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EFFECT OF TANK PARAMETERS ON RESPONSE OF CONCRETE RECTANGULAR LIQUID STORAGE TANKS

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ABSTRACT

This paper presents the results of parametric studies on seismic response of concrete rectangular liquid storage tanks using the generalized single degree of freedom (SDOF) system. The effects of height of liquid on dynamic response of liquid storage tanks are investigated. The liquid level varies from empty condition to full tank. Also, instead of the commonly used ratio of length of tank to the liquid height, L_x/H_L, the ratio of length of tank to the full height of tank wall, L_x/H_W is used as a characteristic parameter of tank to study the effect of tank size on the dynamic response. The trends of added mass of liquid and effective height for different sizes of tanks are established. The values of the added mass of liquid due to impulsive hydrodynamic pressure and the effective height in relationship with the ratios of L_x/H_W and H_L/H_W are determined and can be used in the seismic design of liquid storage tanks.

Introduction

Housner's model (Housner 1963) is commonly used in dynamic analysis and design of liquid containing structures (LCS). This model approximates the effect of hydrodynamic pressure for a two fold-symmetric-fluid container subjected to horizontal acceleration. The hydrodynamic pressures induced by earthquakes are considered using the impulsive and convective components which are approximated by the lumped added masses. The added mass in terms of impulsive pressure is assumed rigidly connected to the tank wall. The added mass in terms of convective pressure is assumed connected to the tank wall using flexible springs to simulate the effect of sloshing motion. The boundary condition in the calculation of hydrodynamic pressures is assumed to be rigid.

Although Housner's model has traditionally been used as a simple tool for seismic design of LCS, the recent studies show that the lumped mass approach overestimates the base shear significantly (Kianoush et al. 2006, Chen and Kianoush 2005, and Ghaemian et al. 2005). Another issue is that the effect of flexibility of tank wall on seismic design of concrete LCS has generally been neglected in the past. As a result, a simplified method using the generalized single degree of freedom (SDOF) system was proposed to improve the dynamic response of

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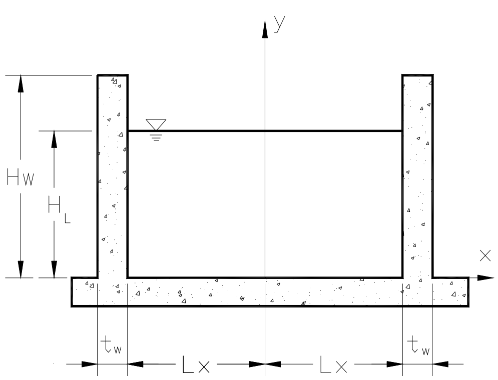
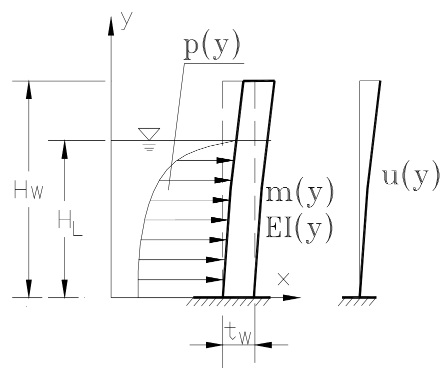
concrete rectangular LCS (Chen and Kianoush 2009). In the generalized SDOF system, only one variable is used in dynamic analysis of distributed mass and stiffness characteristics system for a predetermined mode shape. The consistent mass approach and the effect of flexibility of tank wall on hydrodynamic pressures are considered.

When engineers design liquid containing systems, generally the process engineers determine the design liquid level based on the hydraulic requirements. However, when the liquid containing structures are in operation, it is possible that the actual liquid level may be less than the design maximum liquid level. Also, the liquid level H_L is normally less than the height of wall H_W for free sloshing of open top tanks and may vary for the process and maintenance reasons. In this study, the effect of liquid level on the dynamic response of LCS is investigated using the generalized SDOF system. The liquid level inside tank can vary from the empty condition $H_L=0$ to the full level of tank height, i.e. $H_L=H_W$.

