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PROBABILISTIC MODELS FOR SEISMIC DAMAGE AND SUBSEQUENT LOSSES

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ABSTRACT

This paper develops probabilistic models to predict the cost and time of repair of structures subjected to ground motion. The models will be implemented in a new framework for performance-based earthquake engineering. The two key ingredients of this framework are probabilistic models and reliability analysis. The probabilistic damage models developed in this paper predict the visual damage in components. This is useful because the visual damage determines the repair action. In turn, probabilistic models for the cost and time of the repair actions are developed. In the continuation of the work presented here, both structural and non-structural components are considered.

Introduction

An important objective in the presented work is to account for the unavoidable uncertainties involved in the prediction of damage and loss due to earthquakes. The type of probabilistic models employed in this work account for both irreducible (aleatory) uncertainty in physical parameters and reducible (epistemic) model uncertainty. The uncertainty is characterized by random variables. Given a realization of the random variables, the probabilistic models produce one unique result. Hence, the output from the models is in terms of measurable quantities of damage, instead of the conditional probabilities that are needed in total probability approaches. In the approach adopted in this paper, reliability analysis is used instead to obtain loss probabilities.

To relate structural response to damage, the traditional approach involves damage indices. A damage index is a non-dimensional variable between zero (no damage) and unity (complete failure). The value of a damage index is correlated with states of physical damage. Damage states divide the continuous progression of damage in a structure to a few broad categories such as “slight damage” or “complete damage”. Williams and Sexsmith (1995) give a review of seismic damage indices developed for concrete structures. More recently, the concept of fragility functions has become widespread. These functions provide the conditional probability of being in a damage state for a given ground shaking intensity or structural response. When a structural response is used as input to the fragility function, this response is usually selected as the most significant engineering demand parameter (EDP), such as drift ratio for structural components. Such probability functions are developed based on experimental data and expert opinion and by assuming a distribution type (usually lognormal) to the observed data. Fragilities are central in the ATC-58 approach provided in the recent seismic design guidelines

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from the Applied Technology Council, as well as in the approach adopted by the Pacific Earthquake Engineering Research center (PEER) (Porter 2003). In these methodologies, the development of fragility functions is proposed to relate a structural response, such as drift ratio to the probability of being in a specific damage level such as cracking in a concrete component. Loss fragilities are also suggested to determine the probability of exceeding a certain loss threshold for a given level of damage. Pagni (2003) has developed fragility curves for various repair methods used in older RC components. Discrete damage states representing the progression of damage in older RC beam-column joints are identified based on which the necessary repair methods are selected. The fragility curves are obtained based on existing experimental data. For given values of EDPs, the fragility curves predict the probability of the required repair method. For each repair method, cost of repair is determined as a product of the type of damage, size of the component and the unit cost of repair including cost of labor, material, equipment, *etc*. Markups such as contractor' profit and overhead are then applied on the total repair cost of the building. In the approach taken by Yang *et al*. (2009), the EDPs to generate damage state fragility curves are determined by a limited number of non-linear dynamic analyses and artificially generated random variables. Lookup tables are used to determine the cost associated for the repair of each damage state; however, in order to estimate a final loss value, a uniform random number generator is used to determine the damage state for a given level of demand.

This paper proposes an alternative methodology to the fragility-curve approach. The development of probabilistic models allows for the contribution of several variables rather than only the most significant variable in determining damage. The unavoidable uncertainty involved in the prediction of damage and loss is explicitly taken into account. The outputs of such probabilistic models are deterministic estimates of damage and loss. Probabilistic models have the disadvantage of being complex algebraic or algorithmic functions, rather than visual curves. On the other hand, the implementation of such models is suitable for the reliability analyses that are carried out to obtain the final damage and loss probability curves. A detailed description of such reliability analysis is provided elsewhere (Haukaas *et al*. 2009). It should be noted that these curves determine the probability of exceeding a certain damage and loss threshold and not a conditional probability.

