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VARIATION OF SMALL AMPLITUDE VIBRATION DYNAMIC PROPERTIES WITH DISPLACEMENT IN REINFORCED CONCRETE STRUCTURES

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ABSTRACT

Correlating damage level and changes in dynamic characteristics of a structure forms the basis for damage detection techniques in structural health monitoring. In buildings such correlation is not well established. A vibration-based damage detection technique capable of identifying the structural condition of the system based on small amplitude vibrations is desirable because such data are typically easy to obtain. It is a common practice in engineering applications to estimate dynamic parameters from small-amplitude vibrations assuming a linear behavior of the structure. This simplification causes inaccurate estimations of those dynamic parameters due to the presence of nonlinear behavior in the structure. This study focuses and presents the relationships found between small-amplitude vibration dynamic properties and past levels of maximum displacement in various reinforced concrete structures. One full-scale 3-story building and six small-scale beams were examined. Small displacements are defined as displacements below an overall drift ratio of 0.03%. Approximations in the estimates due to a linear-assumption model are avoided by examining the displacement-dependence of the dynamic properties.

Introduction

Vibration-based structural damage detection is a growing field in different disciplines. Extensive work has been done in the past to provide tools to estimate physical parameters of structures through their vibration records. The usual approach in structural damage detection, also known as structural health monitoring, is to monitor changes in those parameters due to a particular event. However, the relationships between change in dynamic response characteristics and structural condition of reinforced concrete structures are still not established, and the sensitivity of these relationships are not well-understood.

Linear-parameter estimates on nonlinear systems

The equation of motion for a single degree of freedom (SDOF) system can be written as

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$$\ddot{x} + 2\omega\zeta\dot{x} + \omega^2 x = \frac{F(t)}{m} \quad (1)$$

where m, ζ and ω are the mass, viscous-damping ratio and undamped natural frequency of the SDOF and x, $\dot{x}$ and $\ddot{x}$ are the displacement, velocity and acceleration of the system. The external force applied to the system is represented by $F(t)$. In the most general case for civil engineering applications where the mass is constant, representation of an elastic SDOF system through Eq. 1 would include the nonlinear behavior of the structure coming from variable stiffness and damping. Such nonlinear behavior has been identified by several authors in reinforced concrete structures as an amplitude-dependence effect (Clinton et. al. 2006, Celebi 1996, Galambos et. al. 1978, Jeary 1997, Trifunac 1972). While stiffness has been proposed as a displacement-dependent parameter in most of the cases, damping has been proposed as a displacement-dependent and velocity-dependent parameter (Wu et. al. 2007, Feldman 2007).

Estimates of natural frequency and damping in civil engineering applications are typically done through the Fourier Spectrum and the logarithmic decrement technique. These estimations correspond to linear-based formulations due to the stationarity assumption of the Fourier Transform (Oppenheim et. al 1986) and the linear-system assumption on which the logarithmic decrement technique is based (Clough 1975). In the presented work behavior of nonlinear elastic systems with displacement-dependent stiffness and viscous damping is investigated. Following is an example of frequency and damping estimates obtained through the aforementioned linear-model assuming methods applied on a nonlinear elastic system. The nonlinear elastic system is defined with displacement-dependent stiffness and damping parameters.

Linear-frequency estimate on a nonlinear elastic system

Fig. 1 (a) illustrates the load-displacement curve of an elastic non-linear softening system. Mass was defined as $m = 1$. The displacement-dependent parameters of stiffness (secant stiffness) and damping are given as $k(x) = 4\pi|x|^{-0.5}$ and $\zeta(x) = 0.01+0.01|x|$. The free vibration response of the system starting from an initial displacement and zero velocity is shown in Fig. 1(b). The corresponding Fourier Spectrum of the response is given in Fig. 1 (c). The natural frequency of the system identified by the largest peak in the Fourier Spectrum is 1.1 Hz, however smaller peaks are observed in the spectrum due to the nonlinear nature of the system.

Linear-viscous damping estimate on a nonlinear elastic system

Fig. 2 shows the estimate of damping obtained by applying the logarithmic decrement method on the free vibration response of the nonlinear system of Fig. 1. In Fig. 2 (a) the peaks used to compute the equivalent linear viscous damping ratio are shown and in Fig. 2 (b) the equivalent linear viscous damping is compared to the actual displacement-dependent viscous damping defined for the system. A maximum difference of sixty percent is found between the estimated linear values and the actual values.

