Proceedings of the 9th U.S. National and 10th Canadian Conference on Earthquake Engineering
Compte Rendu de la 9ième Conférence Nationale Américaine et
10ième Conférence Canadienne de Génie Parasismique
July 25-29, 2010, Toronto, Ontario, Canada • Paper No 928

EVALUATION OF PERIOD FORMULAS FOR SHEAR WALL BUILDINGS

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ABSTRACT

There are several published analytical and empirical formulas for the vibration period of buildings. These formulas primarily depend on the building material, building type and overall dimensions. Although these formulas are proposed for use in design process, correlation of estimated periods with the ones obtained from detailed finite element modeling of the structural system appear to be too weak. It is believed that the poor performance is primarily due to inappropriate use of these formulas and inadequacy of the parameters used in these formulas. In this study, the period formulas given in the codes and the literature are evaluated using the developed shear wall building models and the databases employed by other researchers. The adequacy of the available formulas was evaluated through comparisons with the periods measured and analytically calculated for the databases of the building models employed. The parameters used in existing formulations were found to be inadequate for buildings having frame-wall interaction. In buildings where frames contribute to the lateral stiffness a term that incorporates the effect of dominant deflection mode of the structure as a result of frame-wall interaction needs to be incorporated in the formulations.

Introduction

Fundamental period of vibration is one of the most important parameter of structures as it is used to determine the elastic base shear for design as well as approximately predict global seismic demand. The seismic design codes generally present relationships that give conservative estimates for the predominant vibration period for common types of buildings. There are several published analytical and empirical formulas to define the vibration period of buildings (Sozen 1989, Wallace and Moehle1992, Goel and Chopra 1998, Lee et al. 2000, Hong and Hwang 2000, Balkaya and Kalkan 2003, Ghrib and Mamedov 2004). These formulas primarily depend on the building material, building type and overall dimensions. Investigation of proposed empirical formulas, which were derived from established analytical procedures or obtained through regression analysis of measured or calculated structural response, to calculate the fundamental vibration period of shear wall buildings reveals that the building structural type plays a significant role on the period calculation. If we exclude frame-wall building (dual system) that has a different form of interaction among frames and walls, the period of shear wall buildings in which the walls

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are coupled with strong-deep beams and are connected only through rigid floor diaphragms differs significantly. The uncoupled walls in a building system can be treated as cantilevers. Theoretical formulas for the fundamental period of the buildings with coupled shear walls are available in Rutenberg (1975), and for buildings with a combination of shear walls and moment resisting frames in Heidebrecht and Stafford-Smith (1973) and Stafford-Smith and Crowe (1986). Sozen (1989), Wallace and Moehle (1992) and Goel and Chopra (1998) presented formulas for the fundamental vibration period of shear wall buildings with uncoupled walls. The formulas proposed by Sozen (1989) and Wallace and Moehle (1992) were developed based on pure-flexural cantilever idealization of shear wall buildings and ignored the influence of shear deformations. Furthermore, the numerical constant in their formula was determined based on assumed material properties and effective member stiffness equal to one-half of its initial value. The formula developed in the Goel and Chopra (1998) includes both flexural and shear deformations and the numerical constant is determined directly from regression analysis of measured period data.

According to Turkish earthquake code (TEC, 1998) the first natural vibration period of the building with $H_N < 25$m may be calculated by the following approximate expression:

$$T_1 = C_t H_N^{3/4} \tag{1}$$

where the value of the C_t depends on the building structural system and H_N represents the building height. For buildings where seismic loads are fully resisted by reinforced concrete structural walls, the value of C_t shall be calculated by

$$C_t = 0.075 / A_c^{1/2} \leq 0.05 \tag{2}$$

Equivalent area A_c appearing in Eq. 2 is given by Eq. 3 where the maximum value of $(L_{w,j}/H_N)$ shall be taken equal to 0.9, $L_{w,j}$ being the wall length.

$$A_c = \sum_{j}^{NW} A_{w,j} \left[0.2 + \left(L_{w,j}/H_N\right)^2\right] \tag{3}$$

$C_t = 0.05$ shall be taken for frame-wall structures. In UBC-97 (UBC, 1997) there is no upper limit defined in the calculation of C_t for shear wall buildings.

