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CUMULATIVE SPECTRAL ACCELERATION-SPECTRAL DISPLACEMENT MEASURES OF GROUND MOTION INTENSITY

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ABSTRACT

Ground motion spectral quantities and response spectrum analysis have been an indispensable part of earthquake engineering research and practice for the major part of the past four decades. The growth of response spectrum concept paved the way for advancements in ground motion intensity description. In this study, acceleration-displacement demand diagram (*AD*) is used to describe the severity of ground motion records. The area under *AD* diagram is integrated to arrive at a cumulative spectral acceleration-spectral displacement spectrum. The ordinate of this spectrum and its slope at the fundamental vibration period of the structure constitute our proposed intensity measures (*IM*). Through numerical simulation of response of an instrumented building, the ability of the proposed *IM* in identifying damaging power of earthquake records is demonstrated.

Introduction

Defining a robust measure of ground motion intensity has its roots in the early work by George Housner at the California Institute of Technology in the 1950's (Housner 1952). A measure of severity of ground motion shaking, often denoted by *IM* in performance-based earthquake engineering formulations, has three main purposes (Chopra 2007; Trifunac 2006; Baker and Cornell 2005; Krawinkler 2001; Housner and Jennings 1982; Housner 1975):

1. Estimation of seismic hazard is to be expressed in terms of a common *IM* that could be easily integrated into structural response calculations,
2. *IM* is a variable in the correlation functions of input ground motion and structural response, and
3. In selection and scaling of ground motion records for structural design and evaluation where the statistics of response is sought on the basis of a uniform level of input excitation.

In this paper, we propose using the acceleration-displacement demand diagram as a description of ground motion severity for the purpose of defining two *IM*s. The area under *AD*

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diagram is integrated from $T = 0.0$ *sec* to $T = T_o$ *sec*, where T is the elastic vibration period of a single-degree-of-freedom (*SDF*) system, and the resulting cumulative *AD* spectrum is plotted against T_o for each period. The ordinates of the cumulative *AD* spectrum and its slope at the fundamental period of vibration for a given structure constitute our proposed *IM*s. The slope of the cumulative *AD* spectrum at T_o is indicative of the rate of increase in demand due to period elongation prompted by inelastic response. Computation of the proposed *IM* is fast using the Green's theorem in the plane. The proposed *IM* correlates well with common *EDP*s encountered in design and evaluation of structures.

Background

Numerous scalar *IM*s have been proposed since the early days of earthquake engineering research; many of them still in use in current seismic hazard estimation and structural analysis procedures. Spectral acceleration at the fundamental period of vibration of a structure is perhaps the most widely used measure of ground motion intensity, in performance-based formulations, loss calculation, and ground motion selection and scaling (ATC 2007; Cornell and Krawinkler 2000; FEMA 2000; Moehle et al. 2005; Shome et al. 1998). In addition, Housner's earthquake power index, Arias Intensity, Araya and Saragoni, and Riddell and Garcia are a few to name (Kramer 1996; Luco and Cornell 2007; Riddell 2007; Tothong and Cornell 2006). Recently, Riddell examined the correlation between twenty-three scalar *IM*s and linear and nonlinear response variables of *SDF* oscillators. According to his study no index was found to be satisfactory over the entire frequency range, while acceleration-related *IM*s, velocity-related *IM*s, and displacement-related *IM*s were deemed better suited for systems with different mechanical properties (Riddell 2007).

Theory

Ground motion shaking is a complex process (Alimoradi et al. 2005; Corral 2004; Housner 1975). The most accurate description of level of shaking is possible through time-based description of full 6-dof motion of a particle on the surface of the earth at a location of interest. The time series of ground motion records contain all necessary information with regards to amplitude, frequency substance, duration of shaking, and imparted energy to uniquely describe the level of intensity. This way of describing ground motion intensity is computationally expensive and impractical. On the other hand, most common methods of seismic hazard evaluation only rely on a simple measure of ground motion intensity. In lieu of a full description of earthquake ground motion, we postulate the necessary conditions for a measure of intensity to be considered efficient and sufficient for the purpose of seismic hazard estimation and structural damage correlation:

- Dispersion: A good measure of intensity has to predict structural response reliably, viz., the dispersion in structural response given level of *IM* should be practically small (or "*IM* efficiency" as defined by Luco and Cornell (Luco and Cornell 2007)).
- Correlation: Structural response quantities (force-related and displacement-related) have to be highly correlated with the *IM*. This should be rather obvious in the framework of PEER center's calculation of loss (Cornell and Krawinkler 2000; Der Kiureghian 2005):

One way of presenting complex data of high dimensions, such as earthquake ground motion, by a few simple parameters is through optimal feature selection in a classical pattern recognition problem. An optimal measure of ground motion intensity, scalar or vector, is mathematically one that reliably categorizes a large database of ground motion records into a number of distinct yet meaningful clusters. This is often done through maximizing a measure of distance (Abdi 2007). An example of such way of ground motion data presentation was recently demonstrated by Alimoradi et al, (Alimoradi et al. 2005). In this regard, an *IM* is naturally a data feature obtained by feature extraction or by the application of principal component analysis. Although this is a common way of dealing with complex data in many applications such as the problems of fingerprint, voice, and face recognition; here we suffice to assume that data features used to describe the motion are the spectral accelerations and displacements at various periods of interest for any given ground motion record.

Cumulative AD Spectrum

The level of structural response and possible state of damage in a structural system are both affected by acceleration and displacement demands. It may be argued that in fact it is the imparted energy and its rate of change over time that drives structural response to a state of damage (Estes and Anderson 2006; Mahin and Bertero 1981; Park et al. 1987). Therefore *IM* measures based on energy should be more appealing than simplistic measures based solely on a single descriptor of acceleration or displacement. The measures of intensity proposed in this paper consider bound of input spectral energy, higher mode excitation effects, and inelastic demand at the first-mode period by integrating the *AD* diagram in the plane.

We speculate that a ground motion *IM* could be represented by the area enclosed between the *AD* diagram and the spectral displacement and spectral acceleration axes. We use the value and the slope of the cumulative *AD* spectrum at the fundamental period of vibration as two intensity measures. We provide a simple numerical scheme for integrating the *AD* diagram followed by some insight into the implications of the cumulative *AD* spectrum.

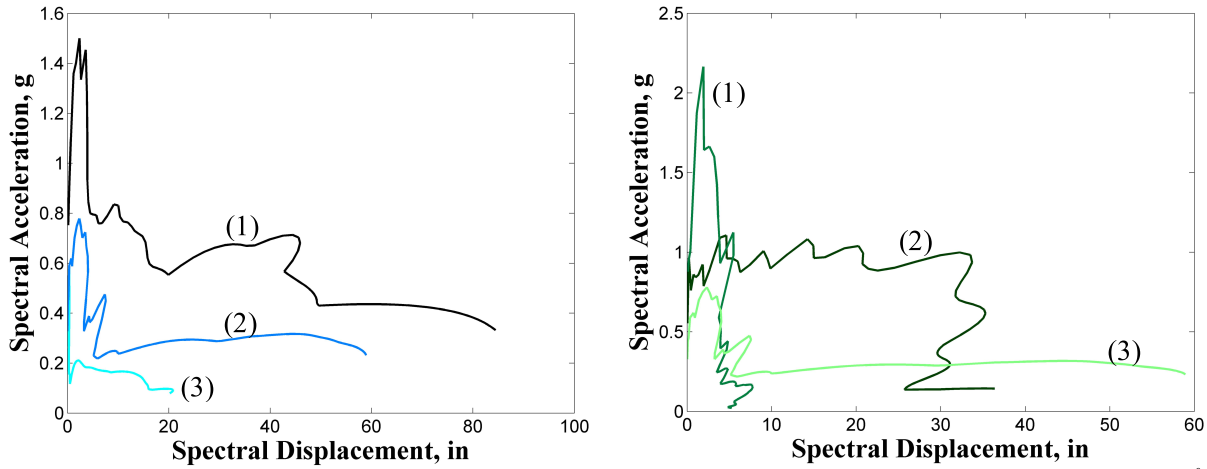
Consider the *AD* diagrams presented in the left panel of Figure 1. Clearly, record (1) has considerably larger acceleration and displacement demands than record (3). On the other hand, various *AD* diagrams exhibit different shapes in the A-D space as depicted in the right panel of Figure 1. The implication is that different degrees of displacement and acceleration demands are expected at different spectral periods. Record (1) in the right panel may be considered a critical record for the design of acceleration-sensitive equipments in a short/stiff structure, whereas record (3) is perhaps suitable for displacement-sensitive damage checks in a more flexible structure. Our proposed vector-based *IM* tries to describe such variability in acceleration and displacement demands within a two-component *IM*.

