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A NEW SIMPLIFIED METHOD FOR EARTHQUAKE ANALYSIS OF GRAVITY DAMS

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ABSTRACT

Dam-reservoir dynamic interactions are complex phenomena requiring advanced mathematical and numerical modeling. Although available sophisticated techniques can handle many aspects of these phenomena, simplified procedures are useful and still needed to evaluate the seismic response of gravity dams. This paper presents and validates an original practical procedure to evaluate earthquake-induced hydrodynamic pressures on gravity dams and their dynamic response. The proposed technique includes the effects of dam geometry and flexibility, fluid-structure interaction, water compressibility, reservoir bottom wave absorption and varying reservoir level. The new formulation can be easily implemented in a computer program and is shown to accurately predict hydrodynamic loads when compared to analytical solutions. We then illustrate how this simplified method can be used to obtain a rigorous and yet simple closed-form expression of the fundamental vibration period of dam-water systems, a key parameter in the assessment of their dynamic or seismic behavior. The new simplified method and proposed closed-form formulation are shown to yield fundamental period predictions in excellent agreement with results obtained when the reservoir is modeled analytically or numerically using potential-based finite elements.

Introduction

Dam failures may result in catastrophic consequences, causing considerable loss of life and property. Research dealing with dam safety has been extensively active during the last 50 years to improve understanding of the complex behavior of dam-reservoir systems and implement more reliable approaches into the codes of practice. Rigorous techniques to investigate dynamic dam-reservoir interactions are relatively new compared to more established static dam analysis tools. The milestone of such rigorous treatment dates back to the pioneering work of Westergaard (Westergaard 1933), who proposed to model water action on a dam subjected to horizontal ground accelerations as an equivalent added mass height-wise distribution applied on the dam face. Although Westergaard's analytical solution neglected dam

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flexibility and water compressibility, it has been widely used for many decades to design earthquake-resistant concrete dams. Although available sophisticated techniques can handle many aspects of these phenomena, simplified procedures are useful and still needed to globally evaluate the effects of dam-reservoir interaction. This paper proposes an original and practical procedure to evaluate earthquake induced dam-reservoir interactions, including the effects of dam flexibility, water compressibility and reservoir bottom wave absorption. In a previous work, the first writer proposed a simplified closed-form formulation to evaluate earthquake-induced hydrodynamic pressures on concrete dams (Bouaanani et al. 2003). The method includes the effects of water compressibility and reservoir bottom wave absorption. The influence of dam deformability was however neglected and therefore the total hydrodynamic pressure exerted on a dam during an earthquake could not be satisfactorily determined. The first part of this paper proposes and validates a newly improved and practical formulation where the rigid dam restricting assumption is waived. We then show how this formulation can be used to obtain a rigorous and yet simple closed-form expression of the fundamental vibration period of gravity dam-reservoir systems. Though several procedures were proposed to estimate this parameter, all were based on extensive approximations of the geometry of the studied dam section. Hatanaka (Hatanaka 1960) developed simplified expressions to estimate the fundamental vibration period of idealized triangular dam with empty reservoirs. Okamoto (Okamoto 1984) proposed simplified formulas to estimate the fundamental vibration periods of dams with empty and full reservoirs using analogy with beam theory. Chopra (Chopra 1978) analyzed several idealized triangular dam cross-sections to obtain an approximate fundamental vibration period and corresponding mode shape of typical gravity dams with an empty reservoir. A more rigorous and general formulation is developed in the present paper to account for the influence of dam geometry and flexibility as well as varying reservoir level. The efficiency and accuracy of the formulation are validated against advanced analytical and finite element techniques.