Depending on the assumption that the fundamental period of a building where flexural behavior of structural walls dominates the lateral load response can be approximated by analysis of an equivalent cantilever, Sozen (1992) derived the following equation to calculate the fundamental period of an uniform concrete cantilever having rectangular cross section and supporting regularly distributed floor load based on a uncracked section stiffness as

$$T = 6.2 \frac{H_w}{L_w} N \sqrt{\frac{wh}{g E_c p}} \tag{4}$$

where N = number of floors; w = unit floor weight including tributary wall height; h = mean story height; E_c = concrete modulus of elasticity; H_w = wall height; and p = ratio of wall area to floor plan area for the walls aligned in the direction the period is calculated ($p = \Sigma A_w / A_f$, where $A_w = L_w.t_w$, L_w is the wall length, t_w is the wall thickness, and A_f is the floor plan area of a typical floor of the building). Eq. 4 can be further simplified by introducing typical values of w = 8.5 kPa (0.84 t/m^2), h_s = 275 cm, g = 981 cm/s^2 and E_c = 25,000 MPa and assuming uncracked section as

$$T = 0.002 \frac{H_w}{L_w} N \frac{1}{\sqrt{p}} \tag{5}$$

As seen in Eq. 5, it is possible to express the fundamental period of structural wall buildings in terms of three nondimensional parameters: the slenderness ratio of the primary walls (H_w/L_w), the number of stories (N), and the ratio of wall are to floor area (p).

Another formula is proposed by Goel and Chopra (1998) using the basic theory on the fundamental period of a cantilever beam as done by Sozen (1989), but this time taking into account both flexural and shear deformations. As a result of regression analysis using measured periods of shear wall buildings Goel and Chopra (1998) proposed the following relation for estimating, conservatively, the period of buildings.

$$T = 0.00623 \frac{1}{\sqrt{\overline{A}_e}} H \tag{6}$$

where H is the building height and $\overline{A}_e$ is the equivalent shear area expressed as a percentage of A_f that is defined as $100A_e/A_f$. Considering symmetric-plan buildings with a lateral-force resisting system comprised of a number of uncoupled (i.e., without coupling beams) shear walls connected through rigid floor diaphragms, Goel and Chopra (1998) calculates the equivalent shear area, A_e, assuming that the stiffness properties of each wall are uniform over its height as

$$A_e = \sum_{i=1}^{NW} \left(\frac{H}{H_{w,i}} \right)^2 \frac{A_{w,i}}{\left[1 + 0.83 \left(\frac{H_{w,i}}{L_{w,i}} \right)^2 \right]} \tag{7}$$

Eq.6 applies only to those buildings in which lateral load resistance is provided by uncoupled shear walls. Eq.6 includes many of the important parameters that influence the fundamental period of concrete shear wall buildings.

Balkaya and Kalkan (2003) developed an empirical equation based on calculated periods of 80 different tunnel-form building configurations analyzed by using three-dimensional finite-element modeling. Shear-wall dominant multistory reinforced concrete structures, constructed by using a special tunnel form technique are commonly built in countries facing a substantial seismic risk, such as Chile, Japan, Italy and Turkey. The walls in tunnel-form construction can be considered as coupled with beams and slabs. The proposed equation was expressed in the form,

$$T = Ch^{b_1} \beta^{b_2} \rho_{as}{}^{b_3} \rho_{al}{}^{b_4} \rho_{\min}{}^{b_5} J^{b_6} \qquad (8)$$

Here, T is the period in seconds; h is the total height of the building in meters; β is the ratio of long-side to short-side dimension; ρ_{as} is the ratio of short-side shear-wall area to total floor area; ρ_{al} is the ratio of long-side shear-wall area to total floor area; ρ_{min} is the ratio of minimum shear wall area to total floor area; C, b_1, b_2, b_3, b_4, b_5 and b_6 are the parameters to be determined by regression analysis; J is the polar moment of inertia of the plan ($J=I_{xx}+I_{yy}$). The predictor coefficients in Eq. 8 determined by using non-linear regression analysis are listed in Table 1. Two sets of coefficients were derived according to the plan dimension ratios. If the ratio of the long-side to short-side dimension is less than 1.5, these plans are accepted as square and those plans having the same ratio greater or equal to 1.5 are accepted as rectangular.

