THREE-DIMENSIONAL STIFFNESS DEGRADATION MODEL FOR PROGRESSIVE COLLAPSE ANALYSIS OF BRIDGES

V. V. Phung¹, D. T. Lau², S. S. Reshotkina³

ABSTRACT

The interaction between flexural and shear behavior can significantly affect the seismic responses of bridges. This paper presents a new three-dimensional stiffness degradation model for progressive collapse analysis of bridges during earthquakes. Stiffness degradations due to both flexural and shear damage are incorporated in the model. The progression of damage and the effect of cumulative damage on structural performance from cyclic load reversals during earthquakes can be accurately followed from initial failure, through gradual stiffness degradation of structural members, to ultimate collapse of the bridge structure. The validity of the proposed model is verified by performing nonlinear earthquake response analyses of three-span reinforced concrete bridges. The stiffness degradation due to accumulated damage significantly affects the response of the structure and should be considered in the analysis.

Introduction

The importance of progressive collapse of bridges has been recognized since many bridge structures have suffered severe damage leading to partial or total collapse during major earthquakes. To devise better methodology for earthquake-resistant design of new bridges, and more efficient retrofit strategies for older deficient bridges, it is essential to understand the progression and accumulation of damage during strong earthquake responses. An accurate assessment of seismic performance of bridges requires realistic nonlinear modeling and analysis tools that can capture the initiation of local damage, the spreading of failure through the elements, to ultimate collapse of the whole bridge structure or a large part of it. In past earthquake reconnaissance studies it has been observed that seismic damage of reinforced concrete bridges typically occurs at the ends of columns where the largest bending moment is located. Considerable research has been done on the development of three-dimensional nonlinear beam-column elements for modeling the plastic hinging behavior of reinforced concrete columns. A degrading stiffness hysteretic model has been proposed on the basis of experimental observations from cyclic loading of reinforced concrete columns (Takeda 1970). For seismic analysis of highway bridges, a 3D

¹Structural Engineer, TranTech Engineering, Bellevue, WA, USA
²Professor, Dept. of Civil Engineering, Carleton University, Ottawa, ON, Canada
³Graduate Research Assistant, Dept. of Civil Engineering, Carleton University, Ottawa, ON, Canada
elasto-plastic flexural column element has been developed to model the inelastic response during strong earthquakes (Tseng and Penzien 1973). Further studies have been conducted on developing a generalized 3D beam-column element (Chen and Powell 1982), which considers the interaction of bending moment and axial force by means of yield interaction surface. The beam-column element consists of an elastic element and two hinges at both ends of the element to model the inelastic behavior of the member. The stiffness of the plastic hinge is allowed to degrade when the member is subjected to load reversals. The degradation of the stiffness is modeled as inverse proportioned to the largest previous plastic hinge deformation. The degrading stiffness has been incorporated into Tseng’s model (Zhang 1999). A 3D non-linear beam-column model has been developed to account for the accumulated damage in bridge columns during major earthquakes, considering strength deterioration and stiffness degradation behavior (Phung 2005).

Most previous studies have focused on stiffness degradation behavior of bridge columns only related to flexural behavior. Shear strength of bridge column members is separately assessed, often assuming elastic shear response. Recent studies have shown that the shear strength of reinforced concrete columns at plastic hinge regions can be significantly reduced when the displacement ductility of the members increases (Wong 1993, Priestley, 1994, etc.). Shear yielding and degradation can be precipitated by flexural yielding in the plastic hinge region of bridge piers. Bridges designed prior to the 1970s can exhibit significant shear degradation behavior due to insufficient transverse reinforcing steel, inadequate detailing and less conservative design requirements for shear strength compared to flexural strength. For seismic response and progressive collapse analysis of older deficient bridges, the interaction between flexural and shear effects can significantly affect the behavior and thus should be included in the model. This paper presents a three-dimensional nonlinear stiffness degradation model for reinforced concrete bridge columns that comprehensively takes into account the accumulated damage and the interaction between flexural and shear mechanisms on the inelastic degradation behavior of reinforced concrete structural members. Earthquake response analyses of three-span reinforced concrete bridges subjected to bidirectional strong earthquake ground motions are performed to validate the capabilities of the proposed model.