Analytical and Simplified Formulations

Review of analytical formulation

The analytical formulation reviewed in this section was originally proposed by Fenves and Chopra (1984). We consider the dam-reservoir system illustrated in Fig. 1. The dam has a total height H_s and it impounds a semi-infinite reservoir of constant depth H_r. The effects of sediments that may be deposited at reservoir bottom are also considered. A Cartesian coordinate system with axes x and y with origin at the heel of the structure is adopted and the following main assumptions are made: (i) the dam and the water are assumed to have a linear behavior; (ii) the water in the reservoir is compressible and inviscid, with its motion irrotational and limited to small amplitudes; and (iii) gravity surface waves are neglected. Under these assumptions, the hydrodynamic pressure $p(x,y,t)$ in the reservoir (in excess of the hydrostatic pressure) obeys the wave equation

$$\nabla^2 p = \frac{1}{C_r^2}\frac{\partial^2 p}{\partial t^2} \tag{1}$$

where ∇^2 is the Laplace differential operator, t the time variable; ρ_r the mass density of water and C_r the compression wave velocity given by

$$C_{\mathrm{r}} = \sqrt{\frac{\mu_{\mathrm{r}}}{\rho_{\mathrm{r}}}} \tag{2}$$

in which μ_r denotes the bulk modulus of water.

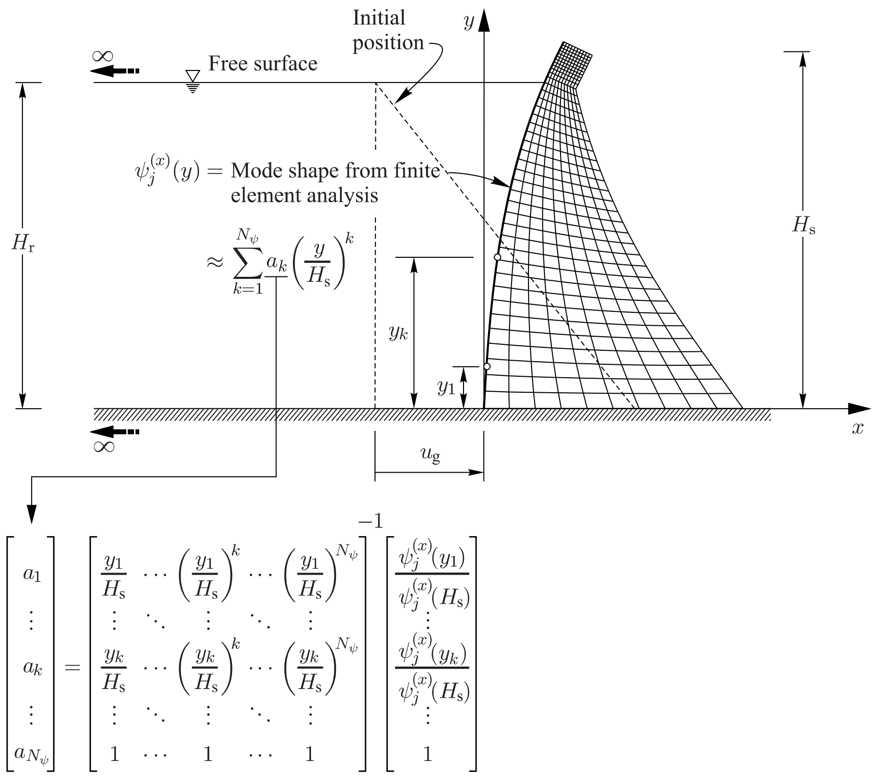

Figure 1. Dam-reservoir system and approximation of dam mode shapes.

Considering harmonic ground accelerations $\ddot{u}_g(t) = a_g \mathrm{e}^{\mathrm{i}\omega t}$, the hydrodynamic pressure in the reservoir can be expressed in the frequency domain as $p(x, y, t) = \overline{p}(x,y,\omega)\mathrm{e}^{\mathrm{i}\omega t}$ where ω is the exciting frequency, and $\overline{p}(x,y,\omega)$ a complex-valued Frequency Response Function (FRF). Introducing this transformation into Eq. 1 yields the classical Helmholtz equation

$$\nabla^2 \overline{p} + \frac{\omega^2}{C_{\mathrm{r}}^2} \overline{p} = 0 \tag{3}$$