An implementation of the proposed methodology is discussed at the end of this paper. Preliminary probabilistic damage models are developed for the total length of epoxy-injectable cracks and area of cover spalling in rectangular RC columns. The proposed damage models quantify the extent of damage as seismic demand increases. A test program is planned as part of this ongoing project to collect extensive observations on damage to RC columns. These models will be revised as new observations become available. The general form of a loss model is also proposed which estimates the associated repair cost based on the selection of a repair action and associated costs, and the effect of earthquake intensity.

Proposed Methodology

Probabilistic Models

The general form of a probabilistic model is shown in Eq. 1 (Gardoni *et al*. 2002):

$$D(\mathbf{x}, \mathbf{\Theta}) = d(\mathbf{x}) + \gamma(\mathbf{x},\boldsymbol{\theta}) + \sigma\varepsilon \quad (1)$$

in which $d(\mathbf{x})$ is an existing deterministic model, $\mathbf{x} = (x_1, x_2, x_3, ...)$ is a set of independent measurable variables, such as engineering parameters for a probabilistic damage model, $\mathbf{\Theta} = (\boldsymbol{\theta}, \sigma)$, $\boldsymbol{\theta} = (\theta_1, \theta_2, \theta_3, ...)$ is the set of unknown model parameters introduced to fit the model to the observations. $\gamma(\mathbf{x}, \boldsymbol{\theta})$ includes additional terms that correct for the bias in $d(\mathbf{x})$ and is defined in terms of independent variables, $\mathbf{x}$, formulated in the explanatory functions, $h_i(\mathbf{x})$:

$$\gamma(\mathbf{x},\boldsymbol{\theta}) = \sum \theta_i h_i(\mathbf{x}) \quad (2)$$

Although $\gamma(\mathbf{x}, \boldsymbol{\theta})$ is linear in the unknown model parameters, the explanatory functions can be in any nonlinear form. σ is the standard deviation of the model error and ε is a random variable with zero mean and unit variance. The $\sigma\varepsilon$ term takes into account the influence of additional missing variables and the remaining error due to inexact model form. As proposed by Gardoni *et al.*, $\mathbf{\Theta}$ values are determined by the application of Bayesian updating rule. The details of this application are found elsewhere (Gardoni *et al*. 2002). To achieve a compromise between model accuracy and model simplicity, an iterative procedure is followed by which the least informative explanatory functions are eliminated. Such functions are recognized by having the largest coefficient of variation (Gardoni *et al*. 2002). In cases where no deterministic model is available, one can still use a probabilistic model form as shown in Eq. 3 (Haukaas *et al.* 2009), in which θ_i parameters in $\gamma(\mathbf{x},\boldsymbol{\theta})$ fit the explanatory functions to the observed data.

$$D(\mathbf{x}, \mathbf{\Theta}) = \gamma(\mathbf{x},\boldsymbol{\theta}) + \sigma\varepsilon \quad (3)$$

Damage Models

Probabilistic models can be developed to predict various measures of loss such as direct and indirect economic losses, casualties, and downtime. The focus of this paper, however, is on the development of probabilistic loss models for repair cost and repair time. It is recognized that repair action is the link from damage to repair cost and time. Hence, the proposed probabilistic damage models predict the visual damage in components based on which the required repair action is selected. Visual damage tracks the progression of damage in a component. Examples of visual damage in a RC column are cracking, cover spalling, core crushing, longitudinal reinforcement buckling, and longitudinal and transverse reinforcement fracture. To select the appropriate repair action, several guidelines and manuals provide details of methods of repair for seismic damaged structures. For example, FEMA 306-308 provide guidelines for structural repair of seismic damage to RC and masonry walls. Following Eq. 1, the general form of a probabilistic damage model is shown in Eq. 4. EDPs from structural analysis are inputs to damage models and the results are deterministic quantities of visual damage.