Frequency and damping estimates considering the nonlinear behavior

As a result of the displacement-dependence of the frequency and viscous damping ratio, an elastic structure exhibits different values of frequency and viscous damping ratio at each instant of its free vibration response. Here two methods used to estimate the instantaneous frequency and damping are described.

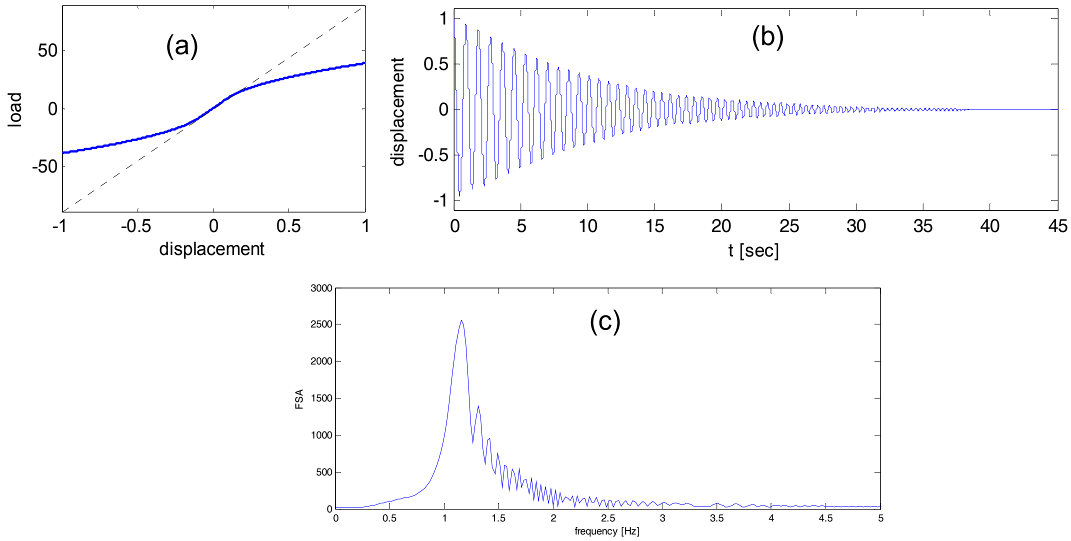

Figure 1. Frequency estimate from free vibration of a nonlinear elastic system. (a) Load-displacement curve. (b) free-displacement response. (c) Fourier spectrum.

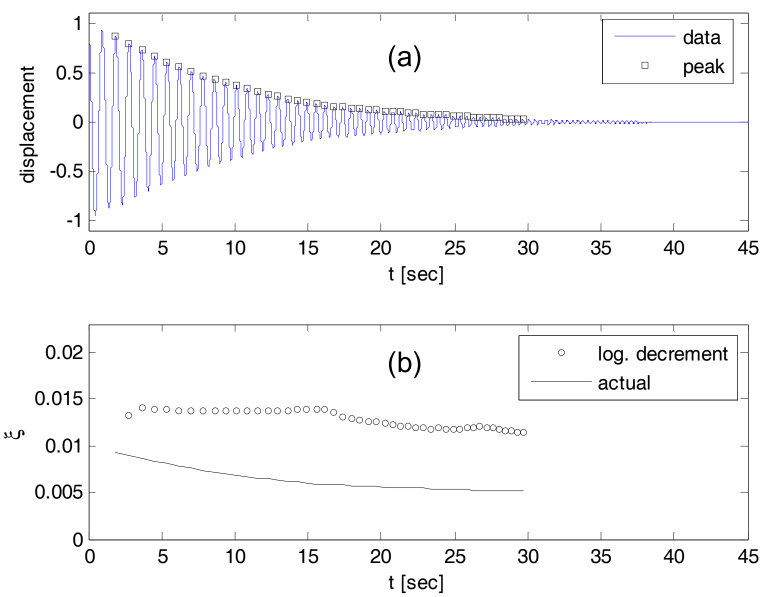

Figure 2. Damping estimate from free vibration of a nonlinear elastic system.
(a) Load-displacement curve. (b) free-displacement response. (c) Fourier spectrum.