The hydrodynamic pressure FRF $\overline{p}$ can be expressed as (Fenves and Chopra 1984)

$$\overline{p}(x,y,\omega) = \overline{p}_0(x,y,\omega) - \omega^2 \sum_{j=1}^{m_{\mathrm{s}}} \overline{Z}_j(\omega)\, \overline{p}_j(x,y,\omega) \tag{4}$$

where $\overline{p}_0$ is the FRF for the hydrodynamic pressure due to rigid body motion of the dam, and where $\overline{p}_j$ is the FRF for hydrodynamic pressure due to horizontal acceleration $\psi_j^{(x)}(y) = \psi_j^{(x)}(0, y)$ of the dam upstream face, $\overline{Z}_j$ is the generalized coordinate, and m_{s} the total number of mode shapes included in the analysis. Throughout this paper, hydrodynamic pressures $\overline{p}_0$ and $\overline{p}_j$ will be referred to as the "rigid" and the "flexible" parts of the total hydrodynamic pressure $\overline{p}$, respectively. The boundary conditions to be satisfied by hydrodynamic frequency response functions translate compatibility of pressures and displacements at dam-reservoir interface and wave absorption at reservoir bottom (Fenves and Chopra 1984). The complex frequency response functions of hydrodynamic pressures $\overline{p}_0$ and $\overline{p}_j$ can then be expressed as the summation of m_{r} response functions $\overline{p}_{0n}$ and $\overline{p}_{jn}$ corresponding each to a reservoir mode n

$$\overline{p}_0(x,y,\omega) = \sum_{n=1}^{m_{\mathrm{r}}} \overline{p}_{0n}(x,y,\omega) = -2\rho_{\mathrm{r}} a_{\mathrm{g}} H_{\mathrm{r}} \sum_{n=1}^{m_{\mathrm{r}}} \frac{\lambda_n^2(\omega)}{\beta_n(\omega)} \frac{I_{0n}(\omega)}{\kappa_n(\omega)} \mathrm{e}^{\kappa_n(\omega)x} Y_n(y,\omega) \tag{5}$$

$$\overline{p}_j(x,y,\omega) = \sum_{n=1}^{m_{\mathrm{r}}} \overline{p}_{jn}(x,y,\omega) = -2\rho_{\mathrm{r}} H_{\mathrm{r}} \sum_{n=1}^{m_{\mathrm{r}}} \frac{\lambda_n^2(\omega)}{\beta_n(\omega)} \frac{I_{jn}(\omega)}{\kappa_n(\omega)} \mathrm{e}^{\kappa_n(\omega)x} Y_n(y,\omega) \tag{6}$$

where λ_n and Y_n are complex-valued frequency dependent eigenvalues and orthogonal eigenfunctions satisfying, for each reservoir mode n

$$e^{2\mathrm{i}\lambda_n(\omega)H_{\mathrm{r}}} = -\frac{\lambda_n(\omega) - \omega q}{\lambda_n(\omega) + \omega q} \tag{7}$$

$$Y_n(y,\omega) = \frac{\left[\lambda_n(\omega) - \omega q\right]\mathrm{e}^{-\mathrm{i}\lambda_n(\omega)y} + \left[\lambda_n(\omega) + \omega q\right]\mathrm{e}^{\mathrm{i}\lambda_n(\omega)y}}{2\lambda_n(\omega)} \tag{8}$$

and where q is a damping coefficient to account for bottom reservoir absorption (Fenves and Chopra 1984) and the terms β_n, κ_n, I_{0n} and I_{jn} are given by

$$\beta_n(\omega) = H_{\mathrm{r}}\left[\lambda_n^2(\omega) - \omega^2 q^2\right] + i\omega q; \qquad \kappa_n(\omega) = \sqrt{\lambda_n^2(\omega) - \frac{\omega^2}{C_{\mathrm{r}}^2}} \tag{9}$$

$$I_{0n}(\omega) = \frac{1}{H_{\mathrm{r}}}\int_0^{H_{\mathrm{r}}} Y_n(y,\omega)\,\mathrm{d}y; \qquad I_{jn}(\omega) = \frac{1}{H_{\mathrm{r}}}\int_0^{H_{\mathrm{r}}} \psi_j^{(x)}(y) Y_n(y,\omega)\,\mathrm{d}y \tag{10}$$

