



AN EMPIRICAL BRACKETED DURATION RELATIONSHIP FOR CENTRAL/EASTERN NORTH AMERICA

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ABSTRACT

Presented herein is an empirical predictive relationship correlating bracketed duration to earthquake magnitude, site-to-source distance, and local site conditions (i.e., rock vs. stiff soil) for stable continental regions (e.g., central/eastern North America; CENA). The correlation was developed from data derived from 620 representative horizontal motions for stable continental regions, consisting of 28 recorded motions and 592 scaled motions (e.g., Boore, 1983; McGuire et al., 2001; Silva and Lee, 1987). The bracketed duration data was comprised of non-zero and zero durations. Non-linear mixed-effects regression technique was performed to fit a predictive model to the non-zero duration data. To account for the zero duration data, logistic regression was conducted to model the probability of zero duration occurrences. Then, the probability models were applied as weighting functions to the NLME regression results. Comparing the predicted durations for CENA motions to those predicted for motions for active shallow crustal tectonic regimes (e.g., western North America: WNA) via an existing relationship, the CENA rock motions have a significantly longer bracketed durations than those for WNA at comparable magnitudes and site-to-source distances. However, for soil sites, the WNA motions tend to have longer durations in the near field and shorter durations in the far field, as compared to CENA motions.

Introduction

Strong ground motion duration is an important parameter for seismic risk assessment because it, along with the amplitude and frequency content of the ground motions, significantly influences the response of geotechnical and structural systems. For example, when the non-linear behavior (i.e., degradation of stiffness) of a system is considered, strong motion duration is a critical feature regarding the amount of potential damage (e.g., Bommer and Martinez-Pereira, 1999). Accordingly, various definitions of strong motion duration have been proposed for quantifying the strong motion phase of earthquake ground shaking, which is the portion of the motion that is of engineering interest.

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Although not necessarily viewed by the seismological community as being the most appropriate quantification of strong ground motion duration, bracketed duration ($D_{bracket}$) has merits and is one of the most commonly used in engineering practice. The bracketed duration is determined using an absolute criterion based on the time interval between the first and last exceedance of ground acceleration above or below a threshold acceleration. Commonly, the threshold acceleration is ± 0.05 g (e.g., Bolt, 1973; Hays, 1975; Page et al., 1972), which is the value used in this study. An example of how bracketed duration is determined is shown in Fig. 1. As may be surmised from this figure, a ground motion will have zero-duration if the peak ground acceleration (pga) of the motion is less than the specified threshold.

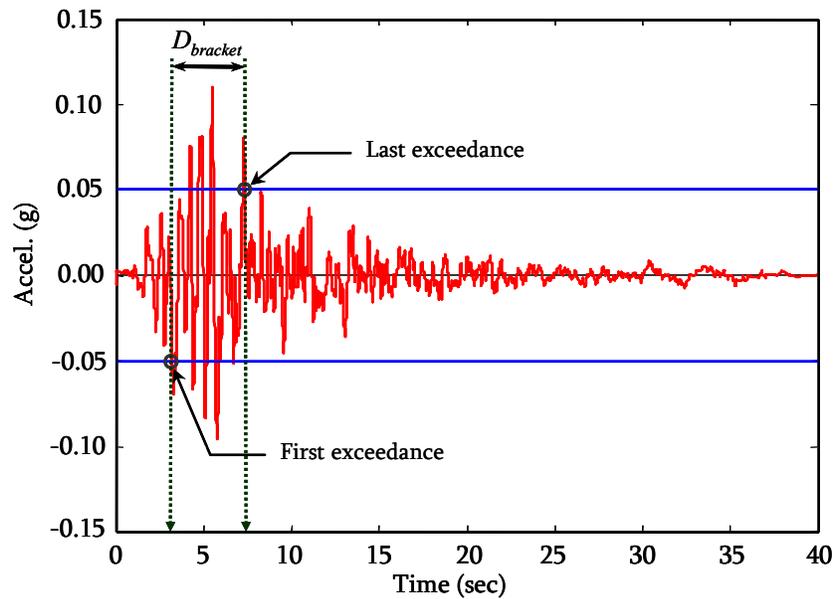


Figure 1. Determination of the bracketed duration for a ground acceleration time history.

