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SEISMIC RESPONSE ANALYSIS OF THE FOLDED CANTILEVER SHEAR STRUCTURE

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ABSTRACT

In this paper, a folded cantilever shear structure with long natural periods and endued with high damping constant is proposed in order to improve seismic performance of middle-multistory building which vibrates mainly as the shear structure. Therefore, the proposed folded cantilever shear structure is designed mainly consisting of two parts namely, fixed shear sub-structure and movable shear sub-structure which are interconnected with each other by connection beam at the top parts of the fixed and movable substructures. Besides, additional viscous dampers connect both fixed and movable shear sub-structure with each other horizontally on the basis of building stories. To investigate period, damping constant and shape of the natural damping vibration modes of the proposed structure analytical model is tested theoretically. As it is expected, while the obtained natural period of ordinary cantilever shear structure is T_1=1.0s, the obtained natural period of proposed folded cantilever shear structure is T_2=2.1s which clarifies that the anticipated seismic performance for proposed structure is acquired. Eventually, the proposed structure provides rich damping coefficients and long natural period for multistory structures.

1. Introduction

Generally the seismic acceleration response of multistory building decreases by extending the natural period of the multistory building in regions which have long natural period exceed about 1 second. The natural period of the multistory building which vibrates mainly as the shear structure increases in proportion to the number of stories. Therefore, it is an efficient method to expand the natural period of the multistory building over about 2 seconds by piling up story in terms of seismic design. The base isolation systems (Soong and Costantinou 1994) which generate the natural sway vibration mode with a very long period can decrease the seismic acceleration response of the multistory building, but the base isolation systems are expensive and are usually applied to the few-multistory buildings with the structural period under about 1 sec.

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Consequently, it is clarified that a new extension technology of the natural period for the middle multi-story buildings is needed. The paper comprises of a proposed folded cantilever shear structure with long natural periods in order to improve the seismic performance of the middle-multistory building which vibrates mainly as the shear vibration structure. At first, the proposed structure is designed within the framework of two main parts namely; fixed shear sub-structure which is fixed on the ground and movable shear sub-structure which is vertically supported by roller bearings on the ground. By doing so, it is aimed that to increase the first natural period of the proposed structure in comparison with the first natural period of the ordinary multistory building which has the same number of stories of the proposed structure. In addition, the both top parts of the fixed and movable sub-structure are interconnected with each other by rigid connection beam. Then installation of damping devices process; additional damping devices connect the both fixed and movable shear sub-structure with each other horizontally on the basis of stories in order to acquire higher seismic performance under effecting loads. Firstly, the structural characteristics of the folded cantilever shear structure are explained on simple illustration of vibration model. Then the period and the shape of the natural vibration modes of the proposed structure are theoretically and numerically investigated by a real and complex eigenvalue analyses. Finally the seismic responses of the proposed structure and ordinary building are obtained under different earthquakes waves, so seismic performance of proposed folded cantilever shear structure is discussed.

2. Vibration Model of the Folded Cantilever Shear Structure

2.1. Outline of Vibration Model

Figure 1 illustrates the vibration model of the folded cantilever shear structure with n stories by simplifying, which is composed of the fixed shear sub-structure and the movable shear sub-structure. The fixed shear sub-structure consists of $2n$ columns, beam-1, beam-2⋯ and beam-n, and the movable shear sub-structure consists of $2n$ columns, beam-n+1, beam-n+2⋯ and beam-2n. By supporting the movable shear sub-structure vertically by means of roller bearings it is possible to make foundation free to move horizontally. The connection beam as beam-n rigidly connects the top of the fixed sub- structure and the top of movable sub-structures. The key idea in modeling the proposed structure to extend the natural period is to interconnect the tops of the fixed and movable sub-structure rigidly by the connection beam. Besides, the symbols which define parameters are given as follows; k_i is the shear spring coefficient for the horizontally deformation performance of the story between beam-i and beam-i+1 (beam-0 is equal to the foundation), m_i is the mass of beam-i and the columns which are connected with beam-i, and c_i is the viscous damping coefficient of the dashpot which connects beam-i and beam-i+1. The dashpots are used to simulate the ordinary structural damping of the fixed and the movable shear sub-structures. The symbol $d_i(i = 0,2,\cdots,n-1)$ is the viscous damping coefficient of the additional viscous damper which horizontally connects beam-i and beam-*2n-i*. The additional viscous dampers are used to improve the damping performance of the folded cantilever shear structure. The symbol H is the total height of the vibration model which consist of n stories with h height. The vibration test model of the folded cantilever shear structure is only shaken in the x direction of Fig. 1.