$$\textit{Visual Damage} = d(\mathbf{x}) + \theta_1 h_1(\mathrm{x}) + \theta_2 h_2(\mathrm{x}) + \theta_3 h_3(\mathrm{x}) + \ldots + \sigma\varepsilon \quad (4)$$

$d(\mathbf{x})$ is an existing deterministic model for the desired damage quantity and the remaining terms are added to account for uncertainties involved in $d(\mathbf{x})$. The variables in the explanatory functions $h_i(\mathrm{x})$ are formulated in terms of mechanics and selected based on engineering parameters and responses. Examples of $h_i(x)$ for a probabilistic damage model of a RC component are P/f'_cA_g and ρ_l in which P, f'_c, A_g and ρ_l are considered as independent variables, x_i (P is the applied axial load, f'_c is the concrete compressive strength, A_g is the gross cross sectional area and ρ_l is the longitudinal reinforcement ratio). As mentioned earlier, the development of probabilistic models is based on the availability of observations. Records of damage at various levels of demand are valuable sources of observations to be used in the development of probabilistic damage models.

Loss Models

Based on the repair actions selected for each component, the total repair cost to a structure includes the summation of costs related to labor, materials, equipment as well as contractors' added profit, economy of scale, *etc*. However, much uncertainty is involved in the estimation of repair cost after occurrence of an earthquake due to the uncertainties involved in the contractor unit cost, overhead and profit, demand surge (amplification in cost of labor, material, *etc*. as a result of sudden increase in demand), the choice of union versus nonunion labor, *etc* (Porter *et al*. 2002). Following Eq. 3, the general form of a probabilistic repair cost, C_{repair}, reads:

$$C_{repair} = \theta_1 h_1(x) + \theta_2 h_2(x) + \theta_3 h_3(x) + \ldots + \sigma\varepsilon \qquad (5)$$

in which C_{repair} is the total repair cost to the structure after occurrence of an earthquake. Similarly, the repair time, T_{repair}, associated with the selected repair action can be determined by:

$$T_{repair} = \theta_1 h_1(x) + \theta_2 h_2(x) + \theta_3 h_3(x) + \ldots + \sigma\varepsilon \qquad (6)$$

in which the explanatory functions include the contribution of factors such as the time associated with the selected repair action, mobilization, *etc*.

Implementation of the Proposed Methodology

Damage Models

The proposed methodology is implemented to develop probabilistic damage models for rectangular RC columns. As mentioned before, the progression of visual damage in a RC component includes cover cracking, cover spalling and core crushing, longitudinal reinforcement buckling and, longitudinal and transverse reinforcement fracture. Much research has focused on seismic behavior of RC columns at failure. Elwood and Moehle (2005) have developed a drift capacity model at shear failure for RC columns with light transverse reinforcement. Berry and Eberhard (2005) have proposed models for drift at onset of bar buckling in flexural dominated RC columns. However, less research has been conducted on quantifying early levels of damage including cracking and spalling associated with small and moderate earthquakes. Hence, this work focuses on the development of probabilistic damage models to measure cracking and cover spalling which initiate in RC columns prior reaching post-peak behavior.

Probabilistic Models of Crack Length

In the literature, numerous cyclic tests have been conducted on various types of RC columns. For all specimens, hairline cracks were initiated on the concrete surface at low levels of demand. As the displacement level increased, new cracks formed and old cracks increased in width and length. These residual cracks developed prior post-peak response can be repaired by the application of epoxy injection (FEMA 306-308). As mentioned earlier, the perceived loss estimates are in terms of cost and time of repair. Cost of epoxy injection is based on crack length. Hence, probabilistic damage models are developed to predict the total length of epoxy-injectable cracks developed in columns prior post-peak behavior. The widths of such residual cracks range between 1mm to 2mm (Hose *et al*. 2000). It is recognized that the underlying mechanics for the development of cracks on perpendicular and parallel faces of a column (with respect to the direction of loading) are different. At higher levels of demand, mainly flexural cracks appear on the perpendicular faces and the cracks on parallel faces turn into shear cracks. Deterministic models of shear and flexural crack length are developed by Igarashi *et al*. (2009) for rectangular RC columns, as shown in Eq. 7 and 8. These models are based on an experimental program including flexural and shear dominated columns. The specimens were identical except in the transverse reinforcement ratio, ρ_s.

