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CORRELATION IN SPECTRAL ACCELERATIONS OF EUROPEAN GROUND MOTION RECORDS

G. P. Cimellaro[1] and A. De Stefano[2]

ABSTRACT

The shape of a Uniform Hazard Spectrum has been criticized to be unrealistic for a site where the spectral ordinates of the UHS at different periods govern by different scenario events and conservative for long-return-period earthquake shaking. Baker and Cornell introduced the Conditional Mean Spectrum (*CMS*), which considers the correlation of spectral demands (represented by values of ε) at different periods, to address these issues. A CMS estimates the median geometric-mean spectral acceleration response of a pair of ground motions given an M, r pair and a target spectral ordinate, $Sa(T_1)$, for which $\varepsilon(T_1)$ is back-calculated using an appropriate attenuation relationship. They developed models for the correlation coefficient $\rho_{\varepsilon(T1)\varepsilon(T2)}$ of $\varepsilon(T_1)$ and $\varepsilon(T_2)$, a key component for developing a CMS, using the PEER Strong Motion Database (http://peer.berkeley.edu/smcat/). This paper develops a model for $\rho_{\varepsilon(T1)\varepsilon(T2)}$ using the European earthquake records from the European ground motion database (http://www.isesd.hi.is/ESD_Local/frameset.htm). Epsilon (ε) for each record is computed using Ambraseys ground motion attenuation relationship. The model can be used to develop CMS for European sites and it can be considered to be incorporated in the seismic European standards (Eurocode 8).

Introduction

One widely used procedure for the scaling of ground motions is spectrally matching ground motions to a uniform hazard spectrum (*UHS*), but the shape of a *UHS* has been criticized to be unrealistic for a site where the spectral ordinates of the *UHS* at different periods govern by different scenarios (Baker and Cornell, 2006). Furthermore, the spectral ordinates of the *UHS* for long-return-period are associated with high values of ε across a wide range of period (Harmsen, 2001). If the geometric mean spectral ordinate of a ground motion pair attains the *UHS* ordinate at a given period, the geometric mean spectrum of the pair is unlikely to have ordinates as large as those of the *UHS* at other periods. To address the above issues, Baker and Cornell (2005, 2006b) introduced the Conditional Mean Spectrum (*CMS*), which consider the correlation of spectral demands (represented by values of ε) at different periods. The parameter

[1]Assistant Professor, Dept. of Structural and Geotechnical Engineering (DISTR), Politecnico di Torino, 10129 Torino, Italy
[2] Professor, Dept. of Structural and Geotechnical Engineering (DISTR), Politecnico di Torino, 10129 Torino, Italy

ε is a measure of the difference between the spectral acceleration of a record and the median spectral demand predicted by an attenuation relationship for the [M, r] pair of the record at a given period, normalized by the logarithmic standard deviation obtained by the attenuation relationship (see Equation (2)). In other words, *ε* specifies the number of logarithmic standard deviations away from the median ground motion model. The *CMS* estimates the median geometric mean spectral acceleration response of a pair of ground motions given a magnitude M and a distance R and a target spectral ordinate $\bar{S}_a\left(T_1\right)$ where T_1 is the first natural period of the structure of interest, where the parameter $\varepsilon(T_1)$ is back-calculated using an appropriate attenuation relationship as it will be described below. The *CMS-ε* has been introduced in an appendix of the 50% draft *Guidelines for the Seismic Performance Assessment of Buildings* developed in the United States ATC-58 project, which is developing the next-generation tools and procedures for performance-based earthquake engineering (ATC-58, 2008).

The scope of the paper is to develop a model for the correlation coefficient $\rho_{\varepsilon(T_1)\varepsilon(T_2)}$ at European sites that is a key parameter for deriving the CMS as shown in Equation (1) below.

The model has been calibrated using 481 earthquake records that have been recorded in Europe and adjacent regions, with surface wave magnitude M_s between 4.0 and 7.9 and focal depth less than or equal to 30 km. A model for the cross correlation coefficients of orthogonal components is also presented. CMS-ε for European sites can be considered to be incorporated in the seismic European standards (Eurocode 8).

