



THE INFLUENCE OF NEAR-FAULT RUPTURE DIRECTIVITY ON LIQUEFACTION

R.A. Green¹ and J. Lee²

ABSTRACT

The objective of the study presented herein is to assess the influence of rupture directivity on the inducement of liquefaction in loose, saturated sand. Rupture directivity is a near fault phenomenon that results in a pronounced double-sided velocity pulse in the strike normal component(s) of motion. Using the Richart-Newmark fatigue theory, the number-of-equivalent cycles were computed for sets of strike normal and strike parallel components of motions, where the former components had the pronounced velocity pulses and the latter did not. Using these results in conjunction with cyclic stresses computed from site response analyses, the cyclic stress ratios adjusted to M7.5 were computed for both components of motions in soil profiles at depths corresponding to ~1 atm of effective vertical stress. The uniqueness of this study is that the Richart-Newmark fatigue theory was employed, which accounts for the sequencing of pulses in the earthquake loading (this was not accounted for in previous studies that used the Palmgren-Miner fatigue theory). Two clear trends were identified. First, the strike normal components tended to induce larger cyclic stresses in the soil than the strike parallel components. However, the strike normal components of motions tended to have fewer numbers of equivalent cycles as compared to the strike parallel components. Although these trends are somewhat compensating in their influence on the inducement of liquefaction, the net result was that the motions containing the rupture directivity pulses had a slightly larger potential to induce liquefaction than motions without the pulses.

Introduction

The objective of the study presented herein is to assess the influence of "rupture directivity" on the inducement of liquefaction in loose, saturated sand. Rupture directivity (or "directivity") is one of two phenomena that can result in a pronounced velocity pulse in near-fault motions; the other phenomenon is referred to as "fling step" or "fling" and is not considered in this paper. Directivity is a Doppler-type phenomenon resulting from the approximate equality of the fault rupture and shear wave velocities and can result in a double-sided velocity pulse in the strike normal component(s) of motion. Several studies have examined the detrimental effects of near fault motions on building structures (e.g., Hall et al., 1995; Sasani and Bertero, 2000; Alavi and Krawinkler, 2001; Makris and Black, 2003; and Luco and Cornell, 2007), but relatively little attention has been given to near fault effects on liquefaction.

¹Associate Professor, Charles E. Via, Jr. Dept. of Civil and Environmental Engineering, Virginia Tech, Blacksburg, VA 24061

²Post-Doctoral Researcher, Charles E. Via, Jr. Dept. of Civil and Environmental Engineering, Virginia Tech, Blacksburg, VA 24061

To assess the influence of rupture directivity on the inducement of liquefaction, a series of site response analyses were performed to determine the cyclic stress ratios (CSR) at depth in a soil profile. The input motions used in the analyses consisted of twenty seven sets of strike normal and strike parallel components of motions, for a total of 54 motions and site response analyses. The strike normal component of each set of motions was identified as having velocity pulses that are likely due to rupture directivity, while the strike parallel component had no identifiable velocity pulse. To account for the influence of ground motion duration on the inducement of liquefaction, the number-of-equivalent cycles were computed for all the motions using the Richart-Newmark fatigue theory (Richart and Newmark, 1948; Green and Lee, 2006) and motion specific magnitude scaling factors (MSFs) determined. This allowed the cyclic stress ratios adjusted to M7.5 (i.e., $CSR_{M7.5}$) for the strike normal and strike parallel components for each set of motions to be computed and compared and the influence of near fault directivity effects to be discerned.

The study presented herein is a follow-on to a previous study that the authors performed examining the influence of directivity on liquefaction (Green et al., 2008). However, in the previous study, the authors used the Palmgren-Miner (P-M) fatigue theory to compute the number-of-equivalent cycles (Palmgren, 1924; Miner 1945; Green and Terri, 2005). In contrast, the study presented herein uses the Richart-Newmark (R-N) fatigue theory, the difference being that the R-N theory accounts for the sequencing of pulses in the earthquake loading while the P-M theory does not. The accounting of pulse sequencing in the earthquake loading is important because it has been shown that pulses preceding a large amplitude pulse have more of an influence on liquefaction than those that follow the large amplitude pulse (e.g., Ishihara and Yasuda, 1975).