Table 1. Empirical equations for predicting fundamental periods of tunnel form buildings.

$T = Ch^{b_1} \beta^{b_2} \rho_{as}{}^{b_3} \rho_{al}{}^{b_4} \rho_{\min}{}^{b_5} J^{b_6}$									
Plan type	C	b_1	b_2	b_3	b_4	b_5	b_6	σ_T	R^2
Square	0.158	1.4	0.972	0.812	1.165	−0.719	0.13	0.025	0.982
Rectangular	0.001	1.455	0.17	−0.485	−0.195	0.17	−0.094	0.025	0.989

Comparison of Empirical Relationships

The building inventory consisting of the sample buildings used by Wallace and Moehle (1992), Goel and Chopra (1998), Lee et al. (2000) and Balkaya and Kalkan (2003) were employed to compare the empirical relationships presented in the previous section. Great majority of these buildings comprise of shear wall buildings where the walls are employed to carry both vertical and lateral loads. The height versus period graph of the building inventory is presented in Fig. 1, where N represents the number of stories and the period T is in seconds. Except the data from Balkaya and Kalkan (2003) all the information given in Fig. 1 belong to the measured response of real life structures. The data by Wallace and Moehle (1992) comes from Chilean shear wall buildings from the city of Vina del Mar as also used by Sozen (1989). These buildings have a wall index of 3 percent in average the lowest value being 2 percent. Data by Lee et al. (2000) is from the measured periods of Korean tunnel-form buildings with an approximate wall index of 2 percent. Balkaya and Kalkan (2003) calculated the periods of 80 different tunnel-form building configurations that are typically applied in Turkey by using three-dimensional finite-element modeling. The average ratios of total wall to floor areas in these buildings are 2-3 percent. The data from Goel and Chopra (1998) covers the measured periods of buildings during eight California earthquakes starting with 1971 San Fernando and ending with the 1994 Northridge earthquake. These buildings have wall indexes ranging from 0.3 to 3 percent.

Examination of Fig. 1 reveals that the presented data generally correlates well with the line represented by 0.05 N that is a well-known approximate relationship between fundamental period and number of stories of shear wall buildings. It is worth noting that these buildings have wall indexes larger than 2 percent. The outlier points lying on $0.15N$ line are from Goel and Chopra's (1998) building inventory. Although they were named as shear wall building, these buildings have special features in terms of structural systems such as flat-plate system with core walls or vertical irregularities in the plan. For example, one of the outliers, the 8-story CSULA Administration Building has shear walls starting from the second floor (first floor being a clear soft story with a height of 7.3 m).

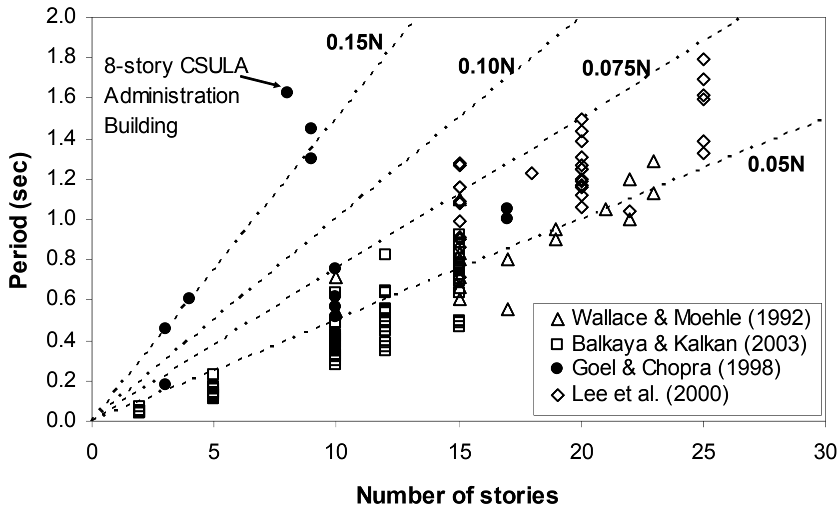

Figure 1. Period versus height relation of all shear wall buildings in the database