Predictive model for spectral shape of European sites

Development of CMS-epsilon

This new target spectrum, called conditional mean spectrum has been developed first for analysis of nuclear facilities (Nuclear Regulatory Commission, 1997), however Baker and Cornell (2006) incorporate also the effect of ε in the procedure, developing the conditional mean spectrum considering ε (CMS-ε) that accounts for the relationship between ε and spectral shape. To develop a CMS-ε, PSHA is used to find the $S_a(T_1)$ value corresponding to the target probability of exceedance at the site of interest. Disaggregation can then be used to find the modal [M, R and ε] values, denoted as $\bar{M}$, $\bar{R}$ and $\bar{\varepsilon}$, respectively, associated with the $S_a(T_1)$ level. Given the value of $\bar{\varepsilon}$ at T_1, denoted $\bar{\varepsilon}\left(T_1\right)$, the conditional mean values of S_a at other periods can be valuated using the following equation

$$\mu_{log_{10}(S_a(T_2))|log_{10}(S_a(T_1))} = \mu_{log_{10}(S_a(T_2))}\left(\bar{M},\bar{R},T_2\right) + \sigma_{log_{10}(S_a)}\left(\bar{M},T_2\right)\cdot\rho_{\varepsilon(T_1)\varepsilon(T_2)}\cdot\bar{\varepsilon}\left(T_1\right) \qquad (1)$$

where $\bar{M}$, $\bar{R}$, and $\bar{\varepsilon}\left(T_1\right)$ are the mean magnitude, distance and epsilon values that come from disaggregation at the site when $S_a(T_1)$ denotes the target spectral acceleration. The terms $\mu_{log_{10}(S_a(T_2))}\left(\bar{M},\bar{R},T_2\right)$ and $\sigma_{log_{10}(S_a)}\left(\bar{M},T_2\right)$ are evaluated from Ambraseys AR (Ambraseys et al., 1996); $\rho_{\varepsilon(T_1)\varepsilon(T_2)}$ is the correlation coefficient for ε at different periods determined from regression of empirical observations in next sections.

Ground motion data

The strong motion recordings used in this study to derive the correlation equations for the European sites, consists of 481 triaxial records extracted from the European ground motion database (http://www.isesd.hi.is/ESD_Local/frameset.htm).