**Post-Elastic Damage Measures**

Damage indices are often employed to quantify the severity of damage sustained by the structure under repeated load reversals typical for seismic events. Post-elastic damage measures are calibrated so that a value of zero indicates no damage, while a value of one represents total damage. Post-elastic damage measures can be divided into local and global damage measures.

**Local Damage Measures**

Flexural damage index: When subjected to seismic excitations, a reinforced concrete member may be damaged by the combined effect of deformation amplitude and repeated cycles of deformation. A comprehensive review of different damage index models has been presented elsewhere (William and Sexsmith 1995). In this study the modified Park and Ang model (Kunnath 1992) is adopted to quantify damage due to flexure:
\[ FD = \frac{\theta_{\text{max}} - \theta_y}{\theta_u - \theta_y} + \frac{\beta}{M_y \theta_y} \int dE_h \]  

(1)

where \( \theta_{\text{max}} \) is the maximum rotation angle sustained during loading history, \( \theta_y \) is the yield rotation angle, and \( \theta_u \) is the ultimate rotation capacity of the section, and \( M_y \) is the yield moment.

Normalized hysteretic energy: To quantify the effect of hysteretic behavior of cumulative dissipated energy on structural damage, normalized hysteretic ratio is adopted as the ratio of the hysteretic energy dissipated through cyclic response of the member normalized to twice the yield strain energy as follows:

\[ \text{NHE} = \frac{\int dE_h}{M_y \theta_y} \]  

(2)

Shear damage index: Shear strength degradation of bridge columns can be evaluated by using the member curvature ductility. The relationship between the displacement ductility and the curvature ductility is given by:

\[ \mu_c = 1 + 3(\mu_o - 1) \frac{L_p}{L} \left( 1 - 0.5 \frac{L_p}{L} \right) \]  

(3)

where \( \mu_c \) and \( \mu_o \) are the displacement and curvature ductility, respectively, \( L_p \) is plastic hinge length and \( L \) is column height (Priestley 1994). The shear behavior of three types of bridge columns is illustrated in Fig. 1. From the shear demand and shear capacity curves shown in Fig. 1, the ultimate displacement ductility of a concrete member of case B can be computed as follows:

\[ \mu_u = \begin{cases} 
< 1 & \text{Case A} \\
\mu_i - \frac{(\mu_{\text{f}} - \mu_i)(V_u - V_f)}{(V_i - V_f)} & \text{Case B} \\
\text{Not applicable} & \text{Case C}
\end{cases} \]  

(4)

\( V_i \) and \( V_u \) are computed following the standard design procedure (Priestley 1994). In this study, a shear damage index is proposed as a function of curvature ductility as follows:

\[ \text{SD} = \begin{cases} 
0 & \text{if } \mu_{\text{max}} < \mu_i \\
\frac{\mu_{\text{max}} - \mu_i}{\mu_u - \mu_i} & \text{if } \mu_i \leq \mu_{\text{max}} \leq \mu_u \\
1 & \text{if } \mu_{\text{max}} > \mu_u
\end{cases} \]  

(5)

where \( \mu_{\text{max}}, \mu_i \) and \( \mu_u \) are the maximum curvature ductility experienced during previous cycles, the curvature ductility where shear strength begins to degrade, and the ultimate curvature ductility, respectively.
Figure 1. Shear demand and shear capacity relationship of reinforced concrete columns.

**Global Damage Measures**

Global flexural damage index: The global flexural damage index is defined as the weighted sum of the local flexural damage indices of all structural members:

\[
GFD = \sum_{i=1}^{N} w_i F_{D_i} \tag{6}
\]

\[
w_i = \frac{H_{E_i}}{\sum_{i=1}^{N} H_{E_i}} \tag{7}
\]

where \(w_i\) and \(H_{E_i}\) are the weighting factor and dissipated hysteretic energy of the i-th damage member, respectively.