Using modal superposition and mode shapes orthogonality, we show that the vector $\overline{\mathbf{Z}}$ of frequency-dependent generalized coordinates $\overline{Z}_j$, $j = 1 \ldots m_{\mathrm{s}}$, can be obtained by solving the system of equations

$$\overline{\mathbf{S}}\,\overline{\mathbf{Z}} = \overline{\mathbf{Q}} \tag{11}$$

in which, for $n = 1 \ldots m_{\mathrm{s}}$ and $j = 1 \ldots m_{\mathrm{s}}$,

$$\overline{S}_{nj}(\omega) = \left[-\omega^2 + \left(1 + \mathrm{i}\eta_{\mathrm{s}}\right)\omega_n^2\right] M_n\, \delta_{nj} + \omega^2 \int_0^{H_r} \overline{p}_j(0,y,\omega)\, \psi_n^{(x)}(0,y)\,\mathrm{d}y \tag{12}$$

$$\overline{Q}_n(\omega) = -L_n + \int_0^{H_r} \overline{p}_0(0,y,\omega)\, \psi_n^{(x)}(0,y)\,\mathrm{d}y \tag{13}$$

with

$$M_n = \boldsymbol{\psi}_n^T \mathbf{M}\, \boldsymbol{\psi}_n\,; \qquad\qquad L_n = \boldsymbol{\psi}_n^T \mathbf{M}\mathbf{1} \tag{14}$$

and where δ_{nj} is the Kronecker symbol, ω_n is the vibration frequency along mode shape ψ_n, η_{s} is the structural hysteretic damping coefficient, **M** is the dam mass matrix and **1** is a column-vector with the same dimension as the vector of nodal relative displacement, containing zeros except along horizontal degrees of freedom which corresponds to the direction of earthquake excitation.

Simplified expressions

The x–component of the structural mode shape ψ_j can be approximated as a polynomial function

$$\psi_j^{(x)}(y) = \sum_k a_k \left(\frac{y}{H_{\mathrm{s}}}\right)^k \tag{15}$$

where y_i the coordinate varying over the height of the dam measured from the bottom, and the a_{k} coefficients can be determined by interpolation as illustrated in Fig. 1. Using this approximation, we show that the rigid and flexible hydrodynamic pressures at acoustical mode n are related by

$$\overline{p}_{jn}(x,y,\omega) = \frac{\overline{p}_{0n}^{(x)}(x,y,\omega)}{a_{\mathrm{g}}^{(x)}}\left[F_{jn}(\omega) + \frac{G_{jn}(\omega)}{I_{0n}(\omega)}\right] \tag{16}$$

in which

$$F_{jn}(\omega) = \sum_k \left\{\left[\sum_{l=0}^{k}(-1)^{k-l}\,\frac{\Lambda_{2l}(\lambda_n H_{\mathrm{r}})}{\Lambda_{2k}(\lambda_n H_{\mathrm{s}})}\right]a_{2k} + \left[\sum_{l=0}^{k}(-1)^{k-l}\,\frac{\Lambda_{2l+1}(\lambda_n H_{\mathrm{r}})}{\Lambda_{2k+1}(\lambda_n H_{\mathrm{s}})}\right]a_{2k+1}\right\} \tag{17}$$

$$G_{jn}(\omega) = -\frac{\mathrm{i}\omega q}{\lambda_n^2 H_{\mathrm{r}}} F_{jn}(\omega) + \frac{1}{\lambda_n H_{\mathrm{r}}}\sum_k \left\{\left[\frac{\mathrm{i}\omega q}{\lambda_n}\frac{(-1)^k}{\Lambda_{2k}(\lambda_n H_{\mathrm{s}})}\right]a_{2k} - \left[\frac{(-1)^k}{\Lambda_{2k+1}(\lambda_n H_{\mathrm{s}})}\right]a_{2k+1}\right\} \tag{18}$$

and where the complex-valued function Λ_m is defined by

$$\Lambda_m(z) = \frac{z^m}{m!} \tag{19}$$

for complex and integer numbers, z and m, respectively. This equation relates the flexible and rigid parts of hydrodynamic pressure for a given reservoir acoustical mode to the vibration of the dam along a given structural mode shape. The rigid pressure can be determined using closed-form expressions similar to those developed by the first writer (Bouaanani et al. 2003, Bouaanani and Perrault 2010). These expressions take account of water compressibility and wave absorption at reservoir bottom and they were shown valid for a wide frequency range.