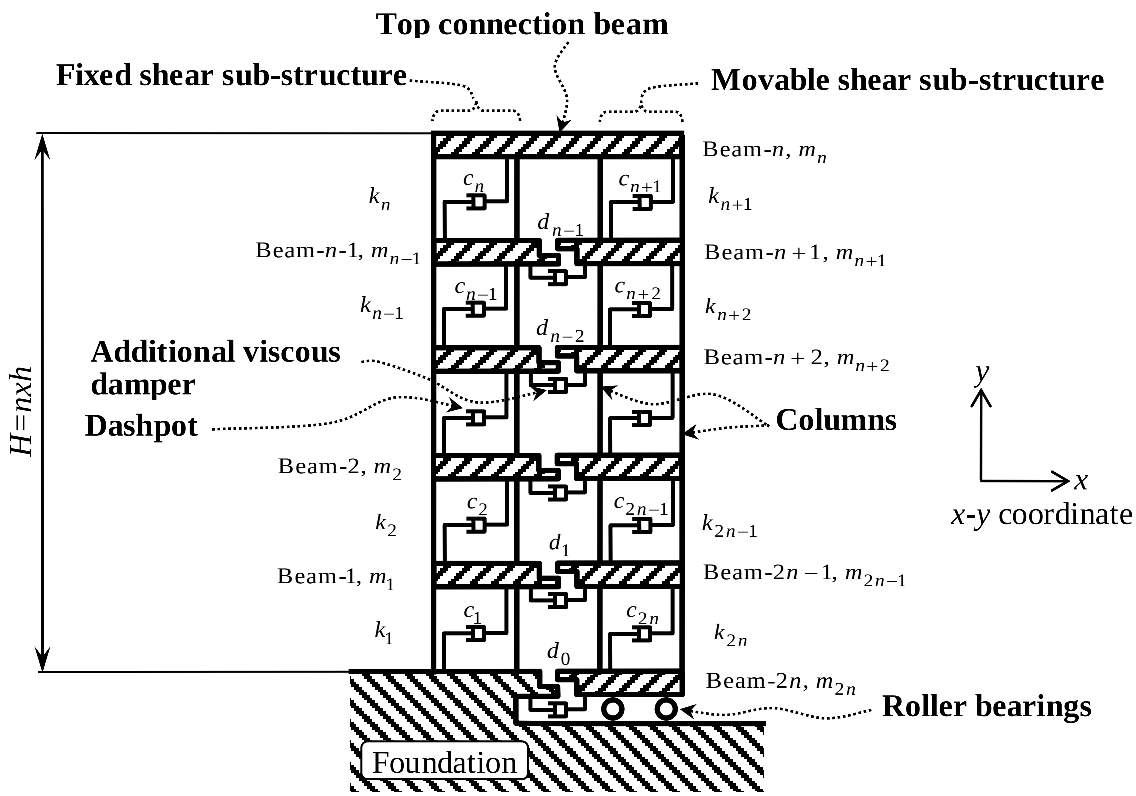


Figure 1. Vibration model of the folded cantilever shear structure with n stories.

2.2. The Equation of Motion for Seismic Vibration

Characteristics of the free vibration of the folded cantilever shear structure is investigated by using the vibration model as shown in Figure 1. The equation of motion of this vibration model can be expressed by the follow,

$$\mathbf{M}\ddot{\boldsymbol{u}} + (\mathbf{C} + \mathbf{D})\dot{\mathbf{u}} + \mathbf{K}\mathbf{u} = -\mathrm{sgn}(\dot{\mathrm{u}}_{2\mathrm{n}})\mathrm{f_d}\mathrm{e}_{2\mathrm{n}} - \ddot{\mathrm{z}}\mathrm{p} \tag{1}$$

, where $\boldsymbol{u} \equiv [u_1, u_2, \cdots, u_{2n}]$ is the displacement vector of size $2n$, $\dot{\boldsymbol{u}} \equiv [\dot{u}_1, \dot{u}_2, \cdots, \dot{u}_{2n}]$ is the velocity vector of size $2n$, $\ddot{\boldsymbol{u}} \equiv [\ddot{u}_1, \ddot{u}_2, \cdots, \ddot{u}_{2n}]$ is the acceleration vector of size $2n$, e_i is the unit vector of size $2n$, of which the i-th element is equal to 1, and f_d is the dynamic frictional force of the roller bearing system. The symbols u_i, $\dot{u}_i$ and $\ddot{u}_i$ are the displacement, the velocity, and the acceleration of beam-i in the x direction, respectively. Then $\boldsymbol{M}$ is the diagonal mass matrix of size $2nx2n$, $\boldsymbol{K}$ is the tri-diagonal stiffness matrix of size $2nx2n$, $\boldsymbol{C}$ is the tri-diagonal structural damping matrix of size $2nx2n$, $\boldsymbol{D}$ is the additional damping matrix of size $2nx2n$. The matrices of $\boldsymbol{M}$, $\boldsymbol{K}$, $\boldsymbol{C}$, and $\boldsymbol{D}$ are defined by the following formulas.