Herein, an empirical relationship correlating bracketed duration to earthquake magnitude, site-to-source distance, and local site conditions (i.e., rock vs. stiff soil) for stable continental tectonic regimes (e.g., central/eastern North America: CENA) is presented. This correlation was developed by performing non-linear mixed-effects (NLME) regression analyses on data derived from 620 representative horizontal motion recordings for CENA. Using the empirical correlation developed in this study, bracketed durations of CENA motions are compared with those predicted using the relationship developed by Chang and Krinitzsky (1977) for active shallow crustal tectonic regimes (e.g., western North America: WNA).

Regarding the organization of this paper, first the strong ground motion dataset used in this study is described. Then, basic concepts of the NLME regression method are reviewed, and an approach for incorporating the effects of zero-durations in the predictive model is presented. Next, the proposed functional form of the predictive model is introduced along with the results of the regression analyses, and a comparison of bracketed durations in stable continental and active shallow crustal regions is presented. It is noted that the acronyms "CENA" and "WNA" in this paper are used in a general sense to refer to "stable continental" and "active shallow crustal"

regions, respectively, not just to the central/eastern NA and western NA.

Strong Ground Motion Data

In total, 620 representative horizontal earthquake motions for CENA were used to develop the empirical bracketed duration relationship in this study. The ground motion dataset was assembled by McGuire et al. (2001). Primarily, this dataset was intended to provide a library of strong ground motion time histories suitable for engineering analyses. Because there are few recorded strong ground motions in stable continental regimes, only 28 of the motions in the dataset are recorded motions, with the remaining 592 motions being "scaled" WNA motions. Dr. Walter Silva scaled the motions using response spectral transfer functions generated from the single-corner point source model (e.g., Boore, 1983; McGuire et al., 2001; Silva and Lee, 1987). The transfer functions account for the differences in seismic source, wave propagation path properties, and site effects between the WNA and CENA regions. The moment magnitudes of these motions range from 4.5 to 7.6, and the site-to-source distances range from 0.1 km to 199.1 km; the site-to-source distance is defined as the closest distance to the fault rupture plane. The recorded motions include motions from the 1988 Saguenay (M 4.5 and M 5.9), the 1985 Nahanni (M 6.8), and the 1989 New Madrid, MO (M 4.7) earthquakes. Fig. 2 shows the magnitude and site-to-source distance distribution.

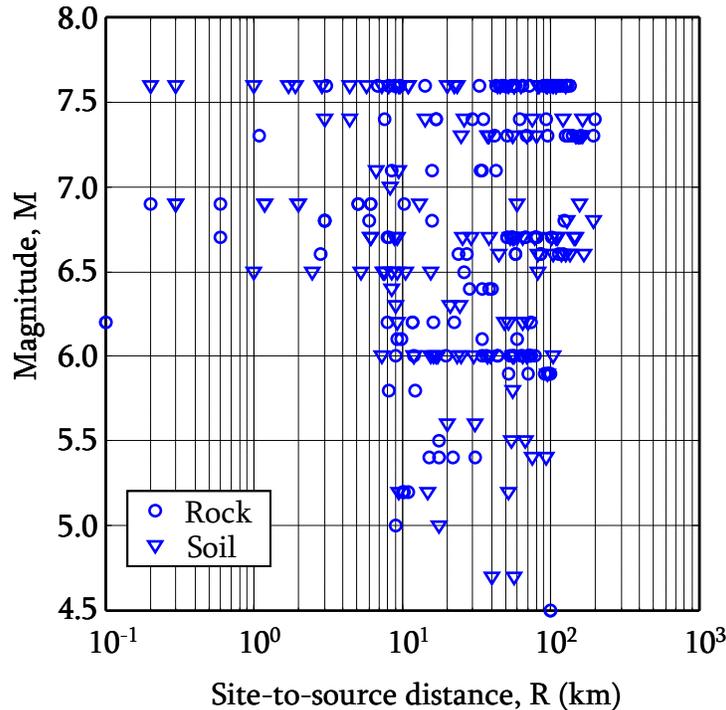


Figure 2. Earthquake magnitude and site-to-source distance distributions.