In the subsequent sections of the paper, first the criteria used to select the ground motions included in this study are discussed. Next, an overview of the site response analyses is presented and trends in the resulting CSRs are discussed. This is followed by a presentation and discussion of the number-of-equivalent cycles for the motions and the corresponding motion specific MSFs. Finally, the $CSR_{M7.5}$ for the strike normal and strike parallel motions are compared and the influence of near-fault directivity on the inducement of liquefaction is discussed.

Selection of Ground Motions

As stated above, twenty seven sets of strike normal and strike parallel motions were used in this study. These motions were selected from a database of ninety one sets of strike normal and strike parallel motions, where all the strike normal components in the database were previously identified as being "pulse-like" by Baker (2007); a "pulse-like" motion is one that has a pronounced velocity pulse, which may or may not be due to near fault effects. To select the sets of motions believed to contain rupture directivity effects, first a visual comparison of the strike normal and strike parallel velocity time histories was made. Although somewhat subjective, in general, a set of motions was considered to contain directivity effects if the strike normal component was pulse-like, but the strike parallel component was not. The basis for this selection criterion is that rupture directivity will only result in a pronounced velocity pulse in the strike normal component(s), not in the strike parallel component. Next, all ninety one set of motions were evaluated using the following predictive equation (Baker, 2007):

$$PI = \frac{1}{1 + e^{-23.3 + 14.6 \cdot pgv_{ratio} + 20.5 \cdot energy_{ratio}}} \quad (1)$$

where, in this study: PI = pulse indicator ($0 < PI < 1$); pgv_{ratio} = the ratio of the peak ground velocities of the strike parallel and strike normal components of motion; and $energy_{ratio}$ = the ratio of the cumulative squared velocities of the strike parallel and the strike normal components of motion. For a set of motions, the closer PI is to unity, the more likely the motions contain directivity effects; sets of motions having $PI \geq 0.85$ were classified as having rupture directivity effects.

Baker (2007) originally developed Eqn. (1) and the $PI \geq 0.85$ criterion for identifying pulse-like motions, where he computed pgv_{ratio} and $energy_{ratio}$ using the "residual" motion in lieu of the strike parallel motion (the residual motion is the strike normal component of motion with the velocity pulse removed). However, with the exception of one set of motions, the initial visual classification and Eqn. (1), as implemented in this study, yielded the same results. And, upon a second visual inspection, the one set of motions in contention was reclassified as containing directivity effects. In total, forty two sets of motions were classified as having directivity effects.

The final criterion used to select sets of motions for this study had nothing to do with the rupture directivity phenomenon, but rather, relates to the limitations of the numerical method used in the site response analyses. Of the forty two sets of motions classified as having directivity effects, only those sets where both components had peak ground accelerations less than 0.5g (i.e., $pga < 0.5g$) were used. In total, twenty seven of the original ninety one sets of motions met all the selection criteria and were used as input motions in the site response analyses to compute the cyclic stress ratios (CSR) at depth in a soil profile. Table 1 lists the selected motions.

Site Response Analyses and CSR

A series of site response analyses were performed using the motions discussed above and CSRs were computed at a depth corresponding to ~ 1 atm vertical effective stress. The shear wave velocity profile for the soil profile used in the site response analyses is shown in Figure 1. The site response analyses were performed using a modified version of SHAKE91 (Idriss and Sun, 1992), with the nonlinear soil characteristics modeled using effective-stress-dependent shear modulus and damping degradation curves proposed by Ishibashi and Zhang (1993). All the motions were treated as rock outcrop motions at bedrock, irrespective of the actual site conditions at the recording seismograph stations.

For each of the analyses, the maximum shear stress (τ_{max}) induced in the soil at a depth of ~ 7.3 m (i.e., the depth corresponding to ~ 1 atm vertical effective stress (σ'_{vo})) was obtained and used to compute the CSR (e.g., Youd et al., 2001):

$$CSR = 0.65 \cdot \frac{\tau_{max}}{\sigma'_{vo}} \quad (2)$$

A plot of the CSRs induced by the strike normal motions versus those for the corresponding

strike parallel motions is shown in Figure 2a. As may be observed from this figure, the strike normal CSRs are generally larger than those for the strike parallel motions, and in some cases significantly so. Furthermore, as may be inferred from Figure 2b, this trend is not simply due to the strike normal input motions having higher pga's than those for the corresponding strike parallel input motions, as the disparity in CSRs is more significant than the disparity in the pga's. Rather, the differences in the CSRs are likely attributed to the rupture directivity velocity pulse in the strike normal motions. Finally, although Figure 2a shows that the strike normal motions tend to induce larger CSRs than the corresponding strike parallel motions, this does not necessarily imply that the former have greater potential to induce liquefaction than the latter, because no consideration has been given to the duration of the respective motions. Duration effects are addressed in the next section.