Comparison of measured and calculated periods of buildings with code formula is given in Fig. 2. While the code formula predicts the period of shear wall buildings in the database of Goel and Chopra (1998) satisfactorily, the period of buildings in Balkaya and Kalkan (2003) can be classified in two separate trend lines. The decomposition is not due to plan form (rectangle or square) as stated by Balkaya and Kalkan (2003) but due to the amount of plan area. The buildings in the database can be broadly grouped into two clusters with respect to the plan area; buildings with plan area smaller than 160 m^2 and buildings with plan area larger than 600 m^2. There is not such clear distinction between the buildings in Goel and Chopra database in terms of the total floor areas of buildings. If the value of $H/\sqrt{\bar{A}_e}$ as defined in Eq. 6 by Goel and Chopra (1998) is plotted against the period of the building the relationship shown in Fig. 3 is obtained. The $H/\sqrt{\bar{A}_e}$ correlates quite well with the period and can differentiate the building type. Evaluating the differences between Figs. 2 and 3 together leads to the conclusion that although very similar expressions are used in the code formula and formula proposed by Goel and Chopra (1998), expressing the equivalent shear area (A_c in code formula and A_e in Goel and Chopra's formula) in terms of ratio of total floor area ($\bar{A}_e$) increases the accuracy of the prediction.

The inadequacy of Sozen's formula is revealed in Fig. 4. In order to investigate the effect of aspect ratio on the period, 4-, 8- and 12-story cantilever wall structures composed of 3, 5 and 8 m walls in length with story height of 3 m are employed to calculate their periods using Eq. 5 and the results are displayed in Fig. 4 for different aspect ratios of the isolated shear walls. Restricting the applicable range of calculated period between 0.05N and 0.1N it can be concluded that the formula given by Eq. 5 yields meaningful results for aspect ratios only up to 5 for a very large portion of wall index range (p>0.005). As the aspect ratio (slenderness ratio) increases the wall index required to provide adequate stiffness also increases. However, as the 12-story case displays, by no means it is logical to assume an isolated wall model with a wall length of 3 m in buildings taller than 10-story, which corresponds to slenderness ratios larger than 10.

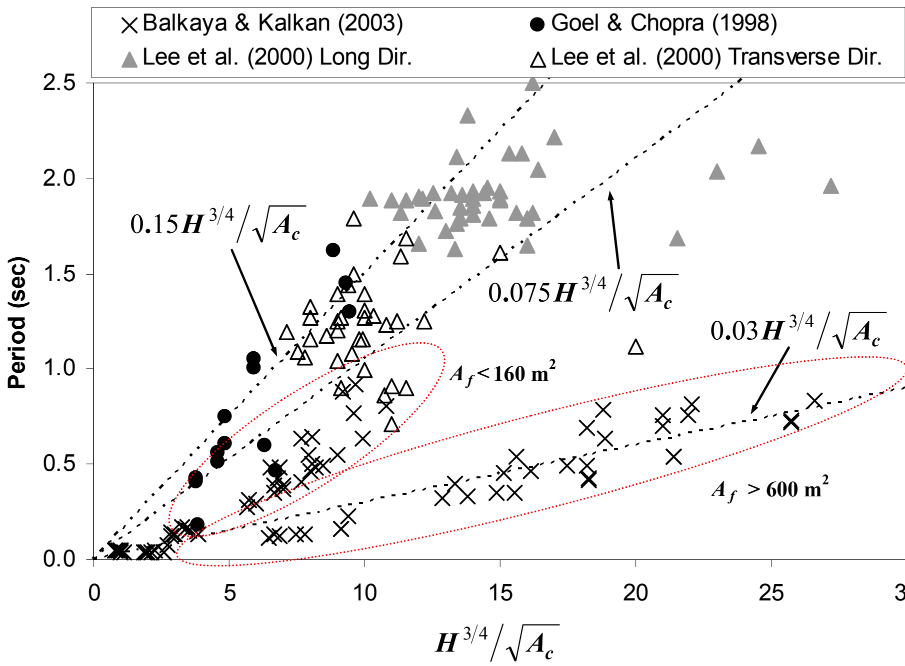

Figure 2. Comparison of measured and calculated building periods by Code formulation.