Green's Theorem in the Plane

The Green's theorem in the plane provides the transformation of double integrals into line integrals performed over the boundaries of a region (Kreyszig 1999). To calculate the area enclosed by an *AD* diagram and the spectral acceleration and spectral displacement axes, the Green's theorem can be stated as follows:

$$A = \frac{1}{2}\oint_C \left(S_A(T,\xi)\,dS_D - S_D(T,\xi)\,dS_A \right) \qquad (1)$$

in which A is the area enclosed by the *AD* diagram, $S_A(T,\xi)$ is spectral acceleration and $S_D(T,\xi)$ is spectral displacement at period T and damping ratio ξ. It should be noted that the Green's theorem is for areas that have continuous boundary functions with continuous partial derivatives. This condition is almost always satisfied for an *AD* diagram.



1. 1999 Chi-Chi, TCU067, NS
2. 1999 Chi-Chi, TCU052, NS
3. 1979 Imperial Valley, Brawley Airport, 225°

1. 1994 Northridge, Sylmar Olive V. Hosp., 360°
2. 1994 Northridge, Jensen Filt. Plant, 022°
3. 1999 Chi-Chi, TCU052, NS

Figure 1. Sample of AD diagrams (5% viscous damping).

A point on the *AD* diagram in Figure 2 may be presented in polar coordinate system by:

$$r^2(T_o,\xi_o) = S_D^2(T_o,\xi_o) + S_A^2(T_o,\xi_o) \qquad (2)$$

the radial coordinate for an elastic *SDF* system with natural period T_o and damping ratio of ξ_o. Equation 2 could be rewritten for spectral displacement, S_D, as:

$$r^2(T_o,\xi_o) = S_D^2(T_o,\xi_o)\left(1+\omega_o^4\right) \qquad (3)$$

for circular frequency $\omega_0 = \frac{2\pi}{T_o}$. The angular coordinate is a function of spectral quantities:

$$\gamma(T_o,\xi_o) = \arctan\left(\frac{S_D(T_o,\xi_o)}{S_A(T_o,\xi_o)}\right) = \arctan\left(\frac{1}{\omega_o^2}\right) \qquad (4)$$

considering:

$$d\gamma(T_o,\xi_o) = \frac{-2\omega_o}{1+\omega_o^4} d\omega_o \tag{5}$$

and integrating the *AD* diagram in the polar coordinate system within a range of periods of interest $[T_{\min}, T_{\max}]$:

$$I_{AD}(T_o,\xi_o) = \int_0^{T_o} r(T_o,\xi_o) d\gamma(T_o,\xi_o) = \int_{\omega_o}^{\infty} \frac{-2\omega_o S_D(\omega_o,\xi_o)}{\sqrt{1+\omega_o^4}} d\omega_o \quad for \quad \begin{cases} \forall T_0 \in [T_{\min}, T_{\max}] \\ or \\ \forall \omega_0 \in \left[\frac{2\pi}{T_{\max}}, \frac{2\pi}{T_{\min}}\right] \end{cases} \tag{6}$$

The period range of integration is problem specific. Here we use the entire range of periods for which spectral quantities are available in the spectra of ground motions used in our database.