Numerical validation

The simplified expressions presented in the previous section are applied to a dam cross-section with dimensions inspired from the tallest non-overflow monolith of Pine Flat dam (Fenves and Chopra 1984) shown in Fig. 3(a). The following dam material properties are selected: a Poisson's ratio $\nu_s = 0.2$ and a mass density $\rho_s = 2400$ kg/m^3. Two values of modulus of elasticity are considered: E_s = 25000 MPa and E_s = 35000 MPa. The water is assumed compressible, with a velocity of pressure waves C_{r} = 1440 m/s and a mass density ρ_r = 1000 kg/m^3. Eqs. 17, 18, 16 and 4 are used to determine the parameters F_{jn} and G_{jn} and hydrodynamic pressure FRFs $\bar{p}$. A sufficient number of structural and reservoir modes are included in the analysis and a structural hysteretic damping ratio η_{s} = 0.1 is adopted. Figure 2 illustrates the obtained FRFs plotted against the frequency ratio ω/ω_1, where ω_1 is the fundamental vibration frequency of the dam on rigid foundation with an empty reservoir. The figure shows an excellent agreement between the simplified formulation and the analytical solution. We also observe that the quality of the approximation does not decay as wave reflection coefficient decreases.

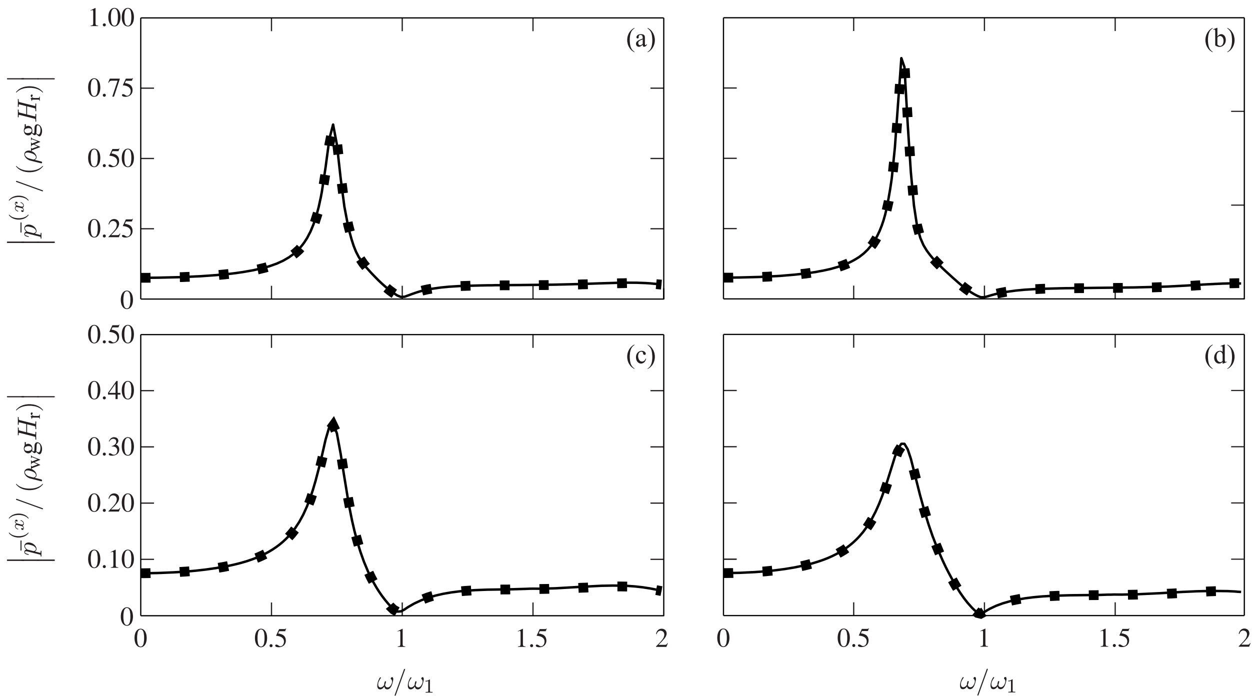

Figure 2. Absolute value of frequency response functions of normalized hydrodynamic pressure at dam heel: (a) α = 0.95 and E_{s} = 25000 MPa; (b) α = 0.95 and E_{s} = 35000 MPa; (c) α = 0.65 and E_{s} = 25000 MPa; (d) α = 0.65 and E_{s} = 35000 MPa; – Analytical solution; –■– Simplified formulation.

Application: Closed-form formulation for fundamental vibration period

The fundamental vibration period of a dam-reservoir system is key factor in the assessment of its dynamic or seismic response. Most seismic provisions and simplified procedures use the fundamental vibration period as an input parameter to determine seismic design accelerations and forces from a site-specified earthquake response spectrum. It is therefore crucial to obtain an accurate and yet practical analytical expressions to evaluate the fundamental period of gravity dams dynamically interacting with their impounded reservoirs. To develop such expressions, we assume that the *x*–component of the fundamental dam mode shape $\psi_1^{(x)}$ can be approximated as a cubic polynomial function