$$\mathbf{M} \equiv \text{diagonal}\,[\mathrm{m}_1, \mathrm{m}_2, \cdots, \mathrm{m}_{2\mathrm{n}-1}, \mathrm{m}_{2\mathrm{n}}] \tag{2}$$

$$\mathbf{K}\equiv\begin{bmatrix} k_1+k_2 & -k_2 & 0 & \cdots & 0 \\ -k_2 & k_2+k_3 & -k_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -k_{2n-1} & -k_{2n-1}+k_{2n} & -k_{2n} \\ 0 & \cdots & 0 & -k_{2n} & k_{2n} \end{bmatrix} \tag{3}$$

$$\mathbf{C}\equiv\begin{bmatrix} c_1+c_2 & -c_2 & 0 & \cdots & 0 \\ -c_2 & c_2+c_3 & -c_2 & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & -c_{2n-1} & -c_{2n-1}+c_{2n} & -c_{2n} \\ 0 & \cdots & 0 & -c_{2n} & c_{2n} \end{bmatrix} \tag{4}$$

$$\mathbf{D}\equiv \mathrm{d}_0\mathbf{e}_{2\mathrm{n}}\mathbf{e}_{2\mathrm{n}}^{\mathrm{T}}+\boldsymbol{\Sigma}_{\mathrm{i}=1}^{\mathrm{n}-1}[\mathbf{e}_{\mathrm{i}},\mathbf{e}_{2\mathrm{n}-1}]\begin{bmatrix} \mathrm{d_i} & -\mathrm{d_i} \\ -\mathrm{d_i} & \mathrm{d_i}\end{bmatrix}[\mathbf{e}_{\mathrm{i}},\mathbf{e}_{2\mathrm{n}-1}]^{\mathrm{T}} \tag{5}$$

3. Free Vibration Modes and Damping

3.1. Un-damped Eigenvalue Problem

If the dynamic frictional force of the roller bearing system, the damping forces of the dashpots, and the damping forces of the additional viscous dampers be neglected on the vibration model in Figure.1, an eigenvalue problem in order to estimate frequencies and shapes of the natural vibration modes of the vibration model can be described by the following homogeneous simultaneous equations,

$$(\boldsymbol{K}-\omega^2\boldsymbol{M})\boldsymbol{\varphi}=0 \tag{6}$$

, where ω and $\boldsymbol{\varphi}^T\equiv[\emptyset_1,\emptyset_2,\cdots,\emptyset_{2n}]$ are the natural frequency and the corresponding eigenvector of size $2n$, respectively, then $\emptyset_i$ is the amplitude of beam-i in the x direction. The natural frequencies and the eigenvectors of Eq.(6) are usually calculated by using the QR method, but analytical solutions exist on the condition that $m_i=\overline{m}$ and $k_i=\overline{k}$ for all i. On this condition, the i-th natural frequency ω_i and the j-th element $\emptyset_{j,i}$ of the eigenvector φ_i corresponding to ω_i are expressed by the following formula,

$$\omega_{\mathrm{i}}=2\overline{\omega}\sin\left(\frac{2i-1}{4x2n+2}\pi\right),\qquad 1\le i\le 2n \tag{7}$$

$$\emptyset_{\mathrm{j,i}}=\frac{2}{\sqrt{2x2n+1}}\sin\left(\frac{2i-1}{2x2n+1}j\pi\right),1\le i\le 2n,\ 1\le j\le 2n \tag{8}$$

, where $\overline{\omega}=\sqrt{\overline{k}/\overline{m}}$ is called the typical story frequency and the eigenvector φ_i satisfies a normalized condition of $\|\varphi_i\|=1$.

3.2. Damped Eigenvalue Problem and Viscous Damping Ratio

If the dynamic friction force of the roller bearing system be neglected on the vibration model in Figure.1, a complex eigenvalue problem to estimate viscous damping ratios of the damped vibration modes of the vibration model can be described by the following homogeneous simultaneous equations,

$$\{\boldsymbol{K} + \lambda(\boldsymbol{C} + \boldsymbol{D})\boldsymbol{u} + \lambda^2 \boldsymbol{M}\}\Psi = 0 \tag{9}$$

, where λ and $\Psi^T \equiv [\Psi_1, \Psi_2, \cdots, \Psi_{2n-1}, \cdots, \Psi_{2n}]$ are the complex eigenvalue and the complex eigenvector of size $2n$ corresponding to the complex eigenvalue, respectively, then Ψ_i is the complex amplitude at beam-i in the x direction. The complex eigenvalue and the complex eigenvector of Eq.(9) are usually calculated by using Foss's method (K. A. FOSS, 1958). The real and imaginary parts of the j-th complex eigenvalue λ_j are denoted by λ_{Rj} and λ_{Ij}, respectively. Then the j-th natural damped frequency ω_{dj} and the viscous damping ratio of ζ_j corresponding to the damped frequency ω_{dj} can be expressed by the following formulas, respectively.

$$\omega_{dj} = \sqrt{\lambda_{Rj}^2 + \lambda_{Ij}^2} \tag{10}$$

$$\zeta_j = \frac{-\lambda_{Rj}}{\sqrt{\lambda_{Rj}^2 + \lambda_{Ij}^2}} \tag{11}$$