Table 1. Sets of motions selected for use in this study.

No.	Event	Year	Station	M _w	Distance* (km)
1	Coyote Lake	1979	Gilroy Array #6	5.7	3.1
2	Imperial Valley-06	1979	Aeropuerto Mexicali	6.5	0.3
3	Imperial Valley-06	1979	Agrarias	6.5	0.7
4	Imperial Valley-06	1979	EC Meloland Overpass FF	6.5	0.1
5	Imperial Valley-06	1979	El Centro Array #4	6.5	7.1
6	Imperial Valley-06	1979	El Centro Array #6	6.5	1.4
7	Imperial Valley-06	1979	El Centro Array #7	6.5	0.6
8	Morgan Hill	1984	Gilroy Array #6	6.2	9.9
9	Taiwan SMART1 (40)	1986	SMART1 C00	6.3	-
10	Taiwan SMART1 (40)	1986	SMART1 M07	6.3	-
11	Whittier Narrows-01	1987	Downey-company maintenance bldg	6.0	20.8
12	Whittier Narrows-01	1987	LB-Orange Ave.	6.0	24.5
13	Superstition Hills-02	1987	Parachute Test Site	6.5	1.0
14	Loma Prieta	1989	Gilroy Array #2	6.9	11.1
15	Loma Prieta	1989	Oakland – Outer Harbor Wharf	6.9	74.3
16	Loma Prieta	1989	Saratoga – Aloha Ave.	6.9	8.5
17	Erzican, Turkey	1992	Erzincan	6.7	4.4
18	Landers	1992	Barstow	7.3	34.9
19	Landers	1992	Yermo Fire Station	7.3	23.6
20	Kocaeli, Turkey	1999	Gebze	7.5	10.9
21	Chi-Chi, Taiwan	1999	TCU075	7.6	0.9
22	Chi-Chi, Taiwan	1999	TCU103	7.6	6.1
23	Chi-Chi, Taiwan	1999	TCU128	7.6	13.2
24	Northwest China-03	1997	Jiashi	6.1	-
25	Chi-Chi, Taiwan-03	1999	CHY080	6.2	22.4
26	Chi-Chi, Taiwan-03	1999	TCU076	6.2	14.7
27	Yountville	2000	Napa Fire Station #3	5.0	-

*Closest distance to the ruptured area on the fault

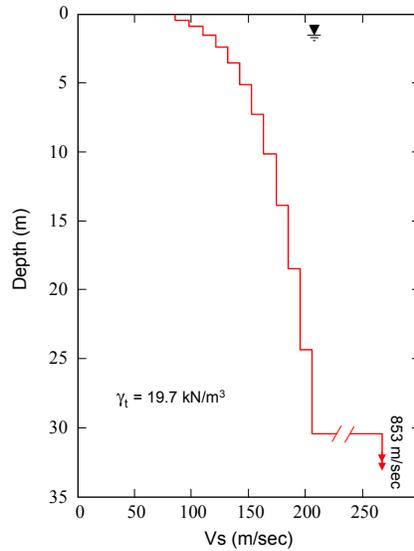


Figure 1. Shear wave velocity profile for the soil profile used in the site response analyses.