$$\psi_1^{(x)}(y) = a_1 \frac{y}{H_\mathrm{s}} + a_2\left(\frac{y}{H_\mathrm{s}}\right)^2 + a_3\left(\frac{y}{H_\mathrm{s}}\right)^3 \tag{20}$$

If water compressibility is neglected, the rigid and flexible parts of the hydrodynamic pressure become frequency-independent. Considering only fundamental vibration mode and neglecting the effect of damping on the fundamental frequency, the generalized coordinate $\overline{Z}_1$ tends to infinity at resonance. Using Eqs. 11 to 13, the fundamental vibration frequency ω_r of the dam-reservoir system is found as

$$\omega_r^2 = \frac{\omega_1^2 M_1}{M_1 - \int_0^{H_r} \overline{p}_1(0,y)\psi_1^{(x)}(0,y)dy} \tag{21}$$

Using the proposed cubic polynomial approximation of the fundamental mode shape and Eqs. 17 and 18, the hydrodynamic pressure $\overline{p}_1$ can be evaluated as the summation of m_r response functions $\overline{p}_{1n}$ defined by Eq. 16. We show that these approximations lead to a simplified form of the integral in Eq. 21, yielding a simple and original expression for the natural period T_r of a dam-reservoir system

$$T_r = T_1\sqrt{1 + \frac{4\rho_r H_s^2 \hat{\varphi}}{M_1}} \tag{22}$$

where T_1 is the natural frequency of the dam, and the function $\hat{\varphi}$ is given by

$$\hat{\varphi} = \left(\frac{H_r}{H_s}\right)^4\left[\hat{\gamma}_1 a_1^2 + \hat{\gamma}_2 a_1 a_2\left(\frac{H_r}{H_s}\right) + \left(\hat{\gamma}_3 a_2^2 + \hat{\gamma}_4 a_1 a_3\right)\left(\frac{H_r}{H_s}\right)^2 + \hat{\gamma}_5 a_2 a_3\left(\frac{H_r}{H_s}\right)^3 + \hat{\gamma}_6 a_3^2\left(\frac{H_r}{H_s}\right)^4\right] \tag{23}$$

in which the coefficients $\hat{\gamma}_1$ to $\hat{\gamma}_6$ are independent of dam-reservoir properties and are given in Table 1. If the dam mode shape is mass-normalized, the generalized mass in Eq. 22 is equal to unity i.e. $M_1 = 1$, otherwise

$$M_1 = \int_0^{H_r} \mu_s \psi_1^{(x)}(0, y)\,\mathrm{d}y \tag{24}$$

where μ_s is the mass of the dam per unit height.