$$\textit{Total length of flexural cracks} = 8N_f\,(B/2 + \beta D) \tag{7}$$

$$\textit{Total length of shear cracks} = 8N_s\,(\sqrt{2}\gamma D) \tag{8}$$

$$N_s = N_f - 1 \tag{9}$$

In above equations, B and D are the perpendicular and parallel sides of column cross section. N_f and N_s represent the total number of flexural and shear cracks (lumped in plastic hinge zone), respectively. γ is a coefficient related to the length of shear cracks in parallel faces of column. β is a constant factor equal to 0.24 accounting for the extension of flexural cracks from perpendicular faces to parallel faces. N_f and γ are determined by Fig. 1. At low levels of demand, however, shear and flexural cracks on each column face are not distinguishable. Furthermore, for the purpose of determining cost of repair, it is not significant if the crack is shear or flexure related. Hence, the above models are modified to determine the total crack length on parallel and perpendicular faces. Namely, βD factor is subtracted from Eq. 7 and added to Eq. 8, as shown in Eq. 10 and 11:

$$\textit{Total length of cracks on perpendicular face} = 4N_f\,B \tag{10}$$

$$\textit{Total length of cracks on parallel face} = 8N_s\,(\sqrt{2}\gamma D + \beta D) \tag{11}$$

Following Eq. 1, the preliminary probabilistic models for length of cracks in perpendicular and parallel faces are shown in Eq. 12 and 13, respectively. These probabilistic models originally include explanatory functions $h_1(\mathrm{x}) = 1$ to account for the bias in the model independent of variables $\mathbf{x}$, $h_2(\mathrm{x}) = \delta^c$ to explicitly account for the effect of drift angle and $h_3(\mathrm{x}) = \rho_s^{\alpha}$. It should be noted that although other variables such as longitudinal reinforcement ratio, axial load, *etc*. are likely significant in the length of cracks developed in a column, such variables are not included in the models due to the limited data set considered. Logarithmic transformation is used to follow the assumption of homoskedasticity (independent model variance with respect to variables, **x**). Total crack length is normalized by the plastic hinge zone length (assumed to be equal to the longer cross section dimension, D) in order to develop a dimensionless model. As described above in "Probabilistic Models", a stepwise elimination process is followed in the Bayesian updating rule to eliminate the least informative explanatory functions. In these models, it is found that h_1(x) is a non-informative function due to its coefficient having a large posterior coefficient of variation and as such is eliminated from the models. The posterior statistics of the model parameters are shown in Tables 1 and 2. Figs. 2-3 compare the results obtained by the median of the probabilistic models with the test observations by Igarashi *et al*. (2009). These probabilistic models will be further revised once additional data are available.

$$\ln (L_{\text{perp}} /D) = \ln (4N_f B /D) + \theta_2\delta + \theta_3\rho_s^4 + \sigma\varepsilon \qquad (12)$$

$$\ln (L_{\text{para}} /D) = \ln (8N_s [\sqrt{2}\gamma D + \beta D]/D) + \theta_2\delta^{-1} + \theta_3\rho_s^3 + \sigma\varepsilon \qquad (13)$$