Analytical Model and Equation of Motion

A 2-D analysis model is shown in Figure 1(a). It is assumed that the width of tank in perpendicular to the direction of earthquake is sufficiently large so that the unit width of tank can represent the tank wall. For an open top concrete rectangular LCS, the wall can be considered in the cantilever condition. Figure 1(b) shows the generalized SDOF system with the distributed mass m(y) and stiffness EI(y) per unit height subjected to earthquake ground motion $u_g(t)$.





(a) 2–D model of rectangular tank (b) Analysis model using generalized SDOF system

Figure 1 Schematic of Rectangular Tank and Analysis Model

The equation of motion for the generalized SDOF system subjected to ground motion for the case of LCS is that:

$$\tilde{m}\cdot\ddot{u}+\tilde{c}\cdot\dot{u}+\tilde{k}\cdot u=\tilde{p} \tag{1}$$

Where $\tilde{m}$, $\tilde{c}$, $\tilde{k}$, $\tilde{p}$ are defined as the generalized system of mass, damping, stiffness and force respectively as shown below:

$$\widetilde{m} = \int_0^{H_W} m(y) \cdot [\psi(y)]^2 \cdot dy + \widetilde{m}_L \tag{2}$$

$$\widetilde{k} = \int_0^{H_W} EI(y) \cdot [\ddot{\psi}(y)]^2 \cdot dy \tag{3}$$

$$\widetilde{p} = \ddot{u}_g(t) \cdot [\int_0^{H_W} m(y) \cdot \psi(y) \cdot dy + m_L] \tag{4}$$

Where $\psi(y)$ is the assumed shape function, and $\widetilde{m}_L$ and m_L are the generalized and effective added mass of liquid due to impulsive hydrodynamic pressure. A damping ratio of 5% of critical is considered for all cases (Chen and Kianoush 2009).

In this study, the prescribed vibration shape function representing the first mode shape for the cantilever wall boundary condition is that:

$$\psi(y) = \frac{3}{2}\frac{y^2}{H_W^2} - \frac{1}{2}\frac{y^3}{H_W^3} \tag{5}$$

It is worth noting that the shape function SF1 provide the most accurate results for the cantilever condition as discussed in the previous study (Chen and Kianoush 2009).

Added Mass of Liquid Due to Impulsive Hydrodynamic Pressure

When using the generalized SDOF system in the dynamic analysis of LCS, the hydrodynamic pressure is incorporated into the coupling analysis through the added mass of liquid in the system. The generalized and effective added masses of liquid due to impulsive hydrodynamic pressure can be calculated using Eqs.6 and 7 (Chen and Kianoush 2009) respectively as follows:

$$\widetilde{m}_L = \sum_{i=1}^{\infty} \frac{2 \cdot \rho_l}{\lambda_{i,n} \cdot H_L} \tanh(\lambda_{i,n} L_x) [\int_0^{H_L} \cos(\lambda_{i,n} y) \cdot \psi(y) dy]^2 \tag{6}$$

$$m_L = \sum_{i=1}^{\infty} \frac{2 \cdot (-1)^{i+1} \rho_l}{\lambda_{i,n}{}^2 \cdot H_L} \tanh(\lambda_{i,n} L_x) \int_0^{H_L} \cos(\lambda_{i,n} y) \cdot \psi(y) dy \tag{7}$$

Where $\lambda_i = (2i-1)\pi/2H_L$. As the series in the above equation convergence very fast, only the first terms of the series are used for practical applications.

The ratios of generalized and effective added masses of liquid due to impulsive hydrodynamic pressure to the half mass of liquid in the containment $\widetilde{m}_L / M_{L1}$ and m_L / M_{L1} can be calculated using Eqs.8 and 9 respectively as follows:

$$\frac{\widetilde{m}_L}{M_{L1}} = \sum_{i=1}^{\infty} \frac{2}{\lambda_{i,n} \cdot H_L^{\ 2} \cdot L_x} \tanh(\lambda_{i,n} L_x)[\int_0^{H_L} \cos(\lambda_{i,n} y) \cdot \psi(y) dy]^2 \tag{8}$$

$$\frac{m_L}{M_{L1}} = \sum_{i=1}^{\infty} \frac{2 \cdot (-1)^{i+1}}{\lambda_{i,n}^{\ 2} \cdot H_L^{\ 2} \cdot L_x} \tanh(\lambda_{i,n} L_x) \int_0^{H_L} \cos(\lambda_{i,n} y) \cdot \psi(y) dy \tag{9}$$