Instantaneous frequency through Hilbert Transform

The Hilbert Transform of a signal *x(t)* is defined as

$$x'(t) = \int_{-\infty}^{\infty} \frac{x(u)}{\pi(t-u)} du \tag{2}$$

Using Eq. 2, it is possible to obtain an analytical signal *X(t)* of the original signal *x(t)* as

$$X(t) = x(t) + i * x'(t) = A(t)e^{i\theta t} \tag{3}$$

where A and θ are the instantaneous amplitude and phase angle, respectively. Using Eq. (3), the instantaneous frequency of the signal can be estimated as time derivatives of the phase angle.

$$f(t) = \left(\frac{1}{2\pi}\right)\frac{d\theta(t)}{dt} \tag{4}$$

Eq. 4 is used to estimate the instantaneous frequency of the freely vibrating SDOF given above (see Fig. 1). The corresponding instantaneous secant stiffness is computed as $k = 4\pi f^2$. Fig. (3) shows a portion of the free vibration response in Fig. (1) and the estimated instantaneous stiffness. In Fig. 3 (b) the actual secant stiffness defined in the system ($k(x) = 4\pi|x|^{-0.5}$) is plotted for comparison. It is seen that instantaneous frequency estimated through Hilbert Transform produces accurate results at instants of peak-amplitude displacements but not elsewhere.

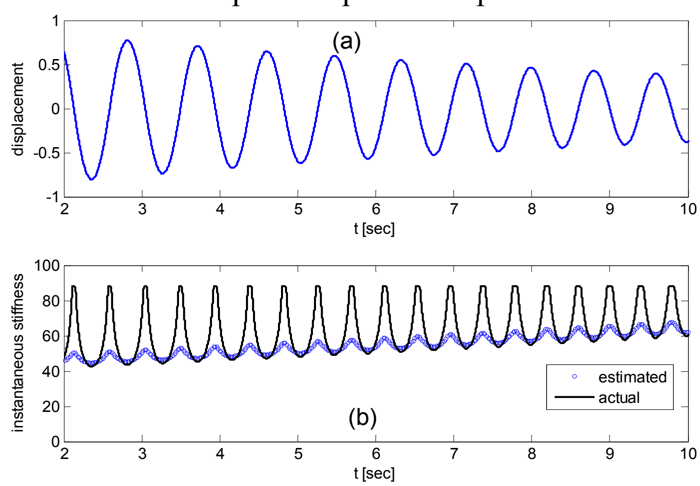

Figure 3. Instantaneous stiffness. (a) Free vibration response. (b) Estimated and actual secant stiffness.

Amplitude-dependent Damping obtained through Energy method

The energy equilibrium between consecutive pair of peaks *i* and *i+1* (peak displacement points) of the free vibration response of a SDOF system can be written as

$$PE_i = PE_{i+1} + DE_i \tag{5}$$

where PE_i and PE_{i+1} are the potential energy at peaks *i* and *i+1* and DE_i is the energy dissipated during the time the system moved from peak *i* to peak *i+1*. The potential energy terms are estimated as the area under the curve of restoring force P versus displacement x as

$$PE_i = \int_0^{x_i} P(x)dx \tag{6}$$

where x_i is the i-th peak displacement. The dissipated energy formulated for the equivalent viscous damper can be written as

$$DE_i = \int_{t_i}^{t_{i+1}} c\dot{x}^2 \, dt \tag{7}$$

where $\dot{x}$ is velocity, t_i and t_{i+1} are the instants at which the peak-displacements i and $i+1$ happen, respectively. The damping coefficient c is expressed through the mass m and the displacement-dependent damping ratio $\zeta(x)$ and circular frequency $\omega(x)$, as $c = 2\cdot\zeta(x)\cdot\omega(x)\cdot m$. The restoring force can be expressed as $P(x) = \omega(x)^2 \cdot x \cdot m$. Eq. 5 can be rewritten using these formulations and Eqs. 6 and 7 as

$$\int_{x_{i+1}}^{x_i} \omega(x)^2 \cdot x \cdot dx = \int_{t_i}^{t_{i+1}} 2\cdot\zeta(x)\cdot\omega(x)\cdot\dot{x}(t)^2 \cdot dt \tag{8}$$