Probabilistic Model of Spall Area

Following the yielding of longitudinal reinforcement at subsequent levels of displacement, spalling initiates and increases in height as displacement demand increases (Lehman *et al*. 2004). Referring to FEMA 308, the repair method of concrete patching is used for repair of spalled cover concrete. The repair cost of concrete patching is based on the area of cover spalled. Based on the recorded observations by Igarashi *et al*. (2009), a probabilistic model for spalled area is developed as shown in Eq. 14. Due to the lack of an existing deterministic model measuring area of spalling in RC columns, Eq. 14 only includes explanatory functions and unknown model parameters (following Eq. 3). Spall area is normalized by the product of column perimeter and assumed plastic hinge zone length, D:

$$A_{\text{sp}} /[2(D+B)D] = \theta_1 + \theta_2\, \delta^2 + \theta_3\, \rho_s + \sigma\varepsilon \qquad (14)$$

Again following the Bayesian updating rule deletion process, h_1(x) and h_3(x) are eliminated due to having large posterior coefficients of variation. The reduced probabilistic model is shown in Eq.15:

$$A_{\text{sp}} /[2(D+B)D] = \theta_2\, \delta^2 + \sigma\varepsilon \qquad (15)$$

Table 3 includes the posterior statistics of the model parameters. Fig. 4 compares the test observations by Igarashi *et al*. (2009) to the results obtained by the median values of Eq. 15. Similar to the crack length models, Eq. 15 is a preliminary model and will be further revised as new data are available.

Loss Models

Preliminary models of repair cost for repair actions of epoxy injection and concrete patching are under development. The general form of such models is shown in Eq. 16 in which DF is the damage factor (the ratio of the repair cost, C_{repair}, to the total replacement cost of the component). Referring to Eq. 5, the estimation of repair cost after occurrance of an earthquake depends on various factors such as the contractor unit cost, overhead and profit, demand surge, the choice of union versus nonunion labor, *etc* (Porter *et al*. 2002). As potentially important as these are, this paper currently focuses only on the effect of unit cost, contractor overhead and profit in addition to the variability in the demand for labor, material, construction equipment, *etc*. after occurrance of an earthquake. To account for the latter, a measure of earthquake intensity is included in the model. Clearly higher intensity levels result in higher demand for repair and lower availability of labor and material. m is the severity index related to Modified Mercalli Intensity (MMI) scale. Due to being an ordinal scale, arithmetic operations cannot be performed directly on MMI scale (Blong 2001). Instead, the severity indices correlated to MMI scale developed by Pacella (1982) are used.

$$(DF)_{\text{after}} = \theta_1 + \theta_2 (DF)_{\text{before}} + \theta_3 m + \sigma\varepsilon \qquad (16)$$

The unknown model parameters, θ_i, in the above model will be determined based on previous records on repair cost before and after occurrence of an earthquake. In this tentative model form, θ_1 is added in order to account for the effect of other contributing factors, which are not explicitly accounted for.

Conclusions

In this paper, the development of probabilistic models is proposed as a new methodology to predict seismic performance of structures. For given realizations of engineering parameters and responses, probabilistic models predict the visual damage in components. The results of probabilistic damage models are linked by physical quantities rather than conditional probabilities to probabilistic loss models. Based on the associated repair action, cost and time of repair can be determined by the probabilistic loss models. The proposed probabilistic damage models predict the extent of cracking and cover spalling in rectangular RC columns. It should be stressed that the proposed models are preliminary and will be revised as new observations on damage become available. Loss models are under development to estimate the repair cost of epoxy injection and concrete patching based on the extent of the observed damage. Such loss models account for the uncertainties in the contributing factors, which affect cost estimation after an earthquake.

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Table 1. Posterior statistics of the coefficients of the preliminary probabilistic model of crack length on perpendicular face.

Coefficients	**Mean**	**Coefficient of Variation**	**Correlation Coefficient**	
			θ_2	θ_3
θ_2	-0.083	0.241	1.00	-0.640
θ_3	7.00	0.318	-0.640	1.00
σ	0.116	0.204	-	-

Table 2. Posterior statistics of the coefficients of the preliminary probabilistic model of crack length on parallel face.

Coefficients	**Mean**	**Coefficient of Variation**	**Correlation Coefficient**	
			θ_2	θ_3
θ_2	0.160	0.133	1.00	-0.462
θ_3	5.49	0.183	-0.462	1.00
σ	0.152	0.204	-	-

Table 3. Posterior statistics of the coefficients of the preliminary probabilistic model of spall area.