It is worth noting that compared to the total mass of liquid in Housner's model, only half the mass of liquid is considered in the generalized SDOF system based on the two-fold symmetric fluid structural model. In addition, when the values of L_x/H_L are relatively large, the ratios of $\widetilde{m}_L / M_{L1}$ and m_L / M_{L1} become minimal. Therefore, it is recommended to use the ratios of the added mass of liquid to that of rigid wall condition in the dynamic analysis of LCS. Therefore, the ratios of $\widetilde{f}_{mass}$ and f_{mass} can be defined as follows:

$$\widetilde{f}_{mass} = \frac{\widetilde{m}_L}{\widetilde{M}_{rigid}} = \frac{\sum_{i=1}^{\infty} \frac{2 \cdot \rho_l \tanh(\lambda_{i,n} L_x)}{\lambda_{i,n} H_L} [\int_0^{H_L} \cos(\lambda_{i,n} y) \cdot \psi(y) dy]^2}{\sum_{i=1}^{\infty} \frac{2 \cdot \rho_l \tanh(\lambda_{i,n} L_x)}{\lambda_{i,n} H_L} [\int_0^{H_L} \cos(\lambda_{i,n} y) dy]^2} \tag{10}$$

$$f_{mass} = \frac{m_L}{M_{rigid}} = \frac{\sum_{i=1}^{\infty} \frac{2 \cdot (-1)^{i+1} \rho_l \tanh(\lambda_{i,n} L_x)}{\lambda_{i,n}^{\ 2} H_L} \int_0^{H_L} \cos(\lambda_{i,n} y) \cdot \psi(y) dy}{\sum_{i=1}^{\infty} \frac{2 \cdot (-1)^{i+1} \rho_l \tanh(\lambda_{i,n} L_x)}{\lambda_{i,n}^{\ 2} H_L} \int_0^{H_L} \cos(\lambda_{i,n} y) dy} \tag{11}$$

where $\widetilde{M}_{rigid}$ and M_{rigid} are the generalized and effective added masses of liquid due to impulsive hydrodynamic pressure using the shape function $\psi(y) = 1$ for the rigid wall boundary condition respectively. It can be found that the values of $\widetilde{M}_{rigid}$ and M_{rigid} are generally the same.

Effect of liquid Level

For the rigid wall boundary condition, the normalized hydrodynamic pressure distribution along the height of tank wall is the same for different height of liquid, i.e. H_L=0~H_W in dynamic analysis. This assumption is only correct for rigid boundary condition and adopted in the current design codes and standards. However, when the flexibility of tank wall is considered, the hydrodynamic pressure distribution along the tank wall is as function of the liquid level which is relative to the height of tank wall. Also, in the generalized SDOF system, the distribution of added mass of liquid along the height of tank wall is based on the prescribed shape functions. Therefore, the effect of variable liquid level inside the tank should be considered in the calculation of the added mass of liquid based on the flexible wall condition.

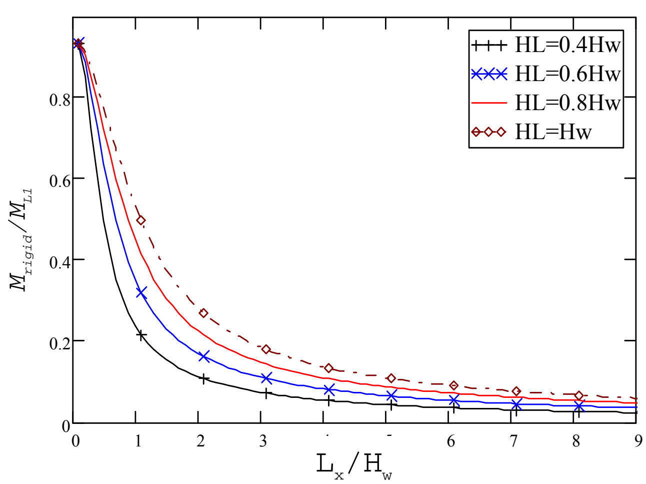
The ratio of length of tank to the liquid height, L_x/H_L is normally used as a parameter to study the effect of tank size and liquid height on the dynamic response of LCS. It is presumed that the tank is full with liquid height equal to height of tank wall. However, as the liquid level may vary, the ratio of L_x/H_L may not remain constant. In this study, the height of tank wall H_w rather than the height of liquid H_L is used to consider the size effect of tank. The advantages of using such an approach are as follows:

1) Both L_x and H_W are fundamental parameters representing the configuration of a tank, and
2) The height of a tank wall is a pre-determined parameter in dynamic analysis while the height of liquid may be considered a variable.

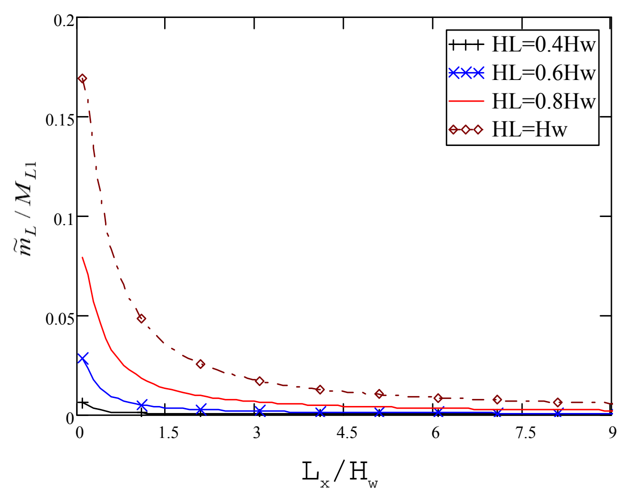
Figures 2(a) to 2(e) show the ratios of added mass of liquid as function of L_x/H_W. The horizontal coordinates L_x/H_W represents the different size of tanks as discussed before. The vertical coordinates show the ratios of added mass of liquid based on the half mass of liquid in tanks, i.e. Figures 2(a) to 2(c), and for the rigid boundary condition $\psi(y)=1$, i.e. Figures 2(d) and 2(e). Also, the Figures present the effect of variable liquid height on the added mass of liquid for which the liquid heights are $0.4H_W$, $0.6H_W$, $0.8H_W$ and $1.0H_W$.

It is worth noting that the ratios of added mass of liquid in the current design codes and standards are based on the total mass of liquid in tank. The design diagrams are similar to Figures 2(a) to 2(c). However, if the length of tank in the direction parallel to earthquake is significantly larger than the depth of liquid, increasing the tank length has no significant effect on the dynamic response of LCS. As a result, it is recommended to use the factors $\tilde{f}_{mass}$ and f_{mass} for design purpose as shown in Figures 2(d) and 2(e).

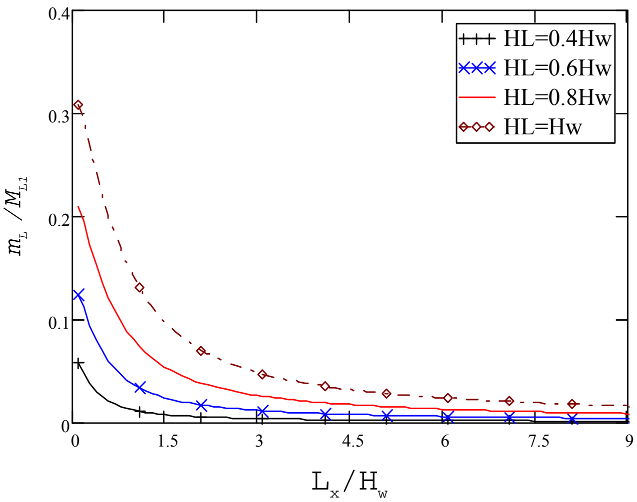
Figure 2 shows that with the increase of liquid level in the tank, the added mass of liquid due to impulsive hydrodynamic pressure increases as expected.