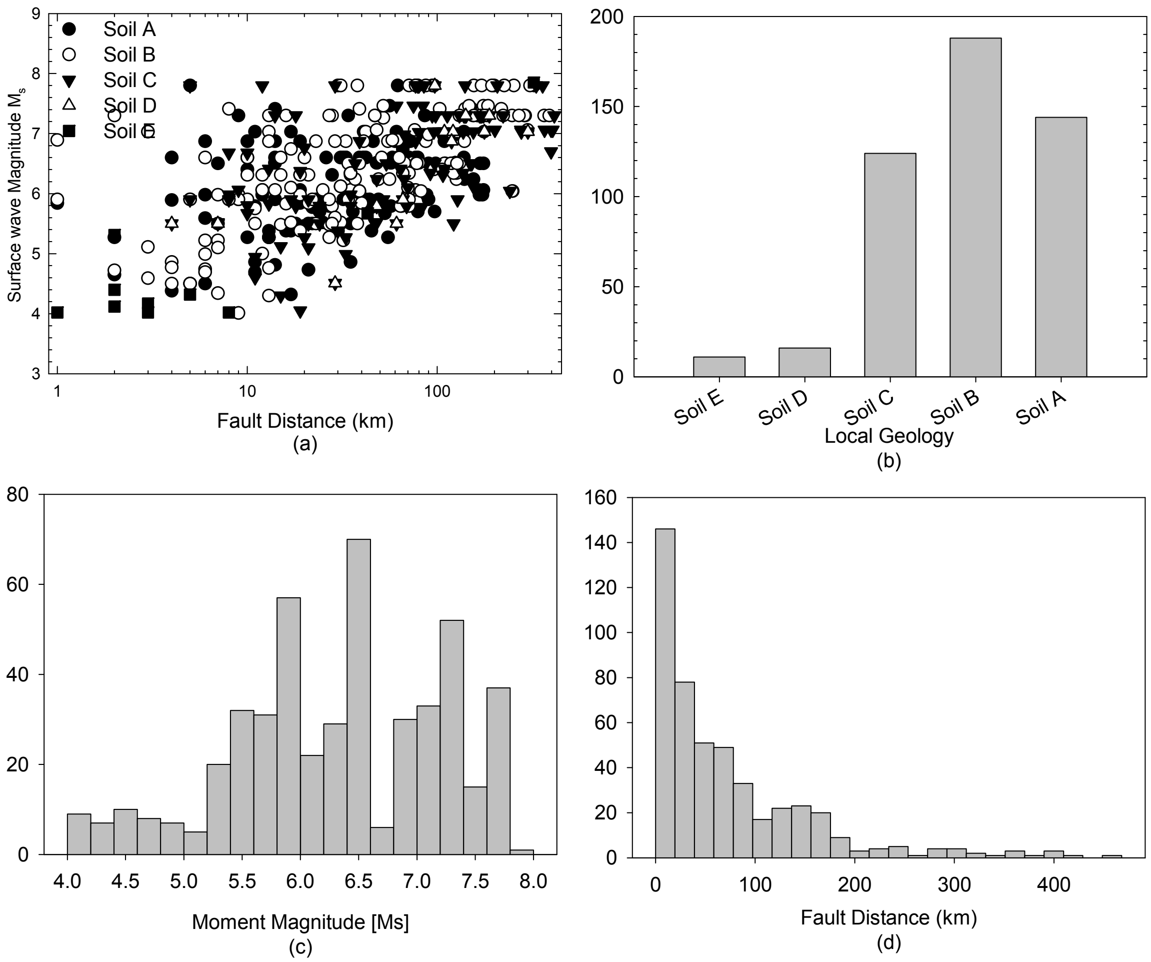

Figure 1 Histograms of strong motion recordings with respect to magnitude, fault distance and site geology for European sites

The records have been recorded in Europe and adjacent regions, with surface wave magnitude Ms between 4.0 and 7.9 and focal depth less than or equal to 30 km.

The lower limit of the magnitude was chosen because smaller earthquakes are generally not of engineering significance. The selected earthquakes have a fault distance between 0 and 450km. The recordings sites were classified according to the soil classification given in the Eurocode 8 where Soil A is rock, and so on (Figure 1). The database mostly coincides with the one used in the study of Ambraseys et al. (1996) to develop the European attenuation relationship of peak ground acceleration. It is important to mention that Ambraseys AR was developed using the shortest distance from the station to the surface projection of the fault rupture, in km and the surface wave magnitude M_S that is commonly used to describe shallow earthquakes that correspond to the dataset used in this study.

Figure 1 summarizes the distributions of the recordings used with respect to magnitude, fault distance and site classification. The ground motions in the selected dataset are within the range of applicability of Ambraseys attenuation relationship.

Development of correlation equations

Attenuation relationships describe the probability distribution of spectral acceleration at an individual period, but do not address the correlations in spectral accelerations at different periods and orientations. Those correlations are needed to develop the *CMS-ε* at a given site, and can be also useful for seismic hazard analysis and ground motion selection.

In a *CMS*, correlation coefficients ρ are used to capture the correlation in the values of ε at different periods. For example, if a recorded spectral acceleration is stronger than expected (i.e. $\varepsilon(T)$ is greater than 0) at a given period, then it is likely to also be stronger than expected at adjacent periods.

In this section is described the methodology for measuring and predicting these correlations coefficients. New correlation equations are developed by the authors and results are examined and compared with previous models (Baker and Cornell, 2006; Baker and Jayaram, 2008) opportunely recalibrated for the European sites. The ε values were computed for each record of the dataset at a range of periods using Ambraseys ground motion prediction model. For each record and period of the dataset, ε values were computed using the following equation

$$\varepsilon(T) = \frac{log_{10}\left(S_a(T)\right) - \mu_{log_{10}(S_a(T))}(M,R,T)}{\sigma_{log_{10}(S_a(T))}(M,T)} \quad (2)$$

where $log_{10}(S_a(T))$ is the logarithm in base *10* of the observed spectral acceleration value, while $\mu_{log10(Sa(T))}$ and $\sigma_{log10(Sa(T))}$ are evaluated from the attenuation relationship.

The empirical correlation coefficients between $\varepsilon(T_1)$ and $\varepsilon(T_2)$ were obtained using the commercial software MATLAB. Several linear and nonlinear equations were fitted to the empirical data of the correlation coefficients and results were sorted using as goodness of fit measure: the r^2 value. Finally, the predictive equations were selected based on the number of parameters adopted, the simplicity and the goodness of fit.