The ground motions were classified as either "rock" or "soil" based on the site conditions at the respective seismograph stations. The site classification scheme used by McGuire et al. is based on the third letter of the Geomatrix 3-letter site classification system shown in Table 1. Site categories A and B were considered to represent rock sites, and site categories C and D were

considered to represent soil sites. This categorization is similar to that of the USGS shown in Table 2, where rock sites encompass site class A and B, and soil sites encompass site classes C and D.

Table 1. Third letter: Geotechnical subsurface characteristics of Geomatrix 3-letter site classification.

Third letter	Site description	Comments
A	Rock	Instrument on rock ($V_S > 600$ m/s) or < 5 m of soil over rock.
B	Shallow (stiff) soil	Instrument on/in soil profile up to 20 m thick overlying rock.
C	Deep narrow soil	Instrument on/in soil profile at least 20 m thick overlying rock, in a narrow canyon or valley no more than several km wide.
D	Deep broad soil	Instrument on/in soil profile at least 20 m thick overlying rock, in a broad valley.
E	Soft deep soil	Instrument on/in deep soil profile with average $V_S < 150$ m/s.

Table 2. USGS site classification.

Site Class	Average shear wave velocity to a depth of 30 m: V_{S30}
A	$V_{S30} \geq 750$ m/s
B	$V_{S30} = 360 - 750$ m/s
C	$V_{S30} = 180 - 360$ m/s
D	$V_{S30} \leq 180$ m/s

Regression Analyses

The non-linear mixed-effects (NLME) regression technique was used to develop the empirical bracketed duration relationship in this study. NLME modeling is a maximum likelihood method based on normal (Gaussian) distribution and is used particularly for analyzing grouped data. The NLME regression method allows regression models to incorporate both fixed-effects that do not vary with the entire population of data and random-effects that vary by group. The random-effects are associated with earthquake events that are considered as a group herein. In comparison to applying a fixed-effects regression technique, which is equivalent to the least squares method to the entire dataset, a mixed-effects regression method allows both inter- and intra-group (i.e., between- and within-event) uncertainty to be quantified. This regression method produces unbiased fittings for each group (i.e., earthquake event). This is important in analyzing earthquake ground motion data because of the varying number of motions from the different earthquakes. The statistical analysis program R (version 2.5.0) was used to implement the NLME regression methods.

A quantile-quantile plot (or Q-Q plot) was used to check whether the data was normally distributed (i.e., if the data points plot approximately as a straight line on a normal Q-Q plot, it indicates that the data is normally distributed.). As may be observed from the normal Q-Q plots shown in Fig. 3, the entire ground motion duration data set does not follow a normal distribution. Also, the presence of zero-duration data precludes the data from being log-normally distributed.

Furthermore, the zero-durations do not correlate well to the independent variables (e.g., magnitude and site-to-source distance) in the regression analyses.

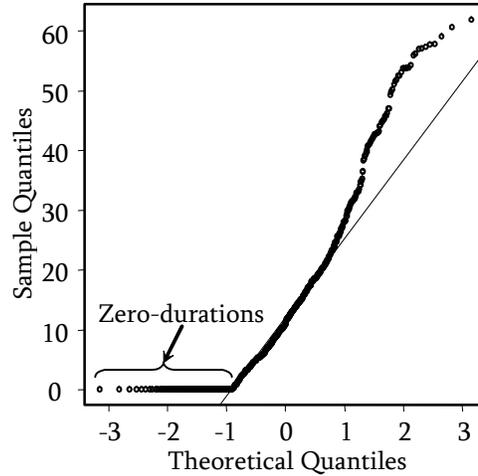


Figure 3. Normal Q-Q plot of bracketed duration data.