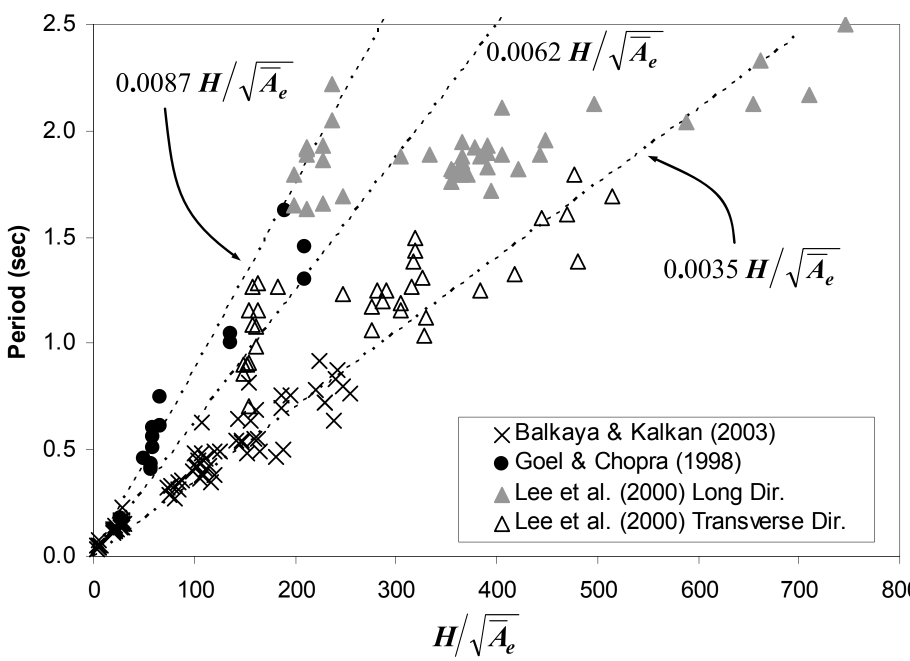

Figure 3. Comparison of measured and calculated periods by Goel and Chopra's formulation.

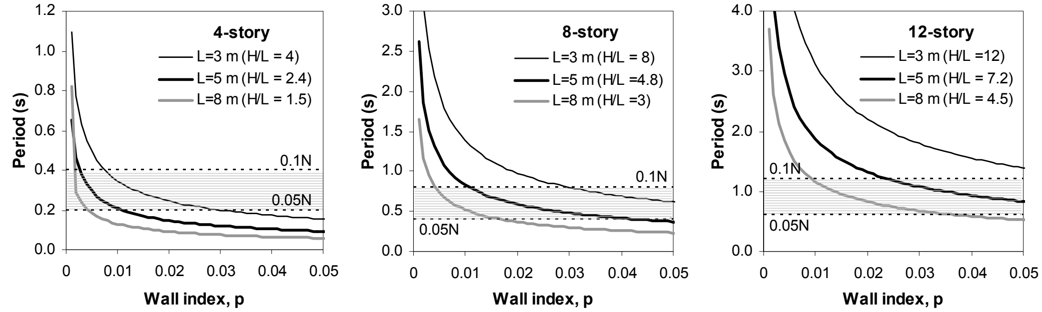

Figure 4. Periods calculated for range of wall indexes as a function of wall aspect ratio.

Proposed Period Relationship

Evaluation of existing relationships given in literature revealed that an improved period relationship that incorporates the frame-wall interaction as well as the wall density need to be developed. The basic mathematical model of the shear-flexure beam can also be used to determine the dynamic properties of a tall building structure consisting of uniform shear walls and frames. In Heidebrecht and Stafford Smith (1973) the equation governing the free vibration of shear-flexure beam is given by

$$\frac{d^4y}{dx^4} - \alpha^2 \frac{d^2y}{dx^2} = -\frac{1}{a^2}\frac{\partial^2 y}{\partial t^2} \tag{9}$$

in which

$$\alpha^2 = \frac{GA+\eta}{EI_w} \qquad \text{and} \qquad a^2 = \frac{EI_w}{\rho_d A_f} \tag{10}$$