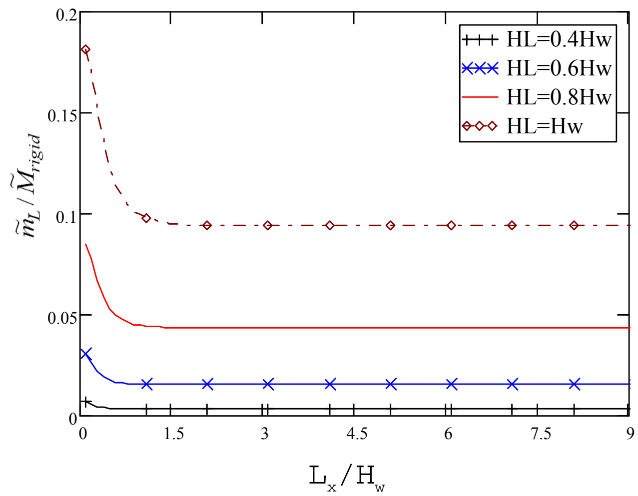
(a) M_{rigid}/M_{L1} vs. L_x / H_W



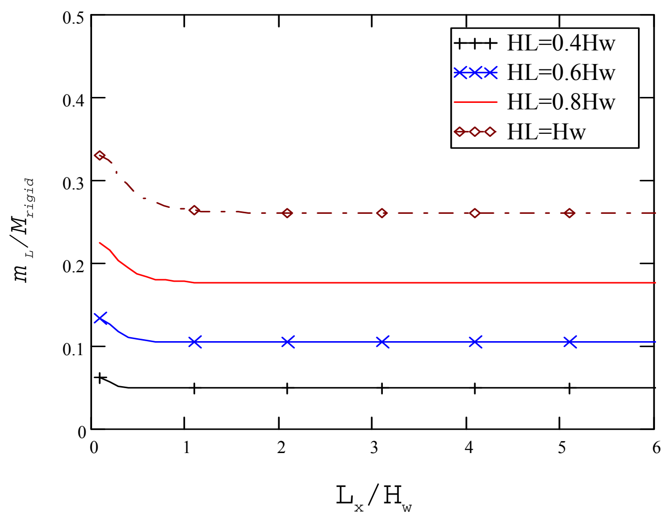
(b) $\tilde{m}_L / M_{L1}$ vs. L_x / H_W



(c) m_L / M_{L1} vs. L_x / H_W



(d) $\tilde{m}_L / \tilde{M}_{rigid}$ vs. L_x / H_W



(e) m_L / M_{rigid} vs. L_x / H_W

Figure 2 Effect of Liquid Level on Added Mass of Liquid

Figure 2(a) shows that with the increase in the values of L_x/H_W up to about 3.0, the ratios of added mass of liquid based on the rigid boundary condition $\psi(y)=1$ to the mass of liquid in tank M_{rigid}/M_{L1} drop significantly for all levels of liquid height. With the increase in the value of L_x/H_W beyond 3.0, the ratio of M_{rigid}/M_{L1} approaches a constant value. In addition, the value of L_x/H_W for the ratio of M_{rigid}/M_{L1} within the constant range is smaller for the lower liquid level as compared to the higher liquid level.

A similar trend to that shown in Figure 2(a) appears in the ratio of generalized and effective added mass of liquid to the half mass of liquid in tank as shown in Figures 2(b) and

2(c). When the value of L_x/H_W exceeds 1.5, the ratio of $\tilde{m}_L / M_{L1}$ approaches a constant value.

It is worth noting that the hydrodynamic pressure is a function of the effective added mass of liquid (Chen and Kianoush 2009). Provided that the acceleration is known, the hydrodynamic pressure can be calculated. Therefore, Figure 2(c) also reflects the trend of the force due to hydrodynamic pressure for different heights of liquid in a tank.

Figures 2(d) and 2(e) show that for values of $L_x/H_W>1$, the factors $\tilde{f}_{mass}$ and f_{mass} remain constant. This means that there is no significant change on the ratio of the added mass in participation of dynamic response based on the rigid wall boundary condition, when $L_x/H_W>1$.