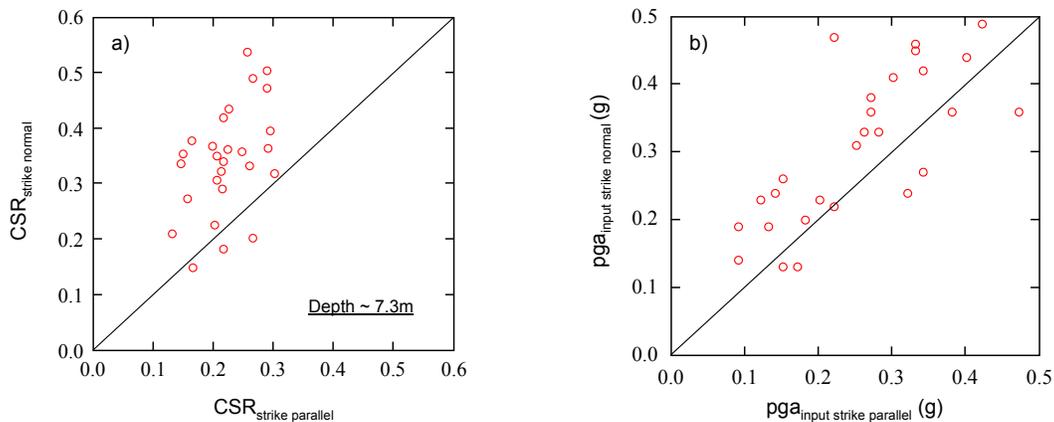


Figure 2. a) Strike normal CSR's vs. strike parallel CSR's for ~7.3m depth; and b) strike normal pga's of input motions vs. strike parallel pga's.

Duration Effects

Per the simplified liquefaction evaluation procedure (e.g., Youd et al., 2001), the influence of ground motion duration on the inducement of liquefaction is accounted for via magnitude scaling factors (MSFs), which can be computed using the equivalent-number-of-cycles concept (e.g., Seed et al., 1975). In this vein, the number-of-equivalent cycles (n_{eqv}) were computed for all the motions using the R-N fatigue theory (Richart and Newmark, 1948). In implementing the R-N theory, the authors first numerically computed the volumetric strains induced in a loose, dry sand subjected to each of the earthquake motions. Then, the same numerical model for loose, dry sand was subjected to a sinusoid motion having an amplitude equal to 65% of the peak amplitude of a given earthquake motion, and n_{eqv} was determined as the number of cycles of the sinusoidal motion that was required to induce the same volumetric strain in the sand as the respective earthquake motion. The numerical model used to compute the

volumetric strains is that proposed by Byrne (1991), calibrated to $N_{1,60} = 10$ blws/ft. This approach for computing n_{eqv} is outlined in detail in Green and Lee (2006) and accounts for both the absolute amplitude of the pulses in the ground motion and the sequencing of the pulses, both of which have been shown to influence the inducement of liquefaction (e.g., Ishihara and Yasuda, 1975). This is in contrast to the use of the P-M theory implemented for high cycle fatigue conditions (e.g., Seed et al., 1975; Liu et al., 2001) in which only the relative amplitudes of the pulses are accounted for or the P-M theory implemented for low cycle fatigue conditions (e.g., Green and Terri, 2005; Green et al., 2008) in which only the absolute amplitudes of the pulses are accounted for.

Although liquefaction is inherently an undrained phenomenon, the basis for using the volumetric strain induced in a loose, dry sand as the "damage" metric in the R-N fatigue theory to compute n_{eqv} is twofold. First, liquefaction is directly related to the tendency of the soil skeleton to contract during shearing (e.g., Martin et al., 1975) and undrained damage metrics such as excess pore pressure ratio (r_u) can "saturate" before the end of loading. In this context, "saturate" means that the damage metric can reach a limiting value (e.g., $r_u = 1$), at which point no additional number of cycles can be computed (e.g., Wer-Asturias, 1982).

Figure 3 is a plot of the computed n_{eqv} values for a depth of ~ 7.3 m. As may be observed from this figure, the strike normal motions tend to have fewer cycles than the corresponding strike parallel motions. This trend is consistent with the findings of Somerville et al. (1997) who found that the duration of motions having rupture directivity effects tend to have shorter durations than motions without these effects. Also shown in Figure 3 are contours of number-of-equivalent cycles for various site-to-source distances (R), defined as the closest distance to the fault, computed using a predictive relation developed by Lee (2009) and Lee and Green (2010), which does not account for near fault effects. Although there is scatter in the data, the contours are somewhat representative of the n_{eqv} for the strike parallel motions, but over predict n_{eqv} for the strike normal motions.