Table 1. Coefficients $\hat{\gamma}_1$ to $\hat{\gamma}_6$

$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$\hat{\gamma}_4$	$\hat{\gamma}_5$	$\hat{\gamma}_6$
25.769 x10^{-3}	31.820 x10^{-3}	10.405 x10^{-3}	22.082 x10^{-3}	15.031 x10^{-3}	5.587x10^{-3}

When water compressibility is included, we show that the fundamental vibration period of a dam-reservoir system can be obtained by simply solving a cubic equation. The mathematical derivations are not given here for the sake of brevity.

Numerical validation

The accuracy of the closed-form formulation is assessed here for the case of incompressible water to compare with Westergaard's solution where the dam is assumed rigid. The Pine Flat dam section described previously is considered. The modulus of elasticity of the dam is chosen as E_s = 25000 MPa. We first compute the period ratios T_r/T_1 where T_1 is the fundamental vibration period of the dam obtained by a finite element analysis of the dam section. Four different analyses are conducted to obtain the fundamental period T_r of the dam-reservoir system as illustrated in Fig. 3:

- Type I: a finite element Fluid-Structure Interaction (FSI) analysis where both the dam and the reservoir are modeled using finite elements using the software ADINA (ADINA 2006).
- Type II: an analytical solution originally developed by Fenves and Chopra (Fenves 1984).
- Type III: a finite element analysis where the dam is modeled using finite elements and the reservoir hydrodynamic loading is modeled approximately using Westergaard added mass formulation (Westergaard 1933). The added mass m_i to be attached to a node i belonging to the dam-reservoir interface can be written as

$$m_i = \frac{7}{8} V_i \sqrt{H_r (H_r - y_i)} \tag{25}$$

  where y_i denotes the height of node i above the dam base and V_i the volume of water tributary to node i.
- Type IV: Using the proposed closed-form expression in Eq. 22.

It is apparent from Fig. 4 that the fundamental period predicted using the FSI finite element model, the analytical formulation proposed by Fenves and Chopra (1984), and our closed-form expression are in perfect agreement regardless of reservoir height ratio H_r/H_s and this even for a dam with a slightly inclined upstream face. In this case, Westergaard's added mass solution predicts the fundamental period of the dam-reservoir system with an error of about 12 per cent due to dam flexibility effects. This error may affect dam earthquake response quantities such as displacements, forces and stresses throughout the dam cross-section.