Figure 3 shows the mass ratios of liquid due to impulsive pressure versus the length of tank to depth of liquid L_x/H_L. The mass ratios are based on Hounser's model and the generalized SDOF system using shape function $\psi(y)=1$ which are both corresponding to a rigid tank wall, and the ratios of effective added mass of liquid, i.e. m_L / M_{L1}. The first two modes are included in order to consider the flexibility of tank wall in dynamic analysis. It is worth noting that Figure 3 is based on full tank condition, i.e. $H_L = H_W$.

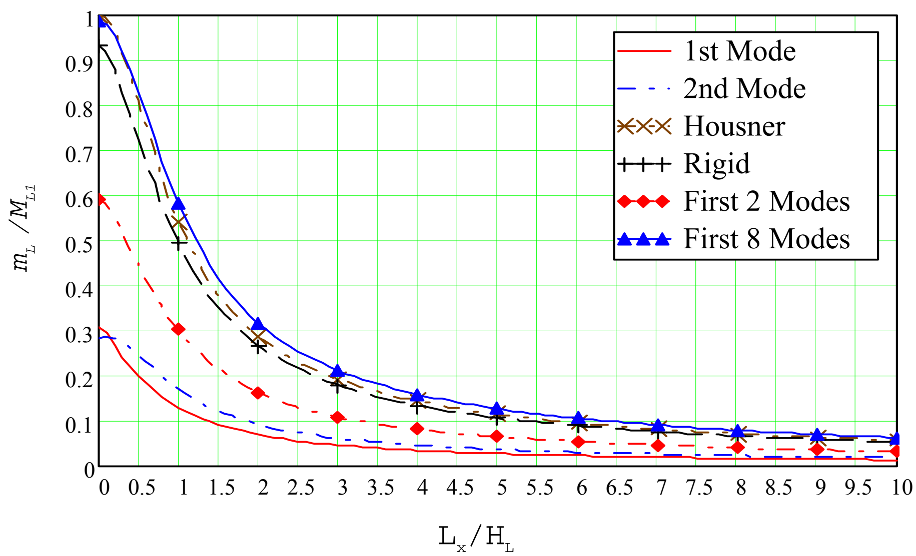


Figure 3 Ratio of Added Mass of Liquid due to Impulsive Hydrodynamic Pressure vs. L_x / H_L

Figure 3 shows that the trend of curves for Housner's model and the generalized SDOF system is similar. It is worth noting that the sum of the effective added mass of liquid for the first two modes is less than that based on the rigid boundary condition as shown in Figure 3. However, if higher modes are considered, the sum of the effective added mass of liquid is larger than that based on the rigid boundary condition. This is consistent with the previous studies that the wall flexibility increases the added mass of liquid due to impulsive hydrodynamic pressure (Yang, 1976).

Effective Height

In the generalized SDOF system, the added mass of liquid due to hydrodynamic pressure can still be treated similar to that of Housner's model as a lumped mass (Chen and Kianoush 2009). The effective height at which the effective added mass of liquid due to impulsive hydrodynamic pressure is applied can be calculated as that:

$$h_i = \frac{\int_0^{H_L} \sum_{i=1}^{\infty} \frac{2\rho_l \tanh(\lambda_{i,n} L_x)}{\lambda_{i,n} H_L} \cos(\lambda_{i,n} y) \int_0^{H_L} \cos(\lambda_{i,n} y) \cdot \psi(y) dy \cdot y dy}{\sum_{i=1}^{\infty} \frac{2 \cdot (-1)^{i+1} \rho_l \tanh(\lambda_{i,n} L_x)}{\lambda_{i,n}^2 H_L} \int_0^{H_L} \cos(\lambda_{i,n} y) \psi(y) dy} \tag{12}$$