The dynamic fictional force of the roller bearing system increases the damping of the proposed structure. A frictional damping by a dynamic frictional force is evaluated so that the total energy dissipated per cycle is the same as for the viscous damping vibration during a steady state of motion. As the result this frictional damping can be evaluated by the following formula,

$$\xi_i = 2f_{\mathrm{d}} \times \frac{|\emptyset_{2n,i}|}{\pi\theta\,\omega_i} \times \frac{|\emptyset_{j,i}|}{a_j} \tag{12}$$

, where ξ_i is called the equivalent viscous damping ratio of the i-th natural vibration mode, θ and a_j are the frequency and the amplitude at beam-j during a steady state of motion, respectively.

4. Elastic Dynamic Response Analysis

4.1. Characteristics of Vibration Test Model

Elastic dynamic response analysis is performed to proposed structure with 15 stories and 50m height. Four different earthquake waves are simulated on vibration model which are known as; Elcentro earthquake (USA, 1940), Taft earthquake (USA, 1956), Hanhinohe earthquake (Japan, 1956) and Miyagi earthquake (Japan, 2005). Hereby shear force, acceleration and relative displacement response diagrams are obtained in order to investigate the effect of changing earthquake waves on the dynamic response behavior. Characteristic parameters for both folded cantilever shear structure (FCSS) and ordinary cantilever shear structures (OCSS) vibration models are displayed in Table 1.

Table 1. Characteristic parameters of Folded cantilever shear structure (FCSS) and Ordinary cantilever shear structure (OCSS)

Parameters	**Folded cantilever shear structure (FCSS)**	**Ordinary cantilever shear structure (OCSS)**
Total height, *H*	50 m	50 m
Number of stories, *n*	15	15
Story height, *h*	3.333 m	3.333 m
Story mass	m_1~m_{14}=50,000kg m_{16}~m_{30}=50,000kg m_{15}=100,000kg	m_1~ m_{30}=100,000kg
Total mass, *m*	1,600,000kg	1,500,000kg
Shear spring coefficient	k_1~k_{30}=1.60×10^8 Nm	k_1~k_{15}=4.85×10^8
Structural damping coefficient	c_1~c_{30}=1.11×10^6 Ns/m	c_1~c_{15}=2.50×10^6 Ns/m
First structural damping ratio	ζ_1=1.0 %	ζ_1=2.0 %
Additional damping coefficient	d_0~d_6=0.25×10^6 Ns/m	-
First additional damping ratio	$\Delta\zeta_1$=34.0 %	-
Frictional coefficient	μ=0.005	-
Frictional force (N)	*f*=41,650 N	-

3.2. Results and Discussions

In this paper, TAFT earthquake wave records in Fig. 2, and response spectrums in Fig. 3, are given as a an illustration as follows among above mentioned earthquakes. These spectrums shows the top point results of the FCSS and OCSS. Acceleration and relative velocity spectrums of FCSS are varies in small range. Besides, relative displacements are stayed within 0.1 m. value range while OCSS exceeds 0.1 m. relative displacement on the top of the structure, in Fig. 3.

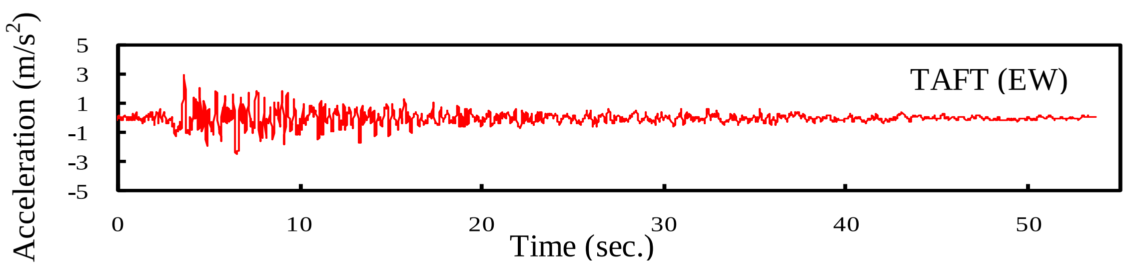
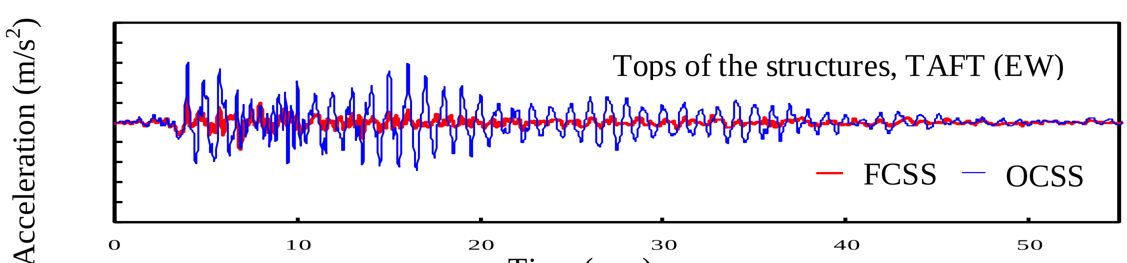
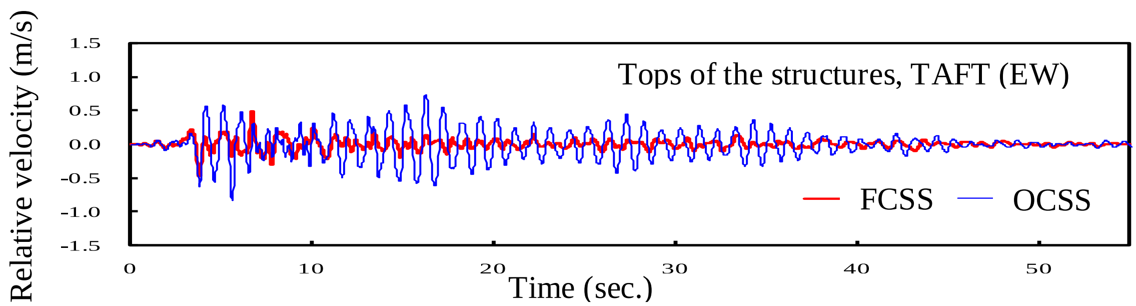