Observed correlation and predictive equations

Empirical correlation coefficients for the attenuation relationship of *Ambraseys* are presented in Figure 2a, using contours as function of T_1 and T_2. These coefficients could be tabulated and used in a look-up table when needed, but the table would be difficult to transfer or reproduce in print. For this reason, the empirical correlation coefficients were fit with an analytical predictive equation that is easier to communicate. First as preliminary study the model proposed by Baker and Cornell (Baker and Cornell, 2006) for the US sites has been used and nonlinear least square regression is used to find the associated coefficients that are suitable for the dataset of the European sites. The predictive correlation coefficient model is then given by

$$\rho_{\varepsilon(T_1)\varepsilon(T_2)} = 1 - \cos\left(\frac{\pi}{2} - \left(0.1608 - 0.3005 I_{(T_{\min} < 0.1377)} \ \ln\left(\frac{T_{\min}}{0.0824}\right)\right)\ln\left(\frac{T_{\max}}{T_{\min}}\right)\right) \tag{3}$$

where $T_{min}=min(T_1,T_2)$ and $T_{max}=max(T_1,T_2)$, and $I_{(Tmin<0.0824)}$ is an indicator function equal to 1 if T_{min}<0.0824s and equal to 0 otherwise. The results of Equation (3) are plotted in Figure 2b. Another analytical predictive equation proposed recently by Baker and Jayaram (2008) for US sites has been also considered and associated coefficients are determined analogously to Equation (3). The predictive correlation coefficient model for European site is then given by

$$\rho_{\varepsilon(T_1)\varepsilon(T_2)} = \begin{cases} C_2 & T_{\max} < 0.0312 \\ C_1 & T_{\min} > 0.0312 \\ \min\left(C_2, C_4\right) & T_{\max} < 0.1922 \\ C_4 & else \end{cases} \tag{4}$$

$$C_1 = 1 - \cos\left(\frac{\pi}{2} - 0.4159 \cdot \ln\left(\frac{T_{\max}}{\max\left(T_{\min}, 0.0312\right)}\right)\right) \tag{5}$$

$$C_2 = \begin{cases} 1 - 0.0203\left(1 - \frac{1}{1 + e^{194 T_{\max} - 3.7540}}\right)\left(\frac{T_{\max} - T_{\min}}{T_{\max} - 0.0152}\right) & T_{\max} < 0.1922 \\ 0 & otherwise \end{cases} \tag{6}$$

$$C_3 = \begin{cases} C_2 & T_{\max} < 0.0312 \\ C_1 & otherwise \end{cases} \tag{7}$$

$$C_4 = C_1 + 0.4274\left(\sqrt{C_3} - C_3\right)\left(1 + \cos\left(\frac{\pi T_{\min}}{0.0312}\right)\right) \tag{8}$$

where $T_{min}=min(T_1,T_2)$ and $T_{max}=max(T_1,T_2)$. The results of Equation (4) are plotted in Figure 2c. Results in Figure 2 clearly show that neither the analytical model in Equation (3) nor Equation (4) is able to describe properly the empirical correlation coefficients. This is somehow expected because both models were built based on US dataset of ground motion records.