Using estimated circular frequency ω from Eq. 4 ($\omega = 2\pi f$) and the given response of the system (velocity and displacement), equivalent viscous damping ratio can be solved from Eq. 8. This energy method is employed to compute the damping from the free vibration response of the nonlinear elastic system with stiffness and viscous damping defined as $k(x) = 4\pi|x|^{-0.5}$ and $\zeta(x) = 0.03+0.01|x|$, respectively. The response is shown in Fig. 4. In Fig. 4 (b) the actual viscous damping ratio (as defined) and that obtained using the logarithmic decrement method are shown for comparison with the viscous damping ratio estimated using the energy method.

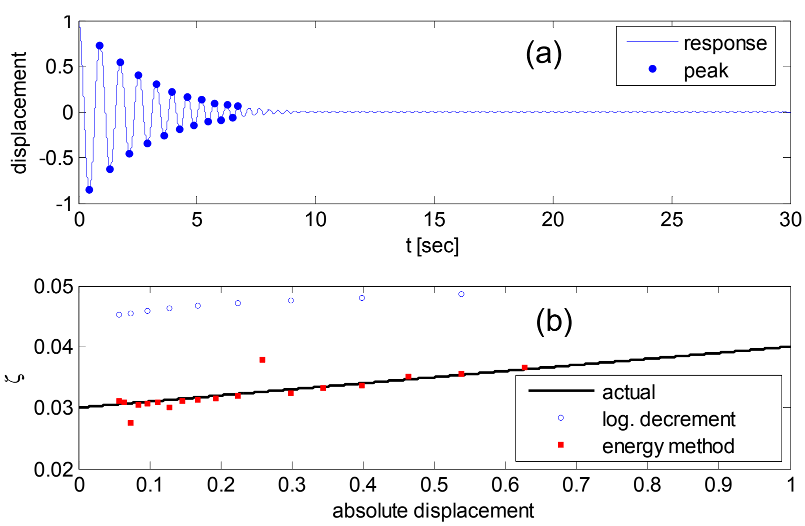

Figure 4. Amplitude-damping obtained estimated through the Energy Method. (a) Free vibration response. (b) Comparison of ζ values.

Experimental measurements in reinforced concrete structures

A full-scale three-story flat-plate building and six small-scale cantilever beams were tested in the laboratory. The dynamic properties were measured at small displacements after different large displacements were induced in the structure. The small displacements at which the dynamic tests were performed were bounded by a maximum overall drift ratio level of 0.03%, which is below the cracking displacement of the structures. The maximum large displacement demand level used was 3.0% overall drift ratio for the full-scale structure (Fick 2008) and 13% overall drift ratio level for the small-scale beams (Consuegra 2009).

Full-scale three-story flat-plate building

The test specimen was a full-scale two-span by one-span three-story reinforced concrete flat-plate structure consisting of six columns spaced at 20 ft in each direction supporting a 7 in. thick slab at each story (see Fig. 5). The story height of the building was 10 ft. A punching-shear failure was observed when the structure was pushed to 3.0% overall drift ratio. Table 1 lists the estimates of frequency and viscous damping ratio from free vibration response the full-scale structure after different large displacement states. Natural frequency has been estimated by the largest peak in the Fourier Spectrum and equivalent viscous damping ratio through the logarithmic decrement. It is seen that dynamic properties measured at a given condition in the structure are sensitive to the initial displacement given at the roof level (Δ_{roof}). However, because of the amplitude decreasing characteristic of the test, dynamic properties estimated via linear-based methods provide only a rough estimate of the displacement-dependent properties of the structure.

Figure 5. Full-scale three-story reinforced concrete flat-plate building at the Bowen Laboratory of Purdue University.