Having the n_{eqv} values, motion specific magnitude scaling factors (MSFs) can be computed using the following relation (e.g., Green, 2001):

$$MSF = \left(\frac{n_{eqv, M=7.5, R=80km}}{n_{eqv}} \right)^m \quad (3)$$

where, $n_{eqv, M=7.5, R=80km} = n_{eqv}$ for a "far field" motion from a M7.5 earthquake ("far field" is assumed in this study to be ~ 80 km); and m is an empirical constant determined from various laboratory studies to range from ~ 0.2 to ~ 0.34 . For this study, $n_{eqv, M=7.5, R=80km} = 13.3$ cycles and $m = 0.22$ were used to compute the motion specific MSFs. The former parameter value was determined using the predictive relation developed by Lee (2009) and Lee and Green (2010) and is somewhat consistent with $n_{eqv} = 15$ cycles for a M7.5 proposed by Seed et al. (1975). The latter parameter value was determined by an iterative approach where the largest value of m was selected that resulted in matching trends in dissipated energy and CSRs adjusted to a M7.5 (i.e., $CSR_{M7.5}$) with depth in the soil profile (Green et al., 2008). The resulting value of $m = 0.22$ falls within the experimentally determined range noted above. The motion specific MSFs are plotted

in Figure 4. Also plotted in this figure are the MSFs proposed by Seed and Idriss (1982), Youd et al. (2001) (range), and Cetin et al. (2004).

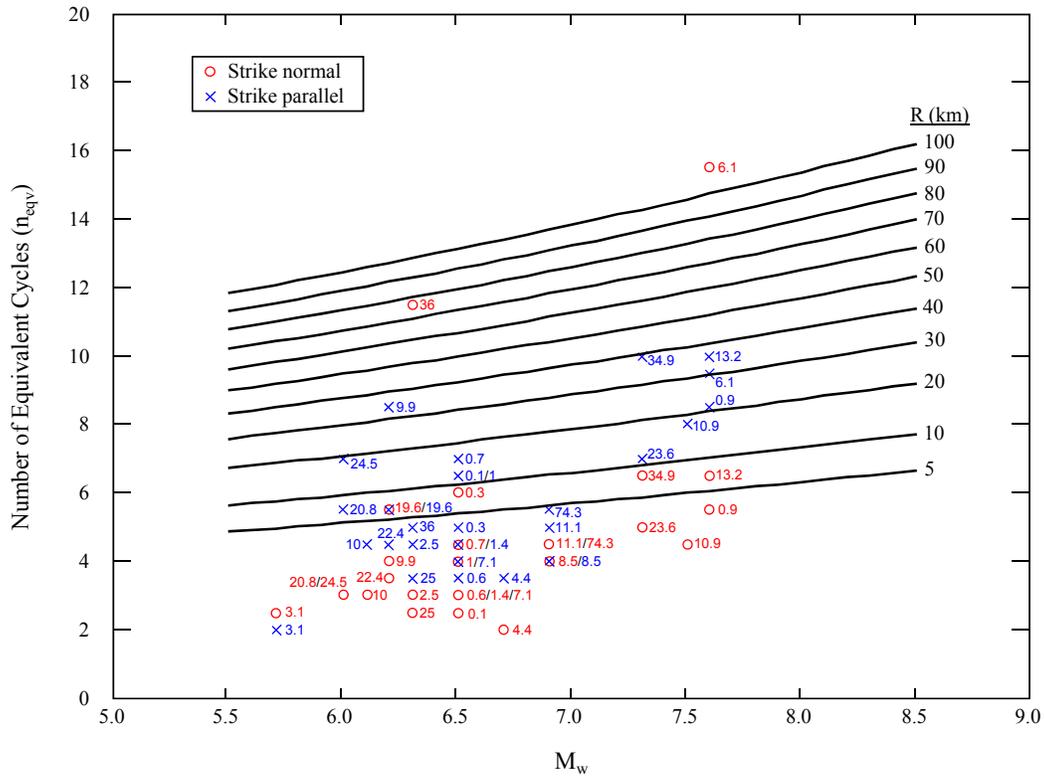


Figure 3. n_{eqv} for the strike normal and strike parallel motions vs. magnitude (numbers by data points is the corresponding site-to-source distance), and n_{eqv} contours for various site-to-source distances computed using a predictive relation by Lee (2009) and Lee and Green (2010).