In these expressions GA is the shear rigidity of equivalent frame component as calculated in Heidebrecht and Stafford Smith (1973), η is the flexural rigidity of beams spanning to walls as defined in Kazaz (2010), EI_w is the total flexural rigidity of walls, and ρ_d is the mass per unit volume of the uniform structure. The solution of the differential equation yields the natural frequencies as

$$\omega = \omega_o \sqrt{1+\left(\frac{\alpha H}{\lambda_o H}\right)^2} \tag{11}$$

in which $\lambda_o H$ takes the values of 1.875, 4.694, 7.855, 10.996 and 14.137 for the first five modes respectively and ω_o is the natural frequency of a flexural beam with the same mass and stiffness properties, as given by $\omega_o = a\lambda_o{}^2$. The first natural period of frame-wall systems can be obtained by using $T = 2\pi/\omega$ as

$$T = \frac{2\pi}{\omega_o \sqrt{1+\left(\frac{\alpha H}{\lambda_o H}\right)^2}} = \frac{2\pi}{\sqrt{\frac{EI}{\rho A_f}}\left(\frac{1.875}{H}\right)^2 \sqrt{1+\left(\frac{\alpha H}{1.875}\right)^2}} \tag{12}$$

and introducing necessary modifications like $A_f = A_w / p = (L_w . t_w) / p$ and $I = L_w^3 . t_w / 12$, and rearranging we get

$$T = \frac{2\pi h}{1.875}\sqrt{\frac{12\rho}{E}}\frac{H_w}{L_w} N \frac{1}{\sqrt{p\left(1.875^2 + (\alpha H)^2\right)}} \tag{13}$$

Upon distribution of typical values used for Eq. 5 into Eq. 13 we get

$$T = 0.00406 \frac{H_w}{L_w} N \frac{1}{\sqrt{p\left(1.875^2 + (\alpha H)^2\right)}} \tag{14}$$

This modified equation is a more general form as compared to Eq. 5 and yields more reliable and realistic period estimations for various types of shear wall buildings due to incorporation of αH term, which can be considered as behavior factor. In order to investigate its efficiency a prototype frame-wall structure shown in Fig. 5 is designed. It is inevitable to check the validity of proposed formula on a prototype structure because the structural plans of database buildings were not available to the authors to calculate the αH parameter. The structure is composed of nine 3-bay frames in the transverse direction and three 8-bay frames in the longitudinal direction. By changing the number of walls allocated in the central bay in the transverse direction different frame-wall arrangements and wall indexes are obtained. The shear wall of lengths of 3, 5 and 8 meters were used. The building heights that were considered to determine the aspect ratio of the shear walls consists of 4, 8 and 12 story structures. The interstory height is considered to be constant along the height of the building, which is 3 m. The dimensions of the columns are 0.6x0.6 m and the beams are 0.6x0.4 m. Robust beam and column elements were used to assure effective development of the desired frame-wall interaction. The αH term effectively characterizes the dominant behavior mode of the frame-wall structures.

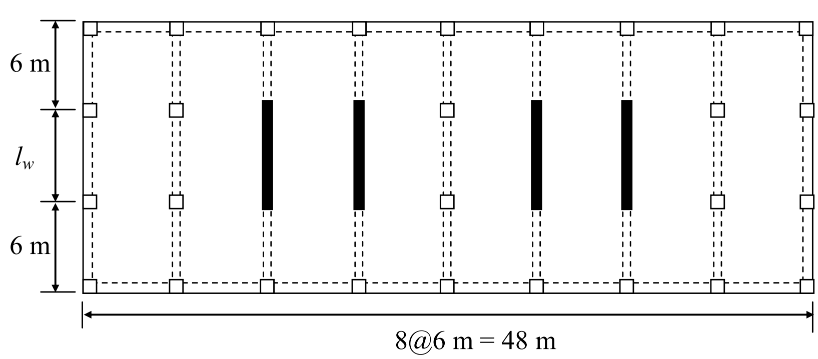

Figure 5. Plan view of bare frame configuration of the prototype frame wall structure

Using Eq. 14 the fundamental vibration period of prototype structures was calculated. The results were plotted in Fig. 6(a). After a certain αH value, which is inversely proportional to the wall index (p), estimated periods of frame-wall structures converge to the values given by $0.1N$. These predicted periods of prototype structures were also compared with more exact values calculated in SAP2000 program by employing the finite element models of the structures. As seen in Fig. 6(b) both predictions agree quite well. The poor predictions of existing formulas in case of frame-wall structures were displayed in Fig. 7. The error increases as the wall index (p) decreases, since the frame-wall interaction effects become more pronounced in cases where the number of frames per wall increases quantitatively.