One can obtain estimates of the dynamic properties in a fashion that allows proper comparison by examining the amplitude-dependence of the dynamic properties using the Hilbert Transform method for frequency estimate and the Energy method for the damping estimate, as described earlier. Alternately, steady-state forced vibration tests could be performed at identical peak displacement levels, regardless of the forcing frequency, to eliminate variation of properties due to peak displacement level during tests. In forced-vibration tests on buildings, it is common practice to set the stroke or generated force of the dynamic linear actuator or the masses of the eccentric-mass shaker exciting the structure to constant amplitude. Then, different excitation frequencies in the vicinity of the natural frequency of interest are swept and a Transfer function (TF) is obtained as the ratio output/input for a given frequency. Eq. 9 shows the analytical TF for a linear SDOF system when the output is displacement and the input is the force (k: stiffness, ω: natural frequency, ζ: viscous damping ratio, Ω : excitation frequency). The type of Transfer Function (TF) obtained from the described, typically employed kind of forced-vibration test, corresponds to that based on a set of different amplitude displacement responses with increasing peak displacement response as the excitation frequency gets closer to the natural frequency. This type of TF is named herein as the "non-constant-displacement-response TF". Another type of TF, named as the "constant-displacement-response TF", could be obtained if the force amplitude is adjusted at each frequency step so the peak displacement response of the structure is kept constant regardless of the forcing frequency. Fig. 6 shows a comparison of the two types of TFs obtained for the building of Fig. 5 after the building has experienced an overall drift ratio of 3.0%. A least-square best fit on each of the experimental TFs shown in Fig. 6 is made using Eq. 9 with proper selection of the parameters k, ω and ζ. The fact that a better fit is obtained on the constant-displacement-response TF demonstrates that the structure exhibits nonlinear behavior even at such small displacements (below 0.03% roof mean drift ratio). This nonlinear behavior cannot be captured by using non-constant displacement response TF (as usually done in building tests) in conjunction with linear assumptions (as that of Eq. 8).

$$|TF(\Omega)| = \frac{1/k}{\sqrt{\left[1-(\Omega/\omega)^2\right]^2 + \left[2\cdot\zeta\cdot\Omega/\omega\right]^2}} \tag{9}$$

Table 2 summarizes the frequency and damping measured at different small displacements (below 0.03% overall drift ratio) after different large drift levels in the full-scale structure. Properties in Table 2 were obtained through constant displacement response TF type of tests. Natural frequency is observed to change as much as 45% in the range of low amplitudes for a given drift level, indicating the extreme dependency of natural frequency to the dynamic test displacement amplitude. It is seen from the dynamic tests done before and after an event, using the same low-amplitude displacement, that the natural frequency is sensitive to the large overall drift level. If the identical dynamic displacement level approach is not followed, however, erroneous conclusions could be made. For example, comparison of fundamental frequency measured at 0.10-in drift after experiencing 1.5% overall drift ratio and measured at 0.01-in after experiencing 3.0% overall drift ratio (i.e. damaged condition) would lead to indifference at best.

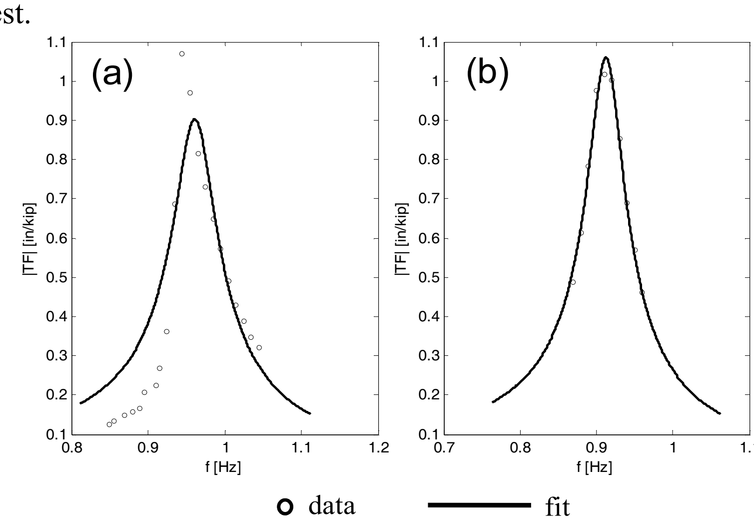

Figure 6. Transfer functions in the full-scale structure. (a) non-constant-displacement-response TF. (b) constant-displacement-response TF.

Table 1. Linear-based dynamic-property estimates from free vibration response in the full-scale structure. Fundamental frequency and viscous damping ratio: f_1 and ζ_1 respectively.