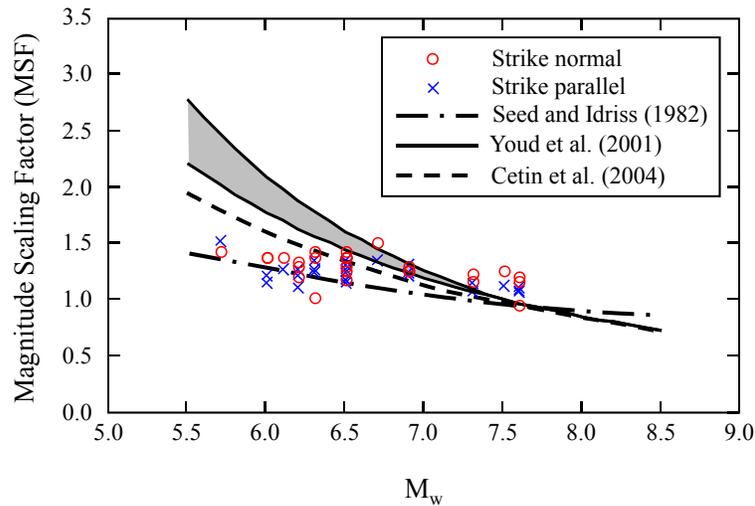


Figure 4. Motion specific MSFs and the MSFs proposed by Seed and Idriss (1982), Youd et al. (2001) (range), and Cetin et al. (2004).

As may be observed from Figure 4, the MSFs for the strike normal motions are generally greater than those for the strike parallel motions. Also, as may be observed from this figure, the motion specific MSFs for both the strike normal and strike parallel motions fall below those proposed by Youd et al. (2001) and Cetin et al. (2004) at lower magnitudes, but are reasonably consistent with the MSFs proposed by Seed and Idriss (1982). At larger magnitudes ($>M7.2$), the motion specific MSFs are higher than the three MSF relations. These trends are the result of several factors, but are primarily due to the three MSF relations being independent of site-to-source distance.

Near Fault Directivity Effects on Liquefaction

Up to this point, two opposing trends have been identified regarding the influence of rupture directivity on liquefaction. First, it was shown that directivity tends to result in an increase in the CSR induced in the soil (i.e., increasing the potential to induce liquefaction). Second, it was shown that directivity tends to result in motions having few number of cycles, or correspondingly, higher MSFs (i.e., decreasing the potential to induce liquefaction). The net effect of these two opposing trends can be examined by comparing the CSRs adjusted to $M7.5$ (i.e., $CSR_{M7.5}$) for the strike normal and strike parallel motions. CSR, MSF, and $CSR_{M7.5}$ are related by the following expression (e.g., Youd et al., 2001):

$$CSR_{M7.5} = CSR \cdot \frac{1}{MSF} \quad (4)$$

The resulting $CSR_{M7.5}$ induced in the soil profile at a depth of $\sim 7.3\text{m}$ for the strike normal and strike parallel motions are plotted in Figure 5. As may be observed from this figure, the strike normal $CSR_{M7.5}$ tend to be larger than the corresponding strike parallel values, implying that near fault directivity does increase the potential to induce liquefaction. However, the trend is not nearly as pronounced as that for CSR (Figure 2a).

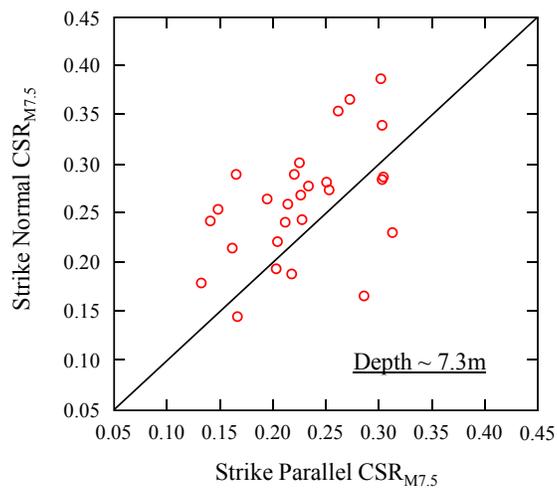


Figure 5. $CSR_{M7.5}$ for strike normal motions vs. $CSR_{M7.5}$ for strike parallel motions.