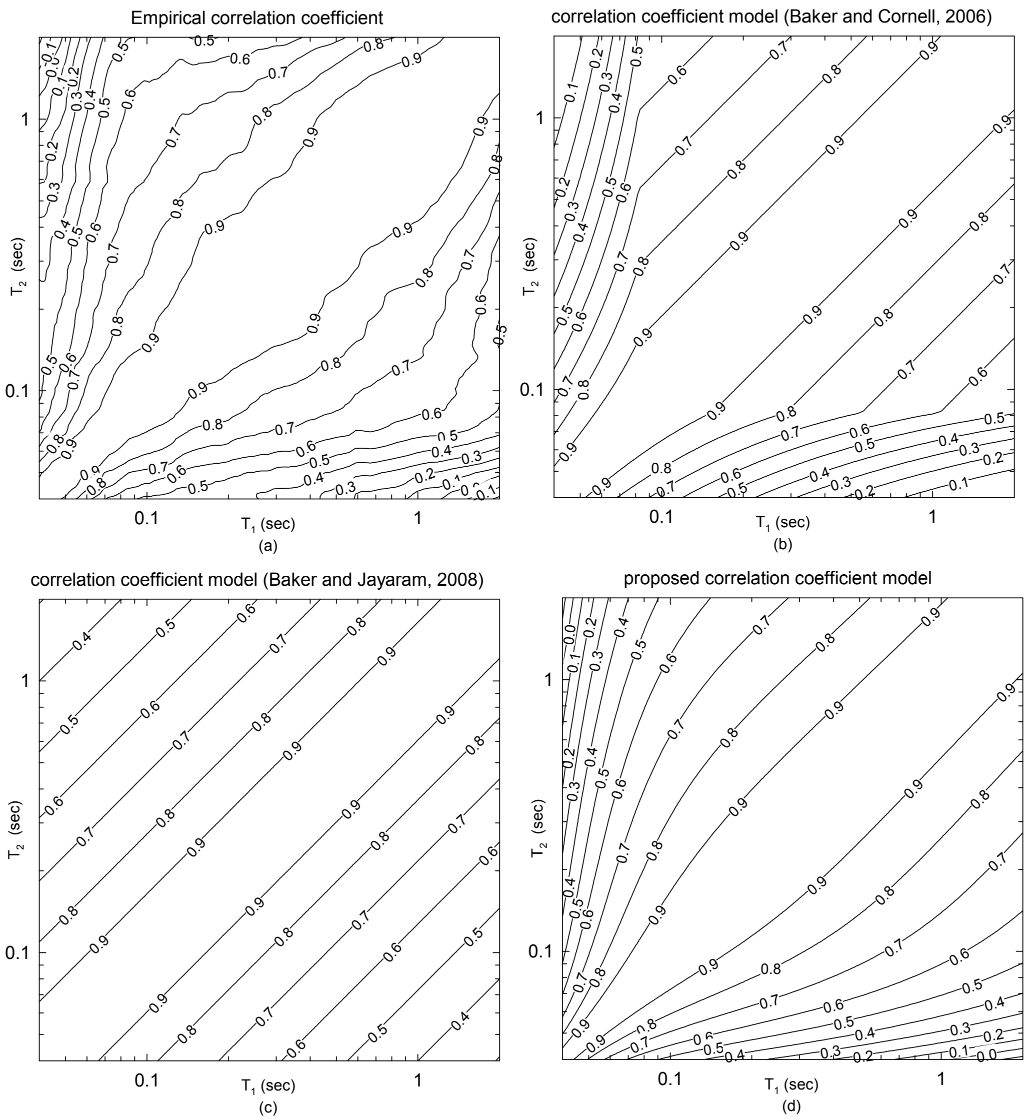

Figure 2 Contours of predicted correlation coefficients versus T1 and T2 for Ambraseys AR. (a) empirical correlation coefficients. (b) Predicted correlation coefficients using Baker and Cornell (2006). (c) predicted correlation coefficients using Chiou and Young (2008); predicted correlation coefficients using equation (9)

Therefore, there is need for a new predictive model. The proposed analytical predictive model is based on Taylor series rationals and is then given by

$$\rho_{\varepsilon(T_1)\varepsilon(T_2)} = 1 - \left(a + \frac{b}{T_1} + \frac{c}{T_2} + \frac{d}{T_1^2} + \frac{e}{T_2^2} + \frac{f}{T_1 T_2} \right) \ln\left(\frac{T_1}{T_2} \right); \tag{9}$$

where *a=-0.1644; b=0.0046; c=0.0090; d=-0.004; e=-0.0003; f=-0.0014*. The form of the equation has no physical interpretation: it is simply a fit to observed data. The results of Equation (9) are plotted in Figure 2d.

The three correlation coefficients analytical models are compared at four selected period T_2, and presented as a function of T_1 between 0.04 and 2 seconds in Figure 3.