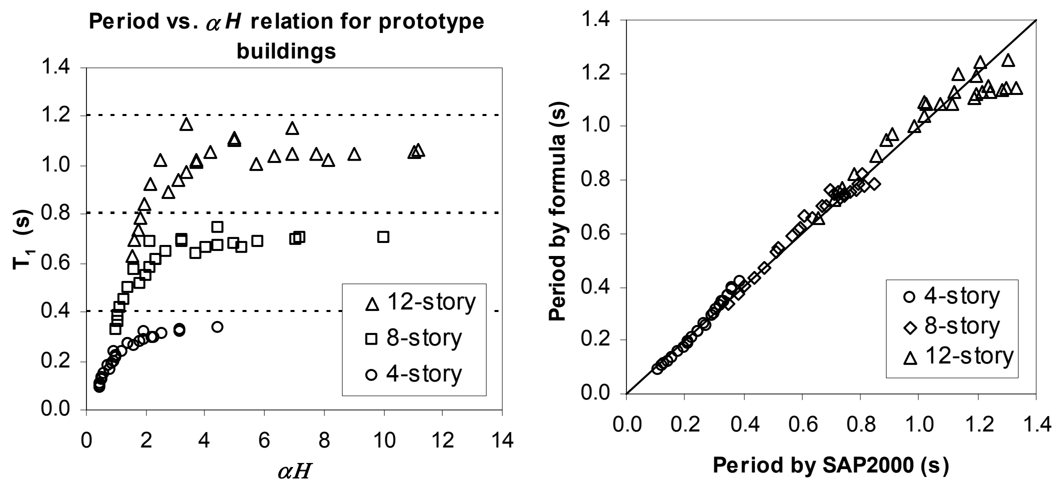

Figure 6. Fundamental period of prototype structures predicted using Eq. 14.

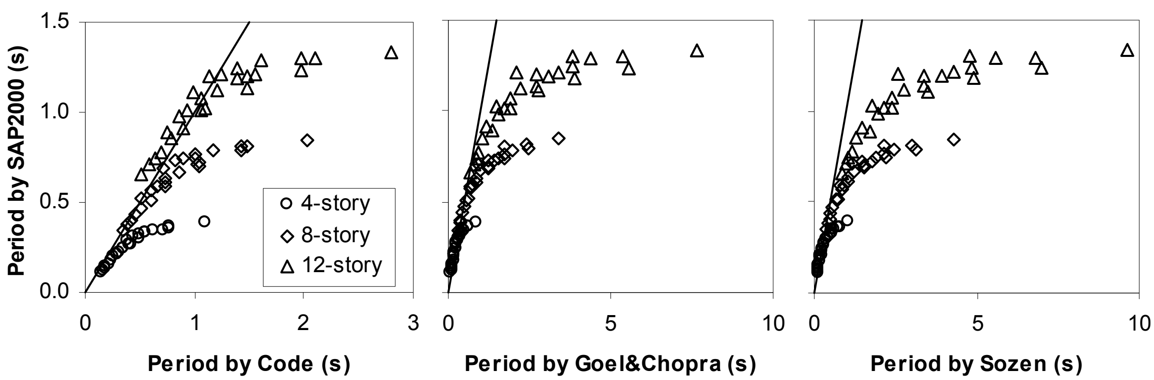

Figure 7. Estimations of existing formulas in case of frame-wall systems.

Conclusions

This study aimed to evaluate the efficiency of existing relationships proposed for calculation of fundamental period of buildings with shear walls. A comprehensive database employed by earlier researchers was used. The results indicated that available relationships are not adequate in certain cases and the period formulas for buildings with shear walls should incorporate the effect of frame-wall interaction in addition to shear wall density and wall aspect ratio. It is clear from the performed comparisons in the manuscript that the key to the correct estimation of structural periods is to use appropriate formula by considering the inherent limitations in the derivation of these formulas. For this reason, a modified equation that was shown to yield satisfactory predictions was proposed. The formula can satisfactorily predicts the period of frame-wall structures ranging from wall dominant to frame dominant systems.

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