For a structure with natural circular frequency of $\omega_s = \sqrt{\frac{k}{m}}$ and damping ratio of ξ_s, our proposed *IM* is defined as (see Figure3):

$$\left\{ I_{AD}\left(\frac{2\pi}{\omega_s}, \xi_s\right)\right\} = \begin{Bmatrix} \left\| I_{AD}(T,\xi) \right\|_{T=\frac{2\pi}{\omega_s}, \xi=\xi_s} \\ \left. \frac{\partial I_{AD}(T,\xi)}{\partial T} \right|_{T=\frac{2\pi}{\omega_s}, \xi=\xi_s} \end{Bmatrix} \tag{7}$$

In a simple system of an un-damped linear elastic *SDF* oscillator vibrating freely with mass *m*, the total energy at time *t* is proportional to the displacement and acceleration of the mass which are bound by their corresponding spectral quantities:

$$E(t) = \frac{1}{2} k[u(t)]^2 + \frac{1}{2} m[\dot{u}(t)]^2 \Rightarrow E(t) \propto \left\{ [u(t)]^2 + \frac{[\ddot{u}(t)]^2}{\omega_o^{\ 2}} \right\} \le r^2(T_o,\xi_o) \tag{8}$$

implying that $\left\| I_{AD}(T,\xi) \right\|$ is an upper-bound to the total imparted energy.

Example

Thirty near-fault strong motion records were used in evaluation of the proposed *IM* (Kalkan and Kunnath 2006). These ground motions were recorded from earthquakes having a magnitude range from 6.1 to 7.6 and at distances varying from 0.7 to 15.9 km to the causative fault. We computed the five percent damped pseudo spectral acceleration and spectral displacements for the records in the database. Using the integration procedure outlined earlier we also calculated the cumulative *AD* spectra (shown in Figure 4.) Two existing 6- and 13-story instrumented steel moment-resisting frame buildings were considered as test structures for evaluation of the proposed *IM*, however only results from the first model is presented here due to space limitation. Twodimensional nonlinear finite element models were created to study the behavior of the lateral load resisting systems, since the buildings are essentially symmetric. The elevation view of a typical perimeter frame for the building example is depicted in Figure 5. The analytical models use fiber sections with distributed plasticity over the length of the elements in all structural components. A stable bilinear response curve with 3.0% and 2.0% kinematic strain hardening were utilized to simulate the material hysteretic behavior. The properties of the first three elastic vibration modes of the building example are shown in Table 1. Further details of the buildings and modeling information together with calibration studies are reported elsewhere (Kalkan and Kunnath 2006).

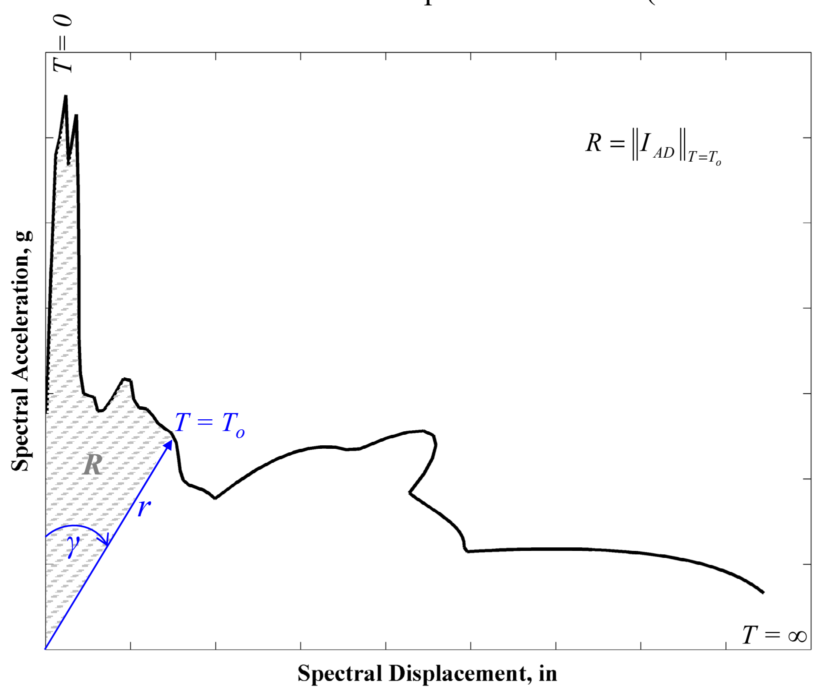


Figure 2. Description of radial integration method for the proposed *IM*.