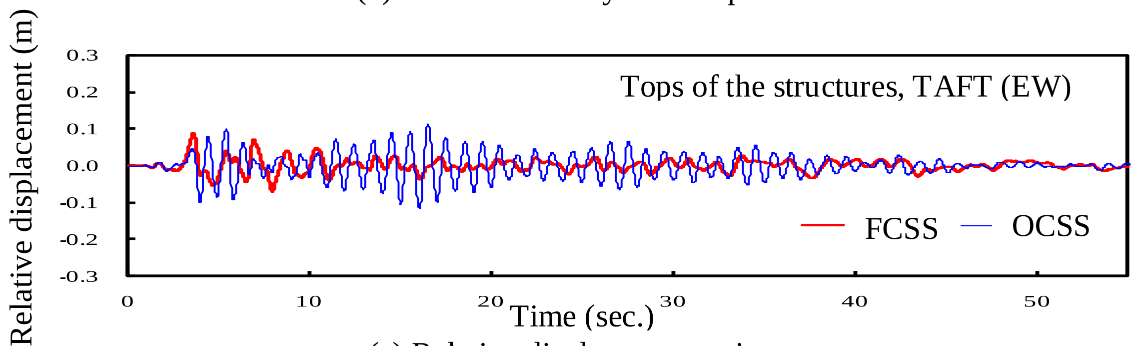
Figure 2. TAFT, earthquake ground acceleration record (max. acceleration 300gal)



(a) Acceleration – time spectrum



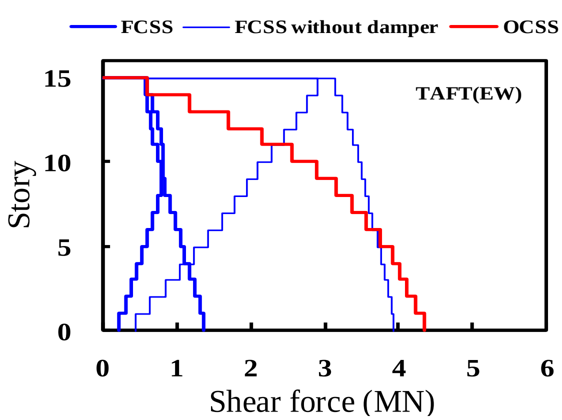
(b) Relative velocity – time spectrum

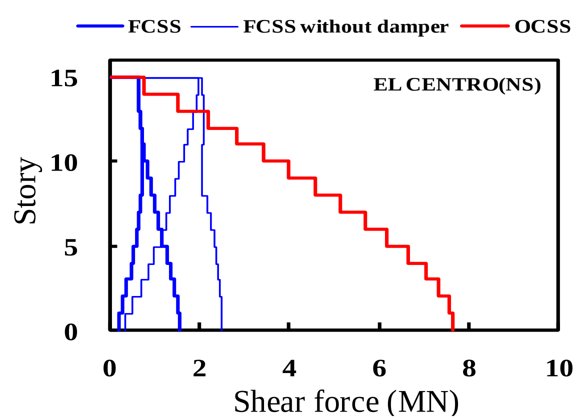


(c) Relative displacement – time spectrum

Figure 3. TAFT Earthquake, response spectrums for tops of the FCSS and OCSS.