To circumvent these issues, the zero- and nonzero-duration data were treated separately, with the nonzero-duration data reasonably following a log normal distribution. The total number of nonzero-duration data used in the NLME regressions was 568; the distribution of zero and nonzero rock and soil motions is shown in Figure 4. The zero-duration data however, needed to be incorporated in the predictive model, otherwise the model would be biased toward longer durations. As a result, a logistic regression method was employed to model the probability of zero-duration occurrence as a function of earthquake magnitude, distance, and site condition. Then, this probability model was applied as a weighting function to the NLME regression result.

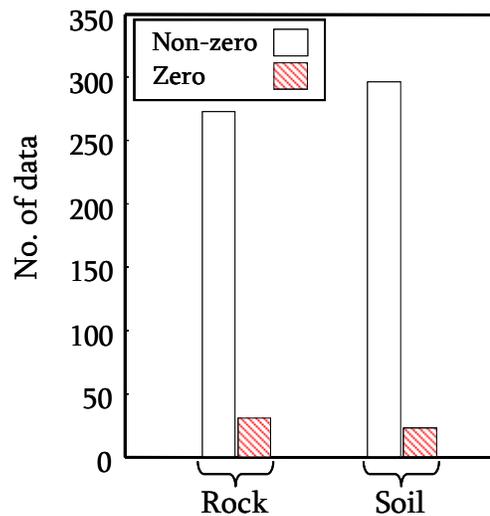


Figure 4. Zero bracketed duration population.

Proposed Model and Regression Results

As mentioned above, the proposed model consists of two parts: one is the non-zero duration model that is developed through the NLME regression analyses using non-zero duration data; the other is a weighting function that represents the probability of non-zero duration occurrence for a given earthquake magnitude, distance, and site condition, which is estimated through logistic regressions.

Non-zero Duration Model

In assessing the normal distribution of the non-zero duration data, it was found that adding one second to the durations contributed to optimizing the overall log-normality of the duration data, as well as the errors. For example, Fig. 5 shows the improvement in the log-normal distributions of the data and errors. As may be observed from this figure, $\ln(D_{\text{bracket}} + 1)$ more closely follows a normal distribution than $\ln(D_{\text{bracket}})$. This optimization of the normality is necessary because normal distribution of data and errors is inherently assumed in the theoretical formulation of the NLME regression. Accordingly, the NLME regression analyses were performed on $\ln(D_{\text{bracket}} + 1)$.