The results shown in Figure 5 are very similar to those in Green et al. (2008) wherein the MSFs were based on n_{eqv} computed using the P-M theory, as opposed to the R-N theory (i.e., pulse sequencing was not accounted for).

Conclusions

From the analysis of twenty seven sets of strike normal and strike parallel ground motions, where the former component had a pronounced velocity pulse due to rupture directivity and the latter did not, two opposing trends were identified relating to the potential of these motions to induce liquefaction. First, the strike normal components tended to induce larger cyclic stresses in the soil than the strike parallel components. However, the strike normal components of motions had fewer number-of-equivalent cycles, or correspondingly, had higher MSFs, as compared to the strike parallel components. These trends are somewhat compensating in their influence on the inducement of liquefaction, and the net result was that the motions containing the rupture directivity pulses had a slightly larger potential to induce liquefaction than motions without the pulses, as determined by comparing the $CSR_{M7.5}$ for the respective motions. This finding is similar to that of Green et al. (2008) wherein the MSFs were based on n_{eqv} computed using the P-M theory, as opposed to the R-N theory (i.e., pulse sequencing was not accounted for). Finally, the authors caution that these findings, while arrived at via a logical process, should be viewed as preliminary until they are rigorously tested by a detailed laboratory study and/or field observations.

Acknowledgements

The authors gratefully acknowledge contributions of Professor Jack Baker, Stanford University, and Capt. Theresa White, US Air Force Academy, on the previous study that the authors performed on near fault effects on liquefaction. The authors also appreciate the comments from the anonymous reviewer. This material is based upon work supported, in part, by the National Science Foundation under Grant No. CMMI 0644580. However, any opinions, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

References

- Alavi, B. and H. Krawinkler (2001). Effects of Near-Fault Ground Motions on Framed Structures, Blume Center Report 138, Stanford, CA, USA.
- Baker, J.W. (2007). Quantitative Classification of Near-Fault Ground Motions. *Bulletin of the Seismological Society of America* 97(5), 1486-1501.
- Byrne, P.M. (1991). A Cyclic Shear-Volume Coupling and Pore Pressure Model for Sand. *Proc. Second Intern. Conf. Recent Advances in Geotechnical Earthquake Engineering and Soil Dynamics*, March 11-15, 1991, St. Louis, Missouri, 47-55.
- Cetin, K.O., R.B. Seed, A. Der Kiureghian, K. Tokimatsu, L.F. Harder, R.E. Kayen, and R.E.S. Moss (2004). Standard Penetration Test-Based Probabilistic and Deterministic Assessment of Seismic Liquefaction Potential. *ASCE Journal of Geotechnical and Geoenvironmental Engineering* 130(12), 1314-1340.