Condition	Δ_{roof} [in]	f_1 [Hz]	ζ_1 [%]
As-built	0.020	1.9	0.8
After 1.5% drift	0.020	1.3	2.4
	0.040	1.2	2.4
	0.100	1.2	2.9
After 3.0% drift	0.020	1.0	2.4
	0.040	1.0	2.6
	0.100	0.9	3.0

Table 2. Linear-based dynamic-property estimates from forced vibration response in the full-scale structure. Fundamental frequency and viscous damping ratio: f_1 and ζ_1 respectively.

Condition	Δ_{roof} [in]	f_1 [Hz]	ζ_1 [%]
As-built	< 0.01(ambient)	2.0	-
	0.01	1.9	0.8
After 1.5% overall drift	< 0.01(ambient)	1.5	-
	0.01	1.2	2.1
	0.02	1.2	2.0
	0.04	1.1	2.1
	0.10	1.0	2.0
After 3.0% overall drift	< 0.01(ambient)	1.1	-
	0.01	1.0	2.2
	0.02	0.9	2.6
	0.04	0.9	2.5
	0.10	0.8	2.5

Small-scale beams

An extensive testing program was developed on six small-scale reinforced concrete cantilever beams (Fig. 7). The beams were subjected to pseudo-static cycles of increasing large amplitudes and the small-amplitude dynamic properties were obtained by means of free, forced and ambient vibrations (Consuegra 2009). Due to the limited space and for sake of brevity only results from constant displacement response TFs (obtained through forced vibrations), which permit direct comparison of properties at a given displacement level, are presented. Fig. 8 shows the variation of natural frequency and equivalent viscous damping ratio with large displacement levels (past maximum displacement) in the small-scale beam of Fig. 7. Results from a typical specimen and dynamic properties measured at a constant tip-displacement of 0.003 in (0.01% overall drift ratio), are presented. Fig. 8 (a) and (b) have identical horizontal axis that illustrates tests performed from the pristine condition (zero) to the condition when the beam had experienced 13% overall drift ratio (3.7 in tip-displacement). The natural frequency is observed to decrease continuously with past maximum displacement, with more dramatic decrease in early cycles. On the other hand, the viscous damping ratio exhibits an early increase and then decrease behavior with past maximum displacement starting from pristine state. The past maximum displacement at which the viscous damping ratio goes from increase to decrease behavior corresponds to a displacement near the yield displacement. This displacement was identified during the pseudo-static cycle testing as the displacement at which yield capacity of the member was developed (approximating the nominal moment capacity Mn as $As*f_y*d$).

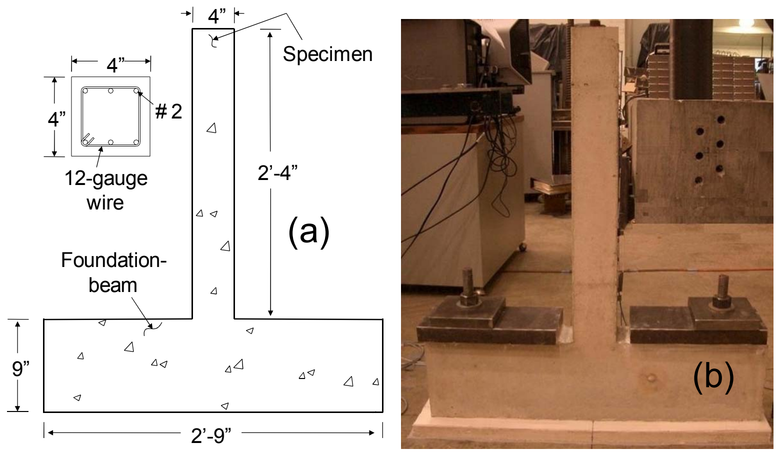

Figure 7. Small-scale cantilever beam. (a) Reinforcing details. (b) General view.