Global normalized hysteretic energy: The global normalized hysteretic energy is defined as the average of the normalized hysteretic energies absorbed by all members that experienced inelastic action, i.e.:

\[
GHE = \frac{1}{N} \sum_{i=1}^{N} N_{H_{E_i}} \tag{8}
\]

Global shear damage index: The global shear damage index is also defined as weighted sum of the local shear damage indices of all the structural members as follows:

\[
GSD = \frac{\sum_{i=1}^{N} S_{D_i}}{\sum_{i=1}^{N} S_{D_i}} \tag{9}
\]
Three-Dimensional Nonlinear Degrading Model

The stiffness of a reinforced concrete member degrades due to the loss of integrity of the concrete material and bonding between concrete and steel reinforcements as a result of repeated inelastic loading reversals. The cumulative damage at the element level is given by the local damage measures as discussed in the previous section. The effects of cumulative damage on the member stiffness are modeled by a new three-dimensional stiffness degradation model which takes into account the cumulative damage on both the flexural and shear stiffness and capacity of the member. The developed model is incorporated in an available beam-column element consisting of an elastic element and two zero-length plastic elements located at the two ends of the element (Tseng and Penzien 1973). The damage model has been implemented in a modified version of the program NEABS - Nonlinear Earthquake Analysis of Bridge Systems (Tseng and Penzien 1973) - developed at Carleton University (Phung 2005). The stiffness matrix of the damaged element is derived from the assumption that the degradation is proportional to the degree of damage at the ends of the element. During elastic loading and unloading responses, the stiffness properties of a beam-column element are the elastic stiffness of the member. After yielding, the reloading stiffness in the opposite direction is degraded to reflect the damage effect on the load resistant behavior of the member. The degrading stiffness of a damaged element is modeled such that the degrading stiffness is between the stiffness of the undamaged state and that of fully damaged state by considering the damage effects to both flexural and shear behavior of the element. The different damage states are illustrated in Fig. 2.

\[
d_{\text{ds}} = d_{\text{ms}} = d_{\text{fm}} = (1-FD)(1-SD)k\]

where \(k\) is the elastic stiffness, \(d_{\text{fm}}\) is the maximum degrading effect of flexure damage on stiffness, \(d_{\text{ds}}\) is the degrading effect of shear damage on the remaining stiffness, \(d_{\text{ms}}\) is the stiffness of the fully damaged state by flexure and shear, FD is the local flexural damage index and SD is the local shear damage index. The detailed derivation of the degrading stiffness is described elsewhere (Phung and Lau, 2008).

Numerical Examples

A three-span example bridge designed for an acceleration coefficient of 0.3g is chosen for earthquake response analyses here. Details of the bridge design can be found in the reference (FHWA 1996). The plan and elevation of the bridge are shown in Fig. 3.
Figure 3. Plan and elevation of the bridge.

Bridge columns detailed with different transverse reinforcement ratio to represent different column shear strength of typical old and more recent bridge design are considered. Three bridge structures are examined in the present study: Bridge A considers no flexural nor shear strength degradation, Bridge B considers only flexural degradation, and Bridge C considers both flexural and shear strength degradation in columns. Nonlinear earthquake response analysis of the bridge is performed using 16 sets of strong ground motions recorded from the 1971 Imperial Valley, the 1989 Loma Prieta, the 1994 Northridge and the 1999 Chi-Chi earthquakes. Table 1 lists the ground motion records selected from the PEER strong ground motion database (2006) used for earthquake response analysis of the bridges. Each input ground motion set is a bidirectional horizontal earthquake excitation. These two components of ground accelerations are applied in the longitudinal and transverse directions of the bridge with the larger magnitude applied in the longitudinal direction.