- Green, R.A. (2001). Energy-Based Evaluation and Remediation of Liquefiable Soils. Ph.D. Dissertation, Department of Civil and Environmental Engineering, Virginia Tech, Blacksburg, VA, USA.
- Green, R.A. and J. Lee (2006). Computation of Number of Equivalent Strain Cycles: A Theoretical Framework, *Geomechanics II: Testing, Modeling, and Simulation* (P.V. Lade and T. Nakai, eds.), ASCE Geotechnical Special Publication 156, 471-487
- Green, R.A., J. Lee, T.M. White, and J.W. Baker (2008). The Significance of Near-Fault Effects on Liquefaction, *Proc. 14th World Conf. on Earthquake Engineering*, Paper No. S26-019.
- Green, R.A. and G.A. Terri (2005). Number of Equivalent Cycles Concept for Liquefaction Evaluations – Revisited. *ASCE Journal of Geotechnical and Geoenvironmental Engineering* 131(4), 477-488.
- Hall, J.F., T.H. Heaton, M.W. Halling, and D.J. Wald (1995). Near-Source Ground Motions and Its Effects on Flexible Buildings. *Earthquake Spectra* 11(4), 569-605.
- Idriss, I.M. and J.I. Sun (1992). SHAKE91: A Computer Program for Conducting Equivalent Linear Seismic Response Analyses of Horizontally Layered Soil Deposits. University of California at Davis, Davis, CA, USA.
- Ishibashi, I. and X. Zhang (1993). Unified Dynamic Shear Moduli and Damping Ratios of Sand and Clay. *Soils and Foundations* 33(1), 182-191.
- Ishihara, K. and S. Yasuda (1975). Sand Liquefaction in Hollow Cylinder Torsion Under Irregular Excitation, *Soils and Foundations* 15(1), 45-59
- Lee, J. (2009). Engineering Characterization of Earthquake Ground Motions, PhD Dissertation, Department of Civil and Environmental Engineering, University of Michigan, Ann Arbor, MI, 246pp.
- Lee, J. and R.A. Green (2010). Predictive Relations for Number of Equivalent Stress Cycles, *in preparation*.
- Liu, A.H., J.P. Stewart, N.A. Abrahamson, and Y. Moriwaki (2001). Equivalent Number of Uniform Stress Cycles for Soil Liquefaction Analysis, *Journal of Geotechnical and Geoenvironmental Engineering* 127(12), 1017-1026.
- Luco, N. and C.A. Cornell (2007). Structure-Specific Scalar Intensity Measures for Near-Source and Ordinary Earthquake Ground Motions. *Earthquake Spectra* 23(2), 357-392.
- Makris, N. and C. Black (2003). Dimensional Analysis of Inelastic Structures Subjected to Near Fault Ground Motions, Earthquake Engineering Research Center, EERC 2003-05, Berkeley, CA, USA.
- Martin, G.R., W.D.L. Finn, and H.B. Seed (1975). Fundamentals of Liquefaction under Cyclic Loading, ASCE, *Journal of the Geotechnical Engineering Division* 101(GT5), 423-438.
- Miner, M.A. (1945). "Cumulative Damage in Fatigue." *Transactions*, ASME, 67, A159-A164.
- Palmgren, A. (1924). "Die Lebensdauer Von Kugella Geru." *ZVDI*, 68(14), 339-341.

- Richart, F.E. and Newmark, N.M. (1948). "An Hypothesis for Determination of Cumulative Damage in Fatigue," *ASTM Proceedings*, 48: 767-800.
- Sasani, M. and V.V. Bertero (2000). Importance of Severe Pulse-Type Ground Motions in Performance-Based Engineering: Historical and Critical Review. *Proc. 12th World Conf. on Earthquake Engineering*, Auckland, New Zealand.
- Seed, H.B. and I.M. Idriss (1982). Ground Motions and Soil Liquefaction During Earthquakes, EERI Monograph, Earthquake Engineering Research Institute, Oakland, CA.
- Seed, H.B., I.M. Idriss, F. Makdisi, and N. Banerjee (1975). Representation of Irregular Stress Time Histories by Equivalent Uniform Stress Series in Liquefaction Analysis, Report No. EERC 75-29, Earthquake Engineering Research Center, College of Engineering, University of California, Berkeley.
- Somerville, P.G., N.F. Smith, R.W. Graves, and N.A. Abrahamson (1997). Modification of Empirical Strong Ground Motion Attenuation Relations to Include the Amplitude and Duration Effects of Rupture Directivity. *Seismological Research Letters* 68(1), 199-222.
- Wer-Asturias, R. (1982). The Equivalent Number of Cycles of Recorded Accelerograms for Soil Liquefaction Studies, M.S. Thesis, Department of Civil Engineering, Rensselaer Polytechnic Institute, Troy, NY.
- Youd, T.L., I.M. Idriss, R.D. Andrus, I. Arango, G. Castro, J.T. Christian, R. Dobry, W.D.L. Finn, L.F. Harder, M.E. Hynes, K. Ishihara, J.P. Koester, S.S.C. Liao, W.F. Marcuson, III, G.R. Martin, J.K. Mitchell, Y. Moriwaki, M.S. Power, P.K. Robertson, R.B. Seed, and K.H. Stokoe, II (2001). Liquefaction Resistance of Soils: Summary Report from the 1996 NCEER and 1998 NCEER/NSF Workshops on Evaluation of Liquefaction Resistance of Soils. *Journal of Geotechnical and Geoenvironmental Engineering* 127(4), 297-313.