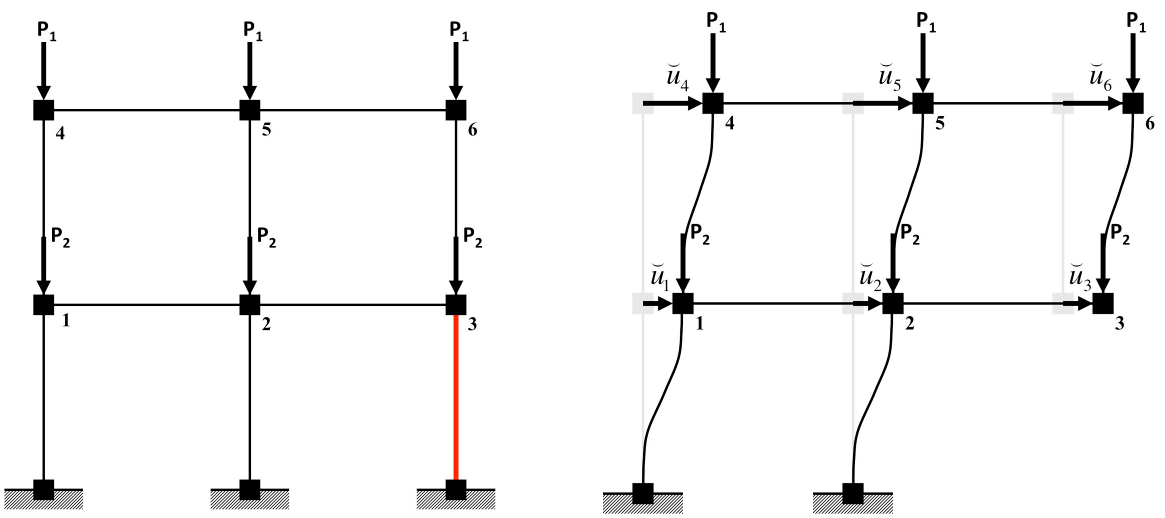

Figure 2. Mixed control example model (left) and deformed model (right) at n + 1.

The forces are summed up in the axial direction at the interface nodes, the nodes where the analytical model is joined to the physical specimen. The interface node is Node 3 in this example as seen in Figure 3 (left). The target force $\breve{r}_{n+1}$ from Eq. 19 is obtained from the unbalance force in the axial direction at the interface node.

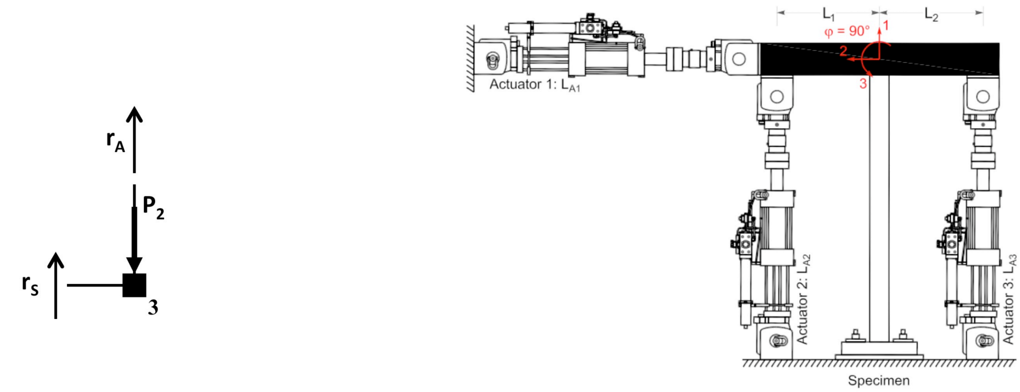

Figure 3. Forces summed up at interface Node 3 at $n + 1$ (left) and three actuator experimental setup (right).

$$\breve{r}_{n+1} = -(r_S + r_A - P_2) \tag{19}$$

The target displacement and force are sent to the OpenFresco. The control system with an experimental setup such as Figure 3 (right) imposes these displacement and force on the specimen. In general, the experimental setups should be such that the force-controlled DOFs are orthogonal to the displacement-controlled DOFs to minimize the contribution of one to the other. The forces are measured in the displacement DOFs and fed back to the FEA software.

The implementation details for this method differ from the previous implementations. The modifications are made mostly in the FEA software and not so much in OpenFresco. The FEA software needs to perform an extra step of computing the target forces via static equilibrium at the interface nodes. Then it sends $\breve{\mathbf{u}}_{n+1}$ and $\breve{\mathbf{r}}_{n+1}$ to OpenFresco. OpenFresco executes the necessary transformations and sends it to the control system without modifying the signals. OpenFresco then receives the measured forces and displacements from the control system. Open-Fresco relays the measure forces to the FEA software to advance the analysis to time step $n + 1$. The measured displacements can be used to ensure the integrity of the specimen. This method requires that the FEA software be highly customizable. This puts a limit on the number of FEA software packages that can be used with this implementation.

Conclusions

Alternative control methods in hybrid simulation were explored. The requirement for other control methods besides position control and the challenges of implementing these methods in hybrid testing were expounded. Two types of force control and mixed control schemes were described. The first type used the tangent stiffness matrix of the experimental substructure. The second more desirable method did not use the tangent stiffness matrix. The implementation of these methods using OpenFresco was also outlined. There still remains more research to be conducted in order to make these schemes more viable in hybrid testing. They need to tested and verified by actual force and mixed controlled hybrid tests.

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