Coefficients	**Mean**	**Coefficient of Variation**
θ_2	0.024	0.049
σ	0.024	0.204

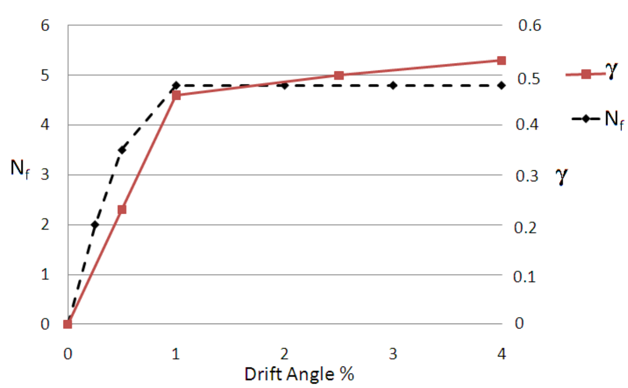

Figure 1. Number of flexural cracks and γ coefficient as a function of maximum rotation angle (adapted from Igarashi *et al*. 2009).

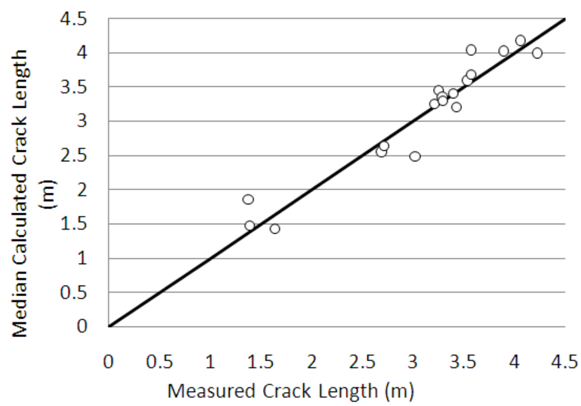

Figure 2. Comparison of test observations with the median results of the crack length probabilistic model for perpendicular face.

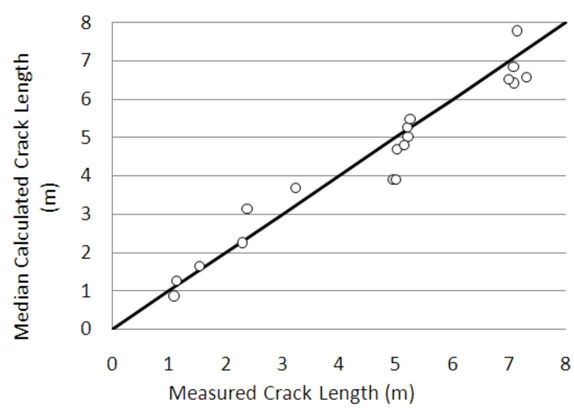

Figure 3. Comparison of test observations with the median results of the crack length probabilistic model for parallel face.

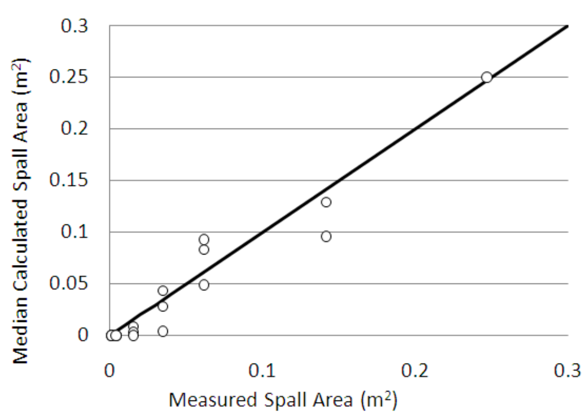

Figure 4. Comparison of test observations with the median results of the probabilistic model of spall area.