The following diagrams obtained for three condition of proposed structure such as; Folded cantilever shear structure (FCSS), Folded cantilever shear structure without damper, and Ordinary cantilever shear structure (OCSS). In terms of shear force, Fig.4, FCSS has acquired considerable amount of seismic performance in comparison with OCSS under each four different earthquake waves. While FCSS has around 1-2 MN shear force at the basement, OCSS shear force values at the basement extended from 4 to 18 MN. In addition, it is obvious that the additional dampers also increased the damping ratio and so seismic performance. As story number increases, high acceleration diagrams stands out for OCSS adversely of FCSS, Fig. 5. By means of movable sub-structure, movements at the basement enabled less relative displacement on the top of the FCSS, despite that the relative displacements on the top of the OCSS has higher values for 3 or 4 times.





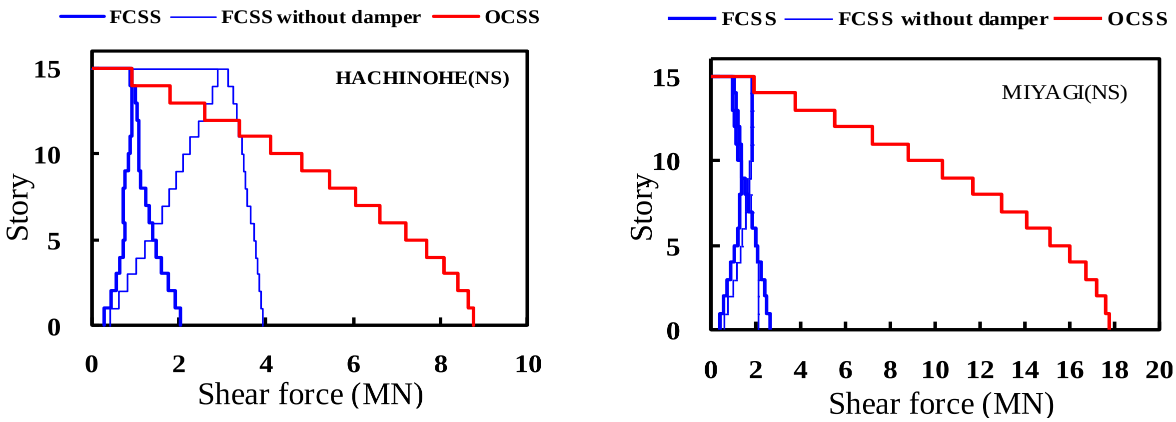


Figure 4. Shear force diagrams of FCSS, FCSS without damper and OCSS for TAFT, Elcentro, Miyagi and Hachinohe earthquakes.