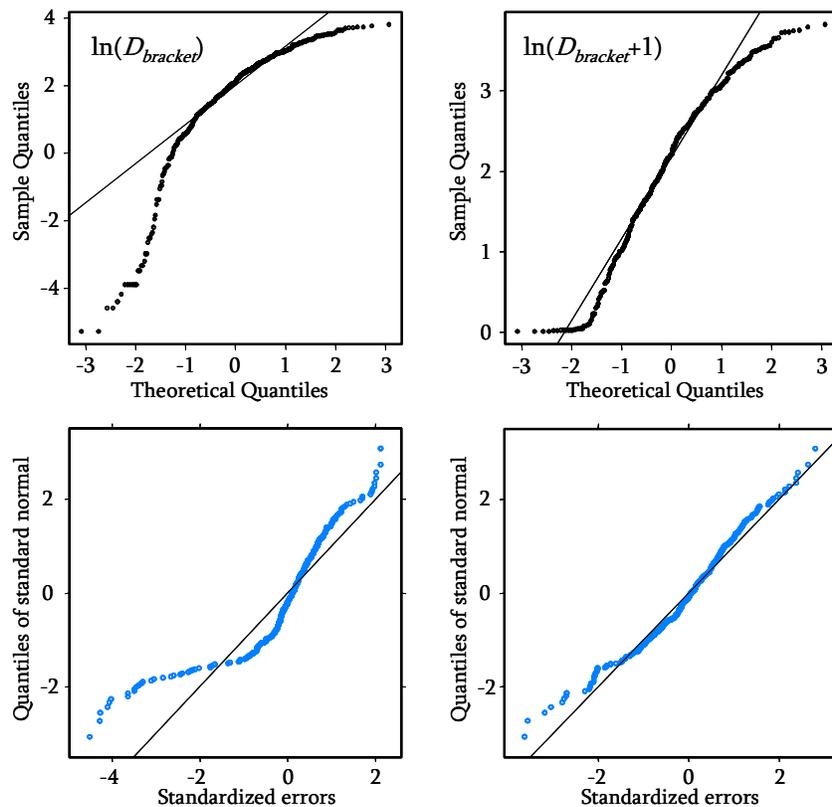


Figure 5. Comparisons of normal Q-Q plots for $\ln(D_{\text{bracket}})$ (left) and $\ln(D_{\text{bracket}} + 1)$ (right) and the errors resulted (bottom).

After considering numerous functional forms of the predictive relationship in the NLME regressions, the proposed model was found to provide the best fit of non-zero duration data, which is given by:

$$D_{bracket} = \exp(C_1 + C_2(M - 6) + C_3R + (S_1 + S_2R)S_S) - 1 \geq 0 \quad (1)$$

where $D_{bracket}$ is bracketed duration (sec); C_1 through C_3 , S_1 , and S_2 are regression coefficients; M is moment magnitude; R is the closest distance to the fault rupture plane (km); and S_S is a binary number representing local site conditions: $S_S = 0$ for rock sites, $S_S = 1$ for soil sites. Note that the proposed model shown as Eq. 1 was rewritten from its original form by taking exponential and subtracting 1 from both sides of the original equation, i.e.: $D_{bracket} = \exp[\ln(D_{bracket}+1)] - 1$. It is also noted that if a bracketed duration obtained from Eq. 1 is less than zero, zero should be used as a final predicted duration. The results from the NLME regression analyses of non-zero duration data are shown in Table 3.

Table 3. NLME Regression results: regression coefficients and standard deviations.

C_1	C_2	C_3	S_1	S_2	σ_{ln}^*
2.67	0.75	-0.0058	-0.16	0.0021	0.67

* The standard deviation values are valid for $\ln(D_{bracket}+1)$.

Combined Model with Weighting Function

To estimate the probability of the occurrence of a zero-duration motion, logistic regressions were implemented separately for each site condition, as a function of M and R . The logistic function is given by:

$$p(D_{bracket} = 0 | M, R) = \frac{e^{\beta_1 + \beta_2 M + \beta_3 R}}{1 + e^{\beta_1 + \beta_2 M + \beta_3 R}} \quad (2)$$

where $p(D_{bracket} = 0 | M, R)$ is the probability of zero-duration for a given M and R ; β_1 through β_3 are the regression coefficients determined from separate logistic regressions for each site condition. Then, the probability of non-zero duration occurrence is determined by subtracting the probability of zero-duration from the total probability of 1 as shown below:

$$p(D_{bracket} > 0 | M, R) = 1 - p(D_{bracket} = 0 | M, R) = \frac{1}{1 + e^{\beta_1 + \beta_2 M + \beta_3 R}} \quad (3)$$

The results of logistic regression are shown in Table 4.

Table 4. Logistic regression coefficients.

Site	β_1	β_2	β_3
Rock	9.47	-2.28	0.042
Soil	4.19	-1.32	0.025

Eq. 3 in conjunction with the regression coefficients for a given site condition is used as

the weighting function that is multiplied with Eq. 1. Finally, the combined model proposed for horizontal durations including zero-durations is given by:

$$D_{bracket} = \{\exp(C_1 + C_2(M - 6) + C_3R + (S_1 + S_2R)S_S) - 1\} \cdot p(D_{bracket} > 0 | M, R) \geq 0 \quad (4)$$

Using Eq. 4 in conjunction with the coefficients listed in Tables 3 and 4, the median bracketed durations predicted for CENA motions are shown in Fig. 6. As may be observed from this figure, the bracketed durations decrease with increasing distance, but increase with increasing magnitude. Significant dependences of durations on magnitude are observed, especially at distances below 50 km where an increase in one magnitude unit results in at least a twofold increase in duration.