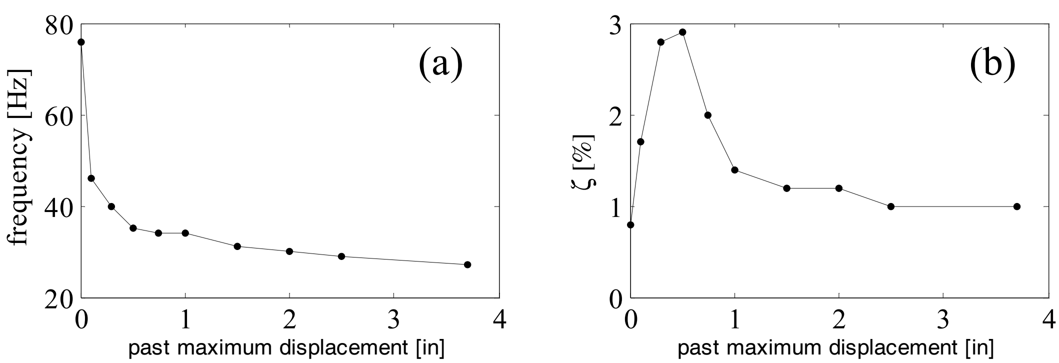

Figure 8. Variation of dynamic properties with the past maximum displacement in a small-scale beam. (a) Frequency. (b) Viscous damping ratio.

Conclusions

Dynamic properties measured at small displacements in reinforced concrete structures (below 0.03% overall drift ratio) are susceptible to nonlinear behavior of the structures. Consequently, it is important to verify the validity of linear elastic model assumptions. Common forced vibration tests performed in buildings, i.e. “non-constant displacement response” type of forced vibration tests, cannot capture the nonlinear behavior of the structures at small displacements unless a nonlinear elastic model is considered. Because of the complexity of such models, alternative testing in which the structure is forced to respond at a certain peak displacement level, i.e. “constant displacement response”, could be used. Such an approach leads to a more consistent implementation of typically employed linear-response based models. Variation of small-amplitude dynamic properties with past large peak displacement levels in reinforced concrete structures could be tracked using test data obtained at identical small displacement levels.

References

Bendat, J. S., and Piersol, A. G. (1986). *Random Data : Analysis and Measurement Procedures*, Wiley, New York.

Celebi, M. (1996). "Comparison of damping in buildings under low-amplitude and strong motions." *Journal of Wind Engineering and Industrial Aerodynamics*, 59(2-3), 309-323.

Clinton, J. F., Bradford, S. C., Heaton, T. H., and Favela, J. (2006). "The observed wander of the natural frequencies in a structure." *Bulletin of the Seismological Society of America*, 96(1), 237-257.

Clough, R. W., and Penzien, J. (1975). *Dynamics of Structures*, McGraw-Hill, New York.

Consuegra, F. (2009). *On the Relationship between Small Amplitude Vibration Dynamic Properties and the Past Maximum Displacement in Reinforced Concrete Structures*, Doctoral Thesis, School of Civil Engineering, Purdue University.

Feldman, M. (2007). "Considering high harmonics for identification of non-linear systems by Hilbert transform." *Mechanical Systems and Signal Processing*, 21(2), 943-958.

Fick, D. (2008). *Experimental Investigation and Analytical Modeling of the Nonlinear Response of Flat-Plate Structures Subjected to Cycling Loading Failure*, Doctoral Thesis, School of Civil Engineering, Purdue University.

Galambos, T. V., and Mayes, R. L. (1978). *Dynamic Tests of a Reinforced Concrete Building*. Report, Washington University.

Jeary, A. P. (1997). "Damping in structures." *Journal of Wind Engineering and Industrial Aerodynamics*, 72(1-3), 345-355.

Li, Q. S., Yang, K., Zhang, N., Wong, C. K., and Jeary, A. P. (2002). "Field measurements of amplitude-dependent damping in a 79-storey tall building and its effects on the structural dynamic responses." *Structural Design of Tall Buildings*, 11(2), 129-153.

Oppenheim, A. V., Schafer, R. W., and Buck, J. R. (1999). *Discrete-Time Signal Processing*.

Trifunac, M. D. (1972). "Comparison between ambient and forced vibration experiments." *Earthquake Engineering and Structural Dynamics*, 1, 133-150.

Wu, Z., Liu, H., Liu, L., and Yuan, D. (2007). "Identification of nonlinear viscous damping and Coulomb friction from the free response data." *Journal of Sound and Vibration*, 304(1-2), 407-414.