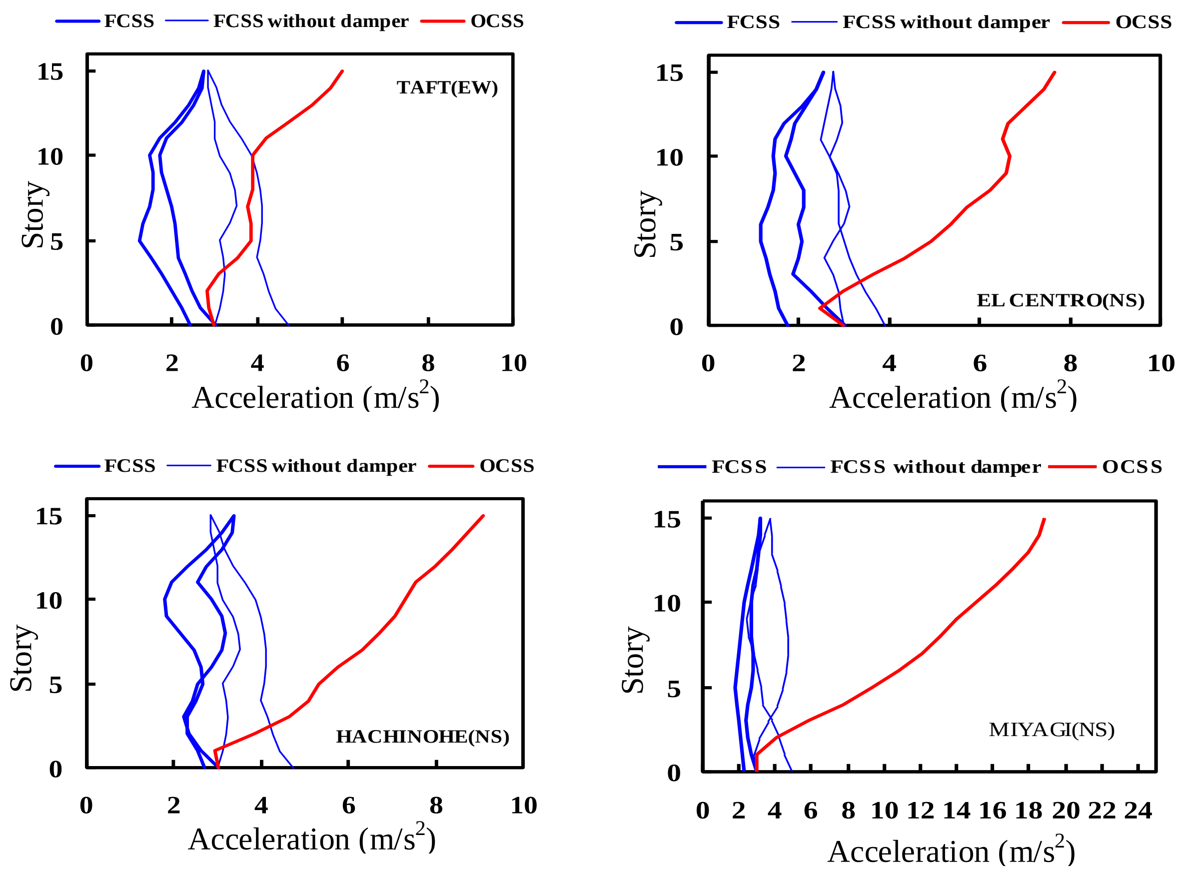


Figure 5. Acceleration response diagrams of FCSS, FCSS without damper and OCSS.

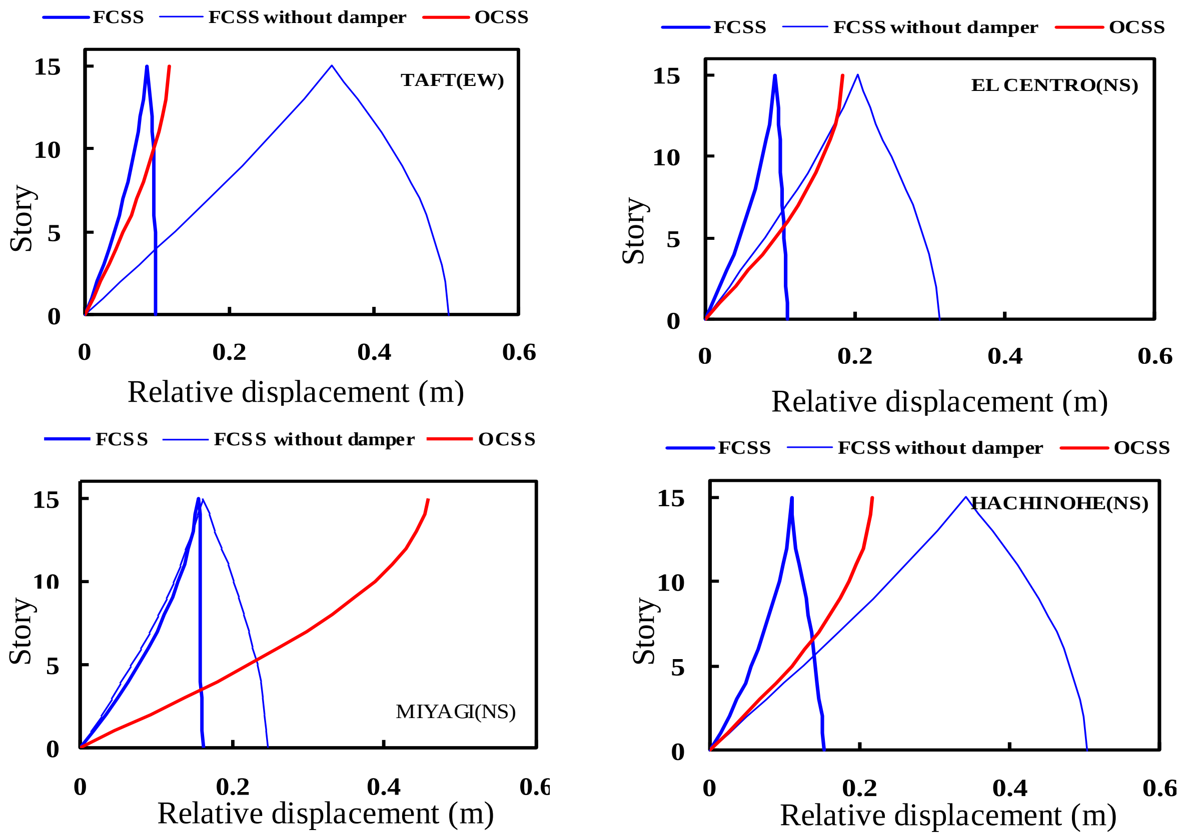


Figure 6. Relative displacement diagrams of FCSS, FCSS without damper and OCSS

Natural vibration modes of FCSS without damper system are obtained $T_1 = 2.30$, $T_2 = 0.77$ and $T_3 = 0.46$ respectively, shown in Figure 5, while the first period of OCSS is obtained $T_1 = 1.0$. Besides natural periods for damped FCSS are $T_1 = 2.10$, $T_2 = 0.80$ and $T_3 = 0.48$. Also complex eigenvalues are obtained λ_{R1}=-1.046, λ_{R2}=-2.825, λ_{R3}=-3.285 and λ_{I1}=±2.796, λ_{I2}=±7.377, λ_{I3}=±12.802. Finally natural frequencies and damping ratio values of the vibration test model are ω_1=11.526, ω_2=33.472, ω_3=56.99, ζ_1=0.350, ζ_2=0.358 and ζ_3=0.249 respectively.