For a tank containing liquid, the overall effective height at which the dynamic force is applied can be expressed using Eq.13. This is obtained by combining the inertial mass of tank wall and the added mass of liquid due to impulsive hydrodynamic pressure as follows:

$$h = \frac{[\int_0^{H_W} m(y) \cdot \psi(y) \cdot y \cdot dy + \int_0^{H_L} \sum_{i=1}^{\infty} \frac{2\rho_l \tanh(\lambda_{i,n} L_x)}{\lambda_{i,n} H_L} \cos(\lambda_{i,n} y) \int_0^{H_L} \cos(\lambda_{i,n} y)\, \psi(y) dy \cdot y dy]}{\int_0^{H_W} m(y) \cdot \psi(y) \cdot dy + \sum_{i=1}^{\infty} \frac{2(-1)^{i+1} \rho_l}{\lambda_{i,n}^2 H_L} \tanh(\lambda_{i,n} L_x) \int_0^{H_L} \cos(\lambda_{i,n} y)\, \psi(y) dy} \tag{13}$$

In addition, the effective height of added mass of liquid due to impulsive pressure and the overall effective height of LCS can also be determined using Hounser's model as follows:

For tanks with $\frac{2L_x}{H_L} < 1.333$,

$$\frac{h_i}{H_L} = 0.5 - 0.09375 \left(\frac{2L_x}{H_L}\right) \tag{14}$$

For tanks with $\frac{2L_x}{H_L} \geq 1.333$,

$$\frac{h_i}{H_L} = 0.375 \tag{15}$$

For liquid containing structures, the effective equivalent height at which the total impulsive dynamic lateral force is applied can be calculated using Eq.16 as follows:

$$h = \frac{m_w \cdot h_w + m_L \cdot h_i}{m_w + m_L} \tag{16}$$

It is worth noting that the boundary condition is assumed to be rigid in Housner's model.

Similar to the added mass of liquid as discussed previously, the ratio of length of tank to height of tank L_X/H_W is used to consider the size effect of tank on the effective heights. Figure 4 shows the normalized effective height at which the effective added mass of liquid due to impulsive hydrodynamic pressure is applied as function of L_x/H_W. The liquid heights considered are $0.4H_W$, $0.6H_W$, $0.8H_W$ and $1.0H_W$. The figure shows that when the value of L_x/H_W is less than 1.0, the values of h_i/H_L decrease at a fast rate. However, for values of L_x/H_W greater than 1.0, the values of h_i/H_L remain constant which indicates that the increase in the tank length in the direction parallel to the direction of earthquake has no significant effect on the effective height of the added mass of liquid.

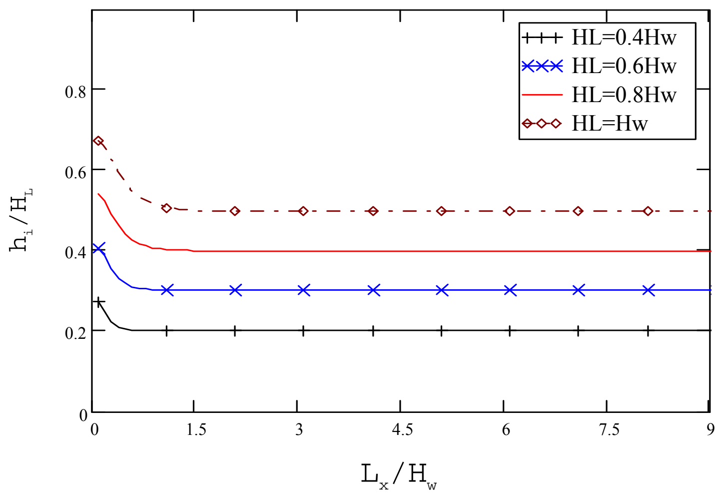


Figure 4 Effective Height

Table 1 shows the effective height at which the hydrodynamic pressure is applied as function of H_L/H_W and L_X/H_W for the first mode shape and the variable liquid level condition. These tabulated values can be used for dynamic response of LCS using the generalized SDOF system which includes the effect of flexibility of tank wall.