Table 1. Ground acceleration dataset

<table>
<thead>
<tr>
<th>Set</th>
<th>Event</th>
<th>Station</th>
<th>Distance (km)</th>
<th>Site class</th>
<th>PGA in X-dir (g)</th>
<th>PGA in Y-dir (g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Chi-Chi, Taiwan</td>
<td>CHY088</td>
<td>42.82</td>
<td>C</td>
<td>0.216</td>
<td>0.144</td>
</tr>
<tr>
<td>C2</td>
<td>1999/09/20</td>
<td>ILA067</td>
<td>46.68</td>
<td>C</td>
<td>0.198</td>
<td>0.171</td>
</tr>
<tr>
<td>C3</td>
<td></td>
<td>TCU034</td>
<td>32.97</td>
<td>B</td>
<td>0.25</td>
<td>0.108</td>
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<tr>
<td>C4</td>
<td></td>
<td>TCU111</td>
<td>22.22</td>
<td>D</td>
<td>0.136</td>
<td>0.099</td>
</tr>
<tr>
<td>II</td>
<td>Imperial Valley</td>
<td>Calexico Fire Station</td>
<td>10.6</td>
<td>D</td>
<td>0.275</td>
<td>0.202</td>
</tr>
<tr>
<td>I2</td>
<td>1979/10/15 23:16</td>
<td>El Centro Array #3</td>
<td>9.3</td>
<td>D</td>
<td>0.266</td>
<td>0.221</td>
</tr>
<tr>
<td>I3</td>
<td></td>
<td>EC County Center FF</td>
<td>7.6</td>
<td>D</td>
<td>0.235</td>
<td>0.213</td>
</tr>
<tr>
<td>I4</td>
<td></td>
<td>Holville Post Office</td>
<td>7.5</td>
<td>D</td>
<td>0.253</td>
<td>0.221</td>
</tr>
<tr>
<td>L1</td>
<td>Loma Prieta</td>
<td>Coyote Lake Dam (Downstream)</td>
<td>22.3</td>
<td>D</td>
<td>0.179</td>
<td>0.16</td>
</tr>
<tr>
<td>L2</td>
<td>10/18/89 00:05</td>
<td>Gilroy Array #1</td>
<td>11.2</td>
<td>A</td>
<td>0.473</td>
<td>0.411</td>
</tr>
<tr>
<td>L3</td>
<td></td>
<td>Gilroy - Gaviilan Coll.</td>
<td>11.6</td>
<td>B</td>
<td>0.357</td>
<td>0.325</td>
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<tr>
<td>L4</td>
<td></td>
<td>UCSC Lick Observatory</td>
<td>17.9</td>
<td>A</td>
<td>0.45</td>
<td>0.395</td>
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<tr>
<td>N1</td>
<td>Northridge</td>
<td>LA - Baldwin Hills</td>
<td>31.3</td>
<td>B</td>
<td>0.239</td>
<td>0.168</td>
</tr>
<tr>
<td>N2</td>
<td>01/17/94 12:31</td>
<td>Manhattan Beach - Manhattan</td>
<td>42.0</td>
<td>C</td>
<td>0.201</td>
<td>0.128</td>
</tr>
<tr>
<td>N3</td>
<td></td>
<td>La Crescenta - New York</td>
<td>22.3</td>
<td>C</td>
<td>0.178</td>
<td>0.159</td>
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<tr>
<td>N4</td>
<td></td>
<td>Stone Canyon</td>
<td>22.2</td>
<td>A</td>
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<td>0.252</td>
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</table>
Numerical results

Numerical results are presented in terms of the maximum displacement, moments at plastic hinge regions, local and global damage indices. In this study, the maximum displacement of bridge deck at the middle point of the second span of the bridge and bending moments at plastic hinges of the bridge columns are computed. Local flexural and shear damage indices at the plastic hinge regions of the column elements and global damage indices of the bridge structure are also computed. The behavior of bridge columns during strong shaking of earthquakes is traced by examining the evolution of the damage indices proposed in this study. The proposed model is capable to more accurately predict the location, extent of seismic damage and cause of collapse of bridges. The earthquake responses of Bridges B and C are compared to study the effects of only shear degradation. Shear strength degradation in columns of Bridge C significantly increases its seismic response. It is noted in many cases, shear strength degradation in columns of Bridge C leads to bridge collapse as a result of insufficient shear capacity. For example, collapse is observed when Bridge C is subjected to any of C3, I1, I2, I3, I4, L1, N2 and N4 ground motions. As a result, the structural mechanism is formed when the shear capacity of both columns in the same bent are reached. Based on overview of all the results, it is found that shear strength degradation is an important factor that greatly impacts the responses and behavior of older deficient bridges (Bridge C). Consideration of the interaction between flexural and shear degradation effects is necessary for the evaluation of the seismic performance of existing bridges with insufficient shear strength of columns.