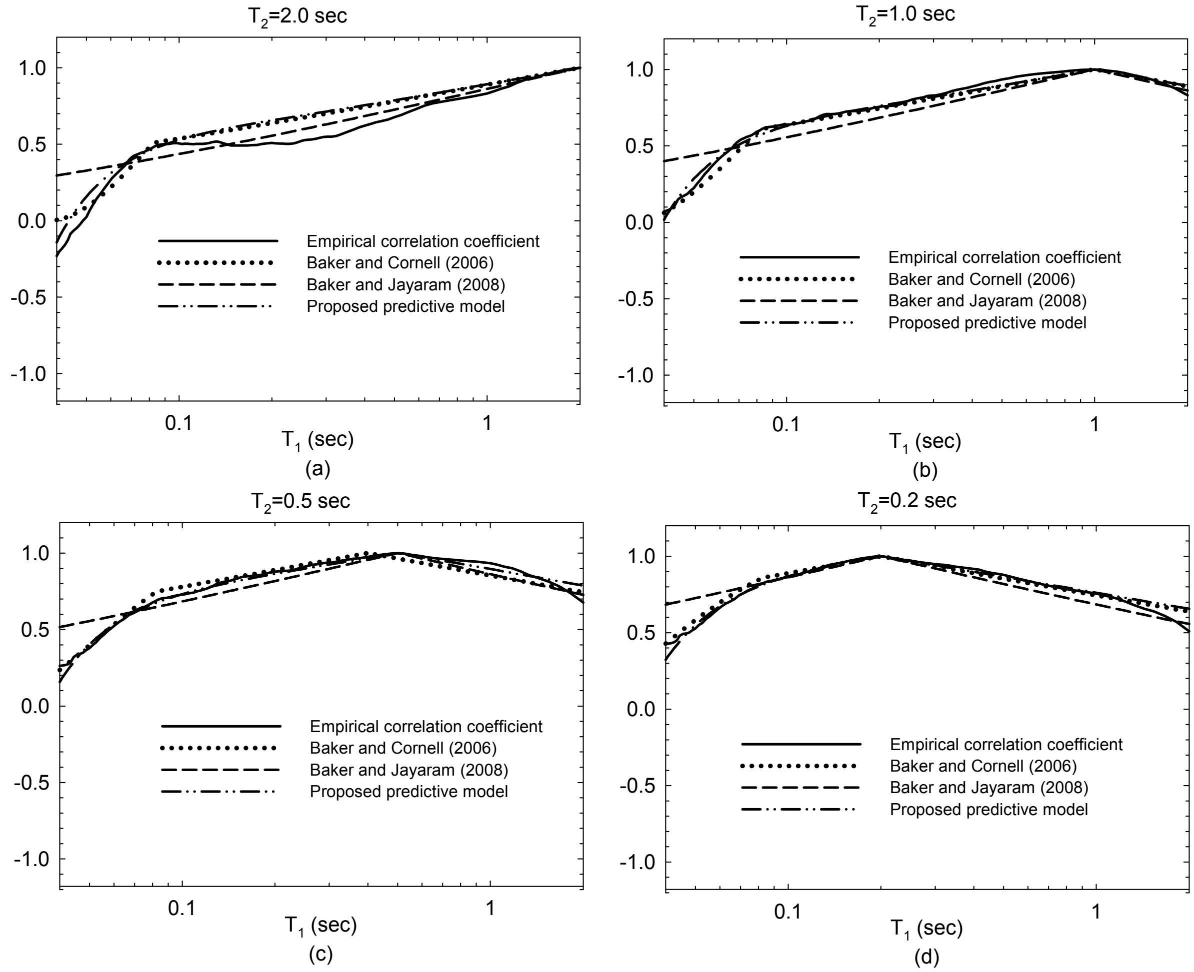

Figure 3 Comparison of the three predictive models for four T2 values using *Ambraseys AR*

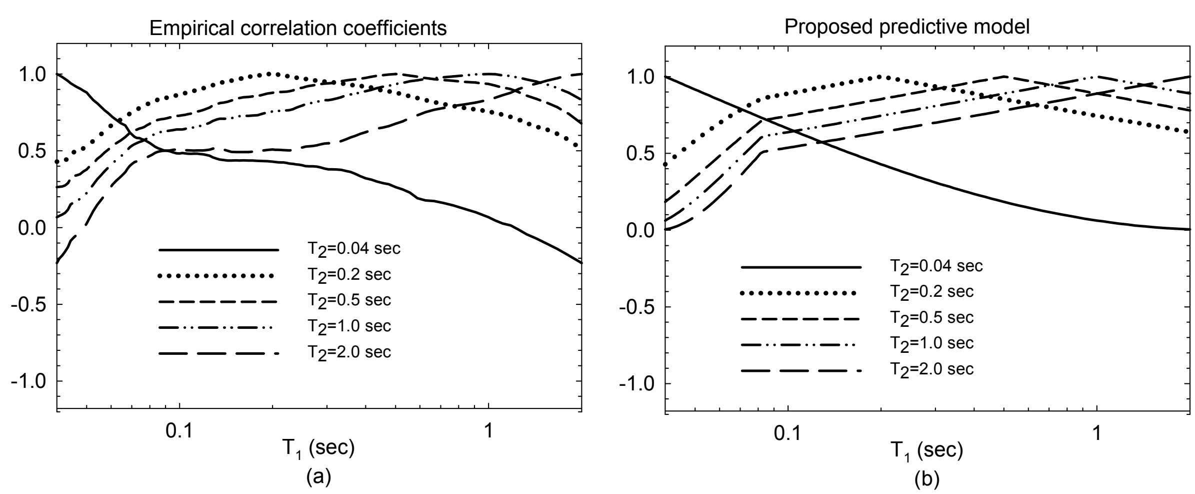

Figure 4 (a) Empirical correlation coefficients and (b) proposed predicted model (2009) versus T_1, for several T_2 values using *Ambraseys AR*

All models except for *Baker and Jayaram* model closely match over most of the periods.
The proposed model and the fitted predictive model of *Baker and Cornell* are able to provide similar results although the proposed model is able to give better estimations at small periods T_1 and for high values of T_2.