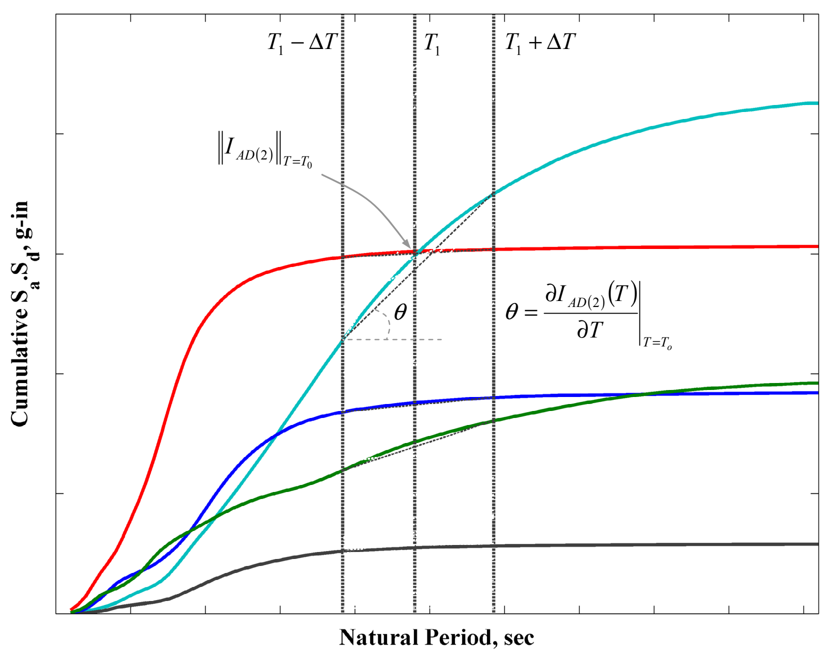


Figure 3. The schematic of typical cumulative *AD* spectra

Calculated *IM*s for the example building and their statistics are presented in Table 2. The results of nonlinear finite element response history analyses, summarized in Table 3 and Figure 6, show that although the variation of cumulative *AD* ordinates and their slopes at the fundamental period of vibration of the example building are relatively large, the ensuing structural response quantities calculated from $\{I_{AD}\}$ are highly correlated with the *IM*s.

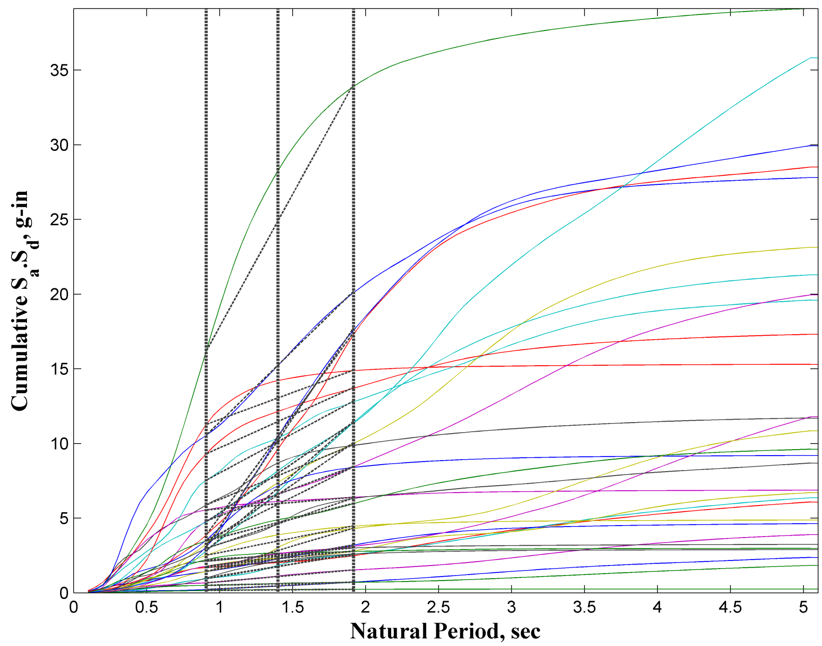


Figure 4. The cumulative *AD* intensity components for the example building (middle vertical line indicates the fundamental period; vertical lines on its both sides define the period range for slope calculation).

Conclusions

Two new intensity measures of ground motion records are introduced in this paper using the concept of *cumulative spectral acceleration-spectral displacement spectrum* to describe the severity of shaking of earthquake records used in nonlinear response history analysis of structures. The new *IMs* are related to the upper bound of input spectral energy and correspond well with both force