Table 1 Ratio of h_i / H_W in relationship with L_x/H_W and H_L/H_W (1st mode)

		Lx/Hw												
		0.0	0.1	0.2	0.3	0.4	0.5	0.8	1.0	1.5	2.0	3.0	4.0	5.0
HL/Hw	0.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
	0.1	0.0280	0.0221	0.0219	0.0218	0.0218	0.0218	0.0218	0.0218	0.0218	0.0218	0.0218	0.0218	0.0218
	0.2	0.0983	0.0823	0.0783	0.0776	0.0775	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774	0.0774
	0.3	0.1906	0.1686	0.1572	0.1538	0.1527	0.1524	0.1522	0.1522	0.1522	0.1522	0.1522	0.1522	0.1522
	0.4	0.2861	0.2635	0.2451	0.2377	0.2347	0.2334	0.2325	0.2324	0.2324	0.2324	0.2324	0.2324	0.2324
	0.5	0.3686	0.3493	0.3278	0.3167	0.3115	0.3089	0.3065	0.3062	0.3061	0.3061	0.3061	0.3061	0.3061
	0.6	0.4256	0.4118	0.3919	0.3793	0.3724	0.3687	0.3647	0.3641	0.3637	0.3637	0.3637	0.3637	0.3637
	0.7	0.4489	0.4415	0.4274	0.4163	0.4093	0.4052	0.4003	0.3993	0.3988	0.3987	0.3987	0.3987	0.3987
	0.8	0.4353	0.4341	0.4284	0.4218	0.4169	0.4136	0.4093	0.4083	0.4077	0.4076	0.4075	0.4075	0.4075
	0.9	0.3856	0.3899	0.3937	0.3939	0.3930	0.3921	0.3905	0.3901	0.3898	0.3897	0.3897	0.3897	0.3897
	1.0	0.3039	0.3126	0.3257	0.3341	0.3388	0.3416	0.3452	0.3461	0.3468	0.3470	0.3470	0.3470	0.3470

Conclusions

In this study, the effect of liquid level on dynamic response of liquid containing structures (LCS) is investigated. It is recommended to use the ratio of length of tank to the height of tank wall L_x/H_W rather than the ratio of length of tank to liquid height, L_x/H_L to study the size effect of tanks. This is because the liquid level may be a variable in the design and operation. In addition, when the flexibility of tank wall is considered in the dynamic analysis of LCS, the liquid level affects the added mass of liquid due to impulsive pressure. This is due to variation of added mass distribution along the height of the tank wall. Therefore, the ratio of height of liquid to the height of wall H_L/H_W is introduced to study the variable depth of liquid inside the tank.

The generalized and effective added masses of liquid due to impulsive hydrodynamic pressure are calculated based on the parameters L_x/H_W and H_L/H_W. The values of the added mass of liquid due to impulsive hydrodynamic pressure presented in this study can be used in the seismic design of LCS.

The effect of liquid level on the effective height of added mass of liquid and the overall effective height of LCS are also investigated. The values of effective height at which the hydrodynamic pressure is applied as function of H_L/H_W and L_X/H_W are presented for the first mode shape and the variable liquid level condition. The results of this study can be used for design of LCS using the generalized SDOF system which includes the effect of flexibility of tank wall.

References

Chen, J.Z. and Kianoush, M.R. 2005. Seismic response of concrete rectangular tanks for liquid containing structures. *Canadian Journal of Civil Engineering* 32(4): 739-752.

Chen, J.Z. and Kianoush, M.R. 2009. Generalized SDOF system for seismic analysis of concrete rectangular liquid storage tanks. *Journal of Engineering Structures*, 31, 2426-2435.

Ghaemian, M., Kianoush, M.R. and Mirzabozorg, H. 2005. Time domain dynamic analysis of rectangular liquid containers in three-dimensional space. *Journal of European Earthquake Engineering*, XIX(2), 3-9.

Housner, G.W. 1963. The dynamic behavior of water tanks. *Bulletin of the Seismological Society of American* 53(2).

Kianoush, M.R., Mirzabozorg, H. and Ghaemian, M. 2006. Dynamic analysis of rectangular liquid containers in three-dimension space. *Canadian Journal of Civil Engineering*, 33(5): 501-507.

Yang, J.Y. 1976. Dynamic Behavior of Fluid-Tank System. Ph.D. Thesis, Civil Engineering, Rice University Houston, Texas.