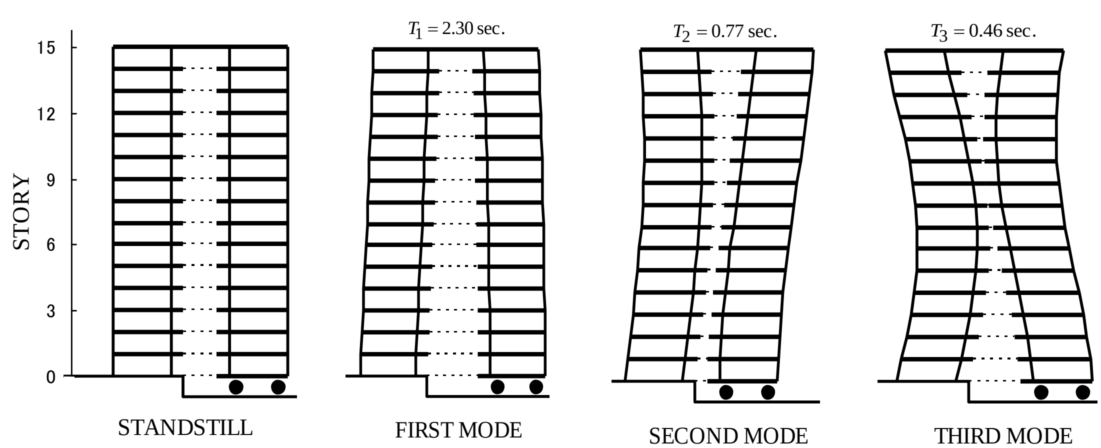


Figure 7. Natural vibration modes of non-damping vibration system.

Earthquake response spectrums for TAFT-EW are also given in Figure 6. In this spectrums, bold line represents FCSS and normal line represents OCSS. Obtained results of damping ratios are 35% for FCSS, which consists of 34% first additional damping ratio and %1 first structural damping ratio and 2% first structural damping ratio for OCSS. During the first natural period of OCSS, T_1 = 1.0 which is marked by red point, absolute acceleration is obtained around 4 m/s^2 but FCSS, T_2 = 2.1 which is marked by blue point, stayed around 1 m/s^2 absolute acceleration. Besides relative displacement values of FCSS remained around 0.08m while OCSS values obtained around 0.1m.

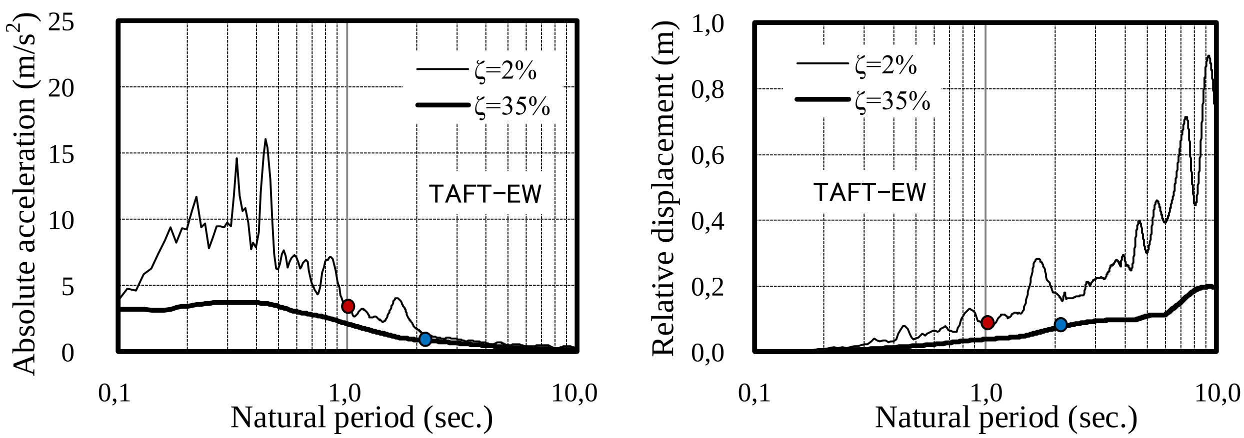


Figure 8. TAFT Earthquake response spectrums.

Conclusions

The proposed folded cantilever shear structure which is designed in order to increase damping performance and acquire highly seismic performance was gave acceptable results. Such as, natural vibration mode of folded cantilever shear structure was extended around two times longer than ordinary cantilever shear structure. In addition, damping constant increased approximately 16 times though coefficient of damper decrease ten times. In conclusion, it is seen that the folded cantilever shear structure with damping system has a great effect on middle multistory buildings to improve seismic performance.

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