Figure 4 shows the empirical correlation coefficients and the prediction of the proposed model for a selected set of period T_2, plotted versus T_1 values between 0.04 and 2 seconds.

Correlation of spectral acceleration values of orthogonal ground motion components

Correlations in spectral accelerations of orthogonal components of ground motions are also of interest for engineers for determining the distribution of spectral amplitudes when analyzing 3-dimensional structures.

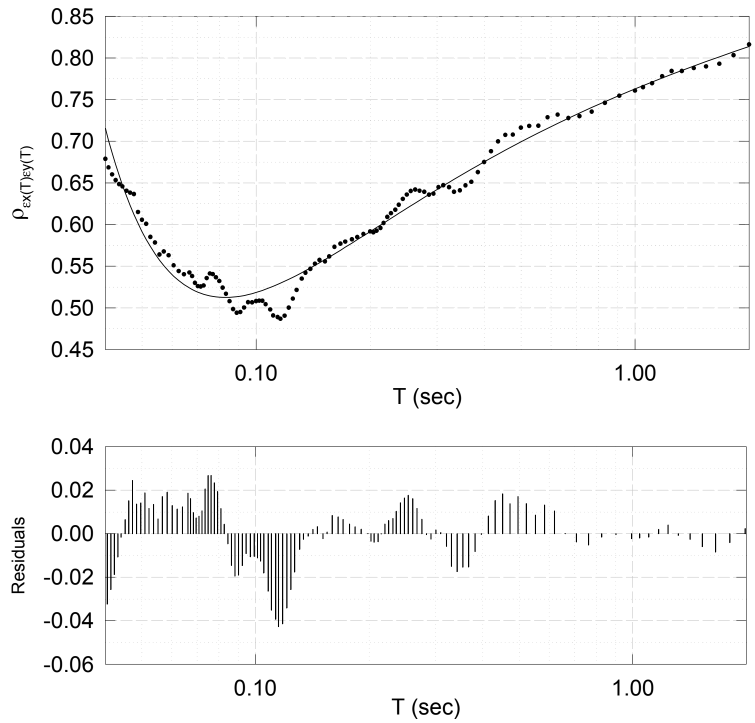

Figure 5 Observed and predicted correlations of ε values from orthogonal components of ground motion at a single period using *Ambraseys AR*

Empirical correlation coefficients are shown in Figure 5 for S_a values of orthogonal ground motion components, when both S_a values have the same period. The predictive model suggested in this case is

$$\rho_{\varepsilon_x(T)\varepsilon_y(T)} = a + \frac{b}{\sqrt{T}} + cT + \frac{d}{T^2} \qquad (10)$$

where $\rho_{\varepsilon x(T)\varepsilon y(T)}$ is used to denote the correlation between two epsilons, ε_x and ε_y, associated with orthogonal ground motion components at a given period T. Using *Ambraseys* AR the coefficients of the model in equation (10) are a=0.906, b=-0.151, c=0.007 and d=0.001. The r-squared

values of the model described in equation (10) is r^2=0.9712.
Results of empirical and predictive model together with residuals are shown in Figure 5.
In the case of spectral acceleration values in orthogonal directions with different periods, the correlations can be estimated using the following equation

$$\rho_{\varepsilon_x(T_1)\varepsilon_y(T_2)} = a + \frac{b}{\sqrt{T_{\min}}} + cT_{\max} + \frac{d}{T_{\max}^2}; \tag{11}$$

where $T_{min}=min(T_1,T_2)$ and $T_{max}=max(T_1,T_2)$ and the coefficients are a=1.1409; b=-0.2033; c=-0.1909; d=0.0011. The r-squared values of the model described in equation (11) is r^2=0.9684.