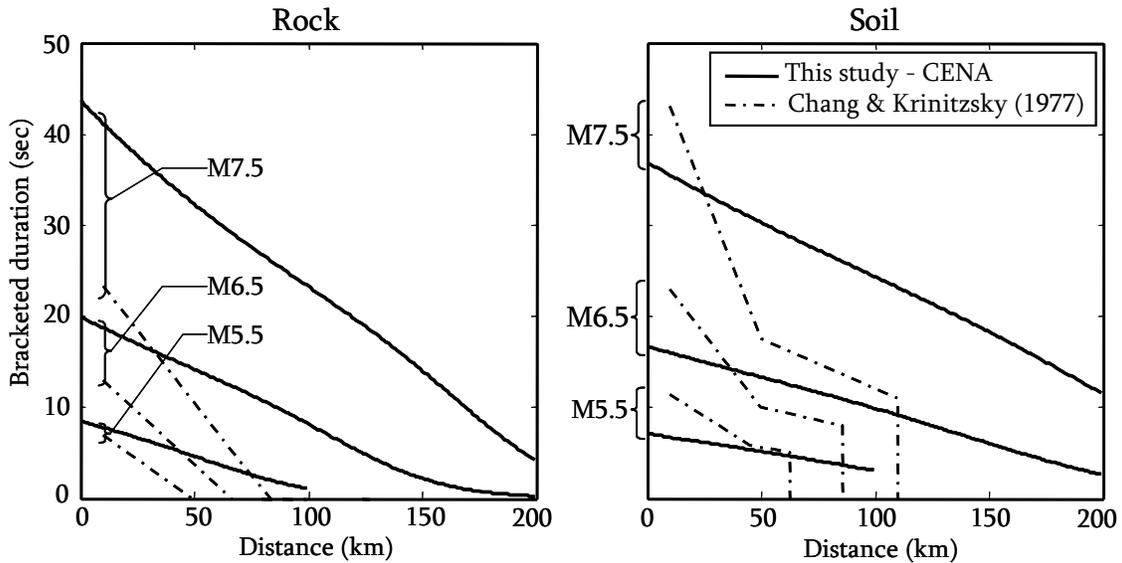


Figure 6. Predicted bracketed durations for CENA motions using Eq. 4. Also shown are the predicted bracketed durations for WNA using the relation proposed by Chang and Krinitzsky (1977).

Comparison with Existing Relationships

The bracketed duration relation proposed in this study is compared with the widely used model proposed by Chang and Krinitzsky (1977). Chang and Krinitzsky determined upper bounds of the bracketed durations for rock data and soil data from a limited ground motion data set of 201 horizontal ground motions from 25 WNA earthquakes, mostly from the 1971 San Fernando earthquake (M6.6). Chang and Krinitzsky (1977) did not give specifics on how they performed their regression analyses. However, they linear-extrapolated or interpolated their relationship developed from magnitude and distance ranges where data was available to ranges for which little-to-no data was available. Also, they truncated the durations for far field soil sites, based on zero-durations observed from the duration data from the 1952 Kern county earthquake (M7.7). Fig. 6 shows the comparison of the bracketed duration relations. Considerable differences exist between the predicted CENA motion durations using the relation developed in this study and those predicted for WNA motions using the relation from Chang and Krinitzsky

(1977). For rock motions, the CENA durations are significantly longer for all magnitudes and site-to-source distances, as compared to the WNA durations. However, for soil motions, the WNA durations are longer in the near field and shorter in the far field, as compared to the CENA durations.

Conclusions

An empirical predictive relationship for bracketed durations of horizontal strong ground motions in CENA has been developed in this study. Zero-durations were incorporated into the model through weighting functions representing the probability of non-zero duration. The bracketed durations were predicted to decrease with increasing distance, but to increase significantly with increasing magnitude. Comparing CENA and WNA motion durations for rock sites, the CENA motions had significantly longer durations than WNA motions at comparable magnitudes and site-to-source distances, where the WNA motion durations were predicted using an existing relation proposed by (Chang and Krinitzky, 1977). However, for soil sites, the WNA durations tended to be longer in the near field but shorter in the far field, in comparison with CENA motion durations at the same magnitude and site-to-source distance.

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