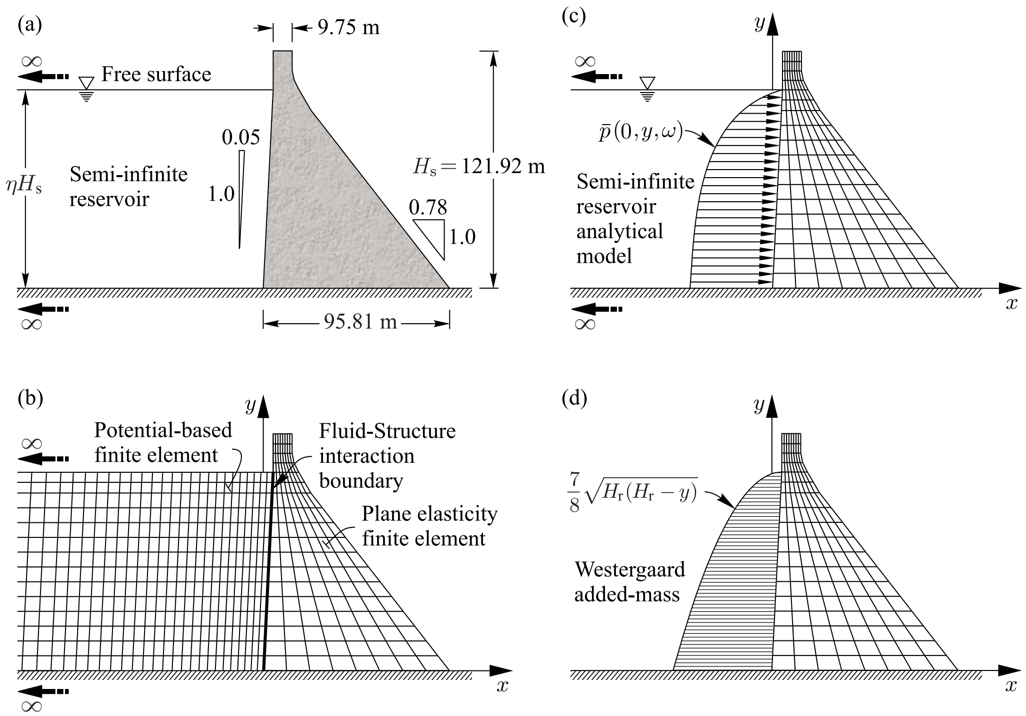

Figure 3. (a) Dam-reservoir system geometry; (b) Analysis type I: Finite element model; (c) Analysis type II: analytical solution; (d) Analysis type III: Westergaard added mass formulation.

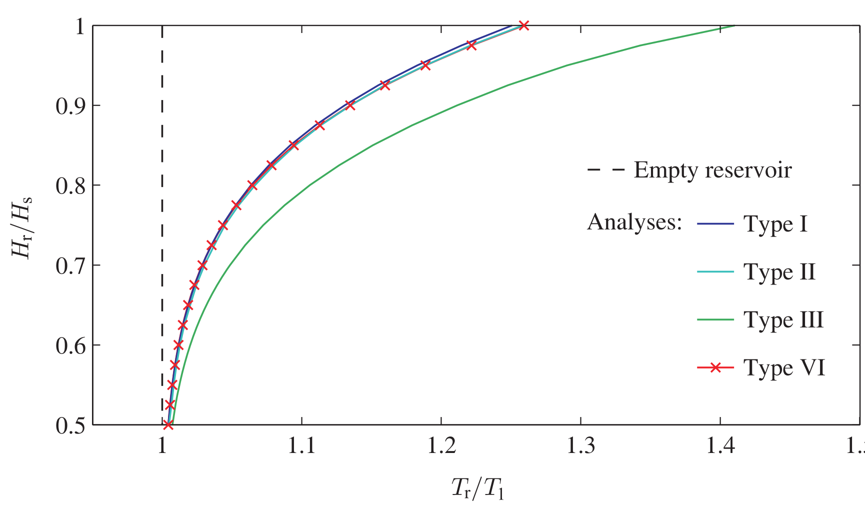

Figure 4. Variation of period ratio T_r/T_1 as a function of reservoir height ratio assuming incompressible water.

We note that the proposed closed-form formulation can also account for dam foundation flexibility by using corresponding dam-foundation modal properties instead of those of the dam on rigid foundation as was done solely for practical purposes in the illustrative example. Westergaard's solution, which is based on structural rigid body motion, would be even less reliable if foundation flexibility effects were to be included. Using the new closed-form formulation, simplified expressions to compute equivalent lateral earthquake forces for simplified earthquake analysis of dams can then be developed. These forces are to be applied at the dam upstream face to determine response quantities of interest, such as displacements and stresses in gravity dam monoliths (Fenves and Chopra 1984, Bouaanani and Perrault 2010).

Concluding remarks

This paper presented and validated a practical and efficient formulation to evaluate the dynamic response of gravity dams impounding semi-infinite reservoirs. The proposed method uses closed-form analytical expressions to relate hydrodynamic pressure due to any deflected modal response of a 2D gravity dam to hydrodynamic pressure caused by a horizontal rigid body motion. Effects of dam flexibility, water compressibility and wave absorption at reservoir bottom are included in the formulation. The new analytical expressions can be easily programmed to conduct frequency-domain analyses of dam-reservoir systems. The formulation was then used to obtain an original and accurate closed-form formulation for the vibration period of dams including hydrodynamic effects. The results obtained were compared to advanced FSI-based finite element solutions and an excellent agreement was found. The proposed new technique is simple, accurate, and can be easily programmed in a spreadsheet for practical purposes. In addition, it presents a significant advantage over conventional Westergaard's added-mass formulation, namely because it can directly account for the flexibility of the dam and its foundation, while Westergaard's solution assumes that both the dam and its foundation are rigid.

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