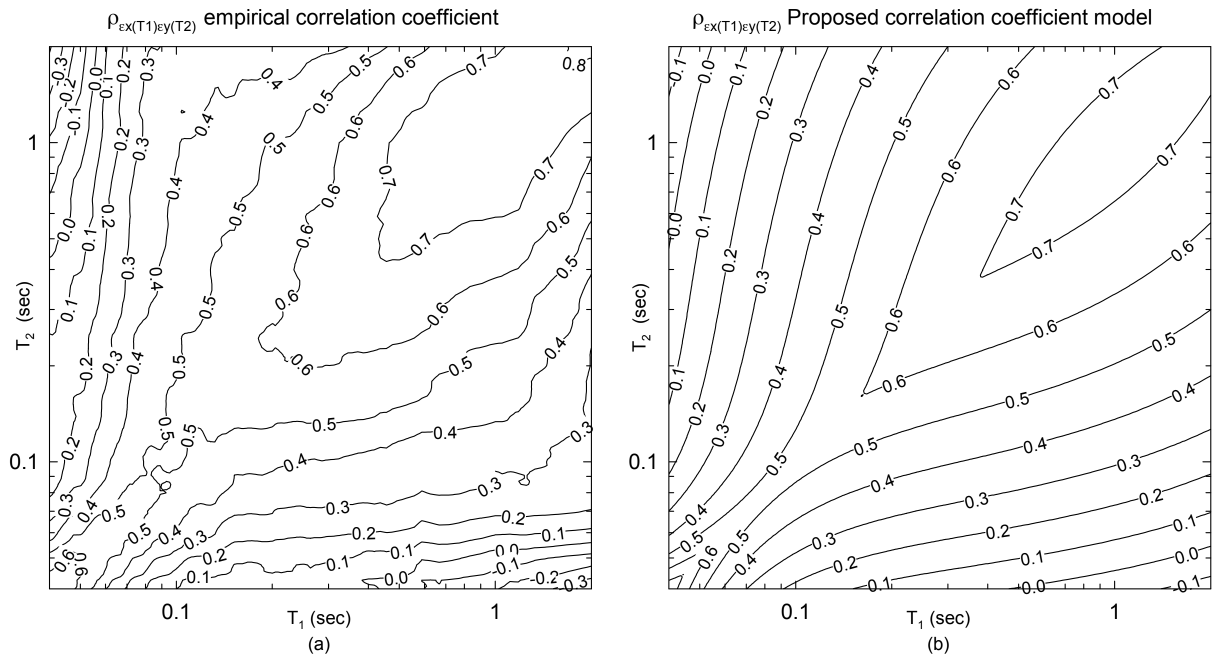

Figure 6 Contours of predicted correlation coefficients versus T_1 and T_2 for orthogonal components of ground motion using *Ambraseys AR*. (a) empirical correlation coefficients. (b) Predicted correlation coefficients using equation 11

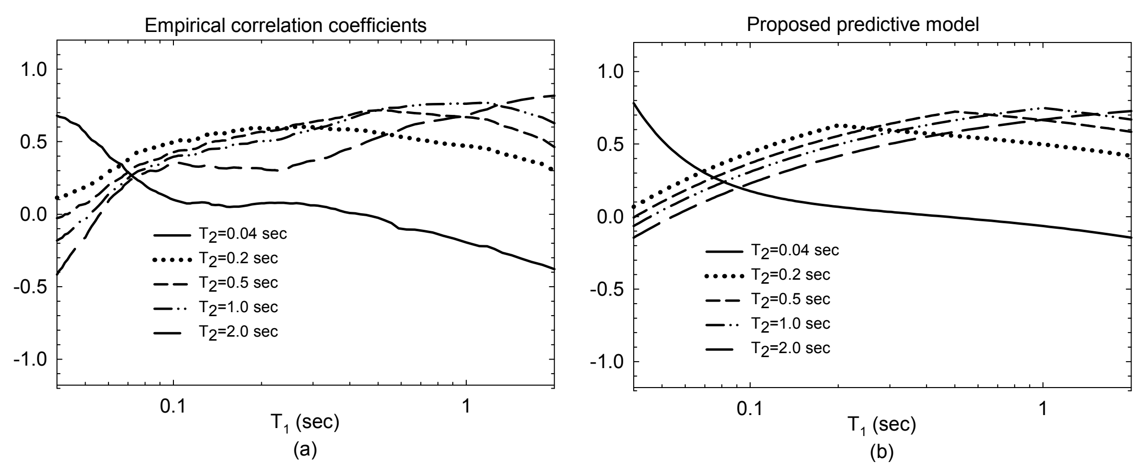

Figure 7 Plot of the empirical (a) and predicted (b) correlation coefficients versus T_1 for several T_2 values using *Ambraseys AR*

Conclusions

Predictive analytical models for the correlation coefficient of spectral accelerations are proposed using European earthquake records extracted from an European ground motion database (http://www.isesd.hi.is/ESD_Local/frameset.htm). The proposed correlation equations developed by the authors are examined and compared with previous models opportunely recalibrated for the European sites. Epsilon (ε) for each record is computed using Ambraseys ground motion attenuation relationship. The model can be used to develop CMS for European sites. In addition, also a predictive model for the correlation coefficients of spectral acceleration values of orthogonal ground motion components are proposed, for the case when the period is the same and for different periods.

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