![Figure 4. Ground motion time histories recorded at LA – Baldwin Hills Station](image)

![Figure 5. Deck displacement response time histories due to ground motion recorded at LA – Baldwin Hills Station during Northridge earthquake](image)

Nonlinear earthquake responses of Bridges B and C subjected to a set of two horizontal ground accelerations recorded at LA – Baldwin Hills Station during the Northridge earthquake are presented in more details to demonstrate the applications of the proposed damage model in tracing the damage evolution and damage location in the bridge structure. Fig. 4 shows the two horizontal components of the ground motion recorded at LA – Baldwin Hills Station. The progress of the cumulative damage as reflected by the evolution of the damage indices proposed...
in this study is discussed. Fig. 5 shows the displacement response of the bridge deck computed at the middle of the second span in the longitudinal and transverse directions of Bridges B and C. It is observed that Bridge C undergoes significantly larger displacements compared to those of Bridge B.

Fig. 6 shows bending moment response at the plastic hinge location. The responses are similar even after the initial formation of the plastic hinge in all columns at 17.94 sec. This is due to the fact that shear damage is small and does not significantly affect the bridge responses. At 19.20 sec., the shear strength capacity of Column 1 of Bent 1 of Bridge C is exceeded. This causes excessive peak and permanent residual displacements of the bridge deck as shown in Fig. 5. Fig. 7 shows the progression of the local flexural damage index of the bridge columns during earthquake response. Flexural damage index at both columns of Bent 1 is equal to one.

Fig. 8 shows the progression of the local shear damage index. It is observed that shear failure occurs at Column 1 of Bent 1 of Bridge C where the shear damage index reaches one. Fig. 9 shows the progress of the global damage indices of the bridges. The global damage indices can be used as an effective indicator to assess the potential severity and extent of overall damage during earthquakes. The global flexural and shear damage index in Bridge B are 0.41 and 0.38, whereas the global flexural and shear damage index in Bridge C are 0.9 and 0.75, respectively. In this case due to inadequate transverse shear reinforcement resulting in insufficient shear capacity of the bridge columns, Bridge C suffers more damage than Bridge B as shown by the global damage indices. Both the local and global damage indices of Bridge C are greater than those of Bridge B when subjected to the same ground motion set. It is also observed that the shear degradation effects on earthquake responses of Bridge C are more significant than the flexural degradation effects on the response of Bridge B. The damage distribution patterns in the bridge columns are different between Bridge B and C. It is expected that by including the shear damage in the computation of seismic response, the seismic responses of bridges can be more accurately predicted. Shear strength degradation should be used in the analysis of bridges with
inadequate shear capacity. For bridges designed in accordance to the modern seismic design criteria, it is not necessary to include the shear strength degradation into the computation of seismic responses since shear damage is negligibly small.

![Figure 8](image)

Figure 8. Local shear damage index response time history of Column 1 of Bent 1 due to ground motions recorded at LA – Baldwin Hills Station, Northridge earthquake.

![Figure 9](image)

Figure 9. Global damage index response time histories due to ground motions recorded at LA – Baldwin Hills Station, Northridge earthquake.

**Conclusions**

This paper presented a three-dimensional stiffness degradation model that considers the interaction of flexural and shear effects during the seismic response of bridge structures. The model can follow the bilateral stiffness degradation due to flexural and shear progression and accumulation of damage in reinforced concrete bridge columns. The stiffness matrix of a damaged element is computed based on the assumption that the degradation is proportional to the degree of damage at the element ends. The progression of damage is followed from the undamaged state, through the accumulation of damage due to reversed load cycles, up to the fully damaged state represented by a roller-end condition of the element. The modified Park and Ang damage model is used to quantify the flexural damage. A shear damage index is developed based on shear strength and shear demand relationship to quantify the degree of shear damage in the member. Nonlinear seismic response analyses of three-span concrete box girder bridges subjected to bi-directional strong ground motions have been performed. Bridge columns detailed with different transverse reinforcement ratio to represent different column shear strength are considered. It is demonstrated that the consideration of the shear degradation is critical for older deficient bridge piers that can exhibit brittle failure in shear. By following the evolution of the damage indices during the response, the damaged state of the bridge structure can be accurately assessed. The developed model is an important analytical tool for the realistic simulation of the progressive collapse behavior of reinforced concrete bridges during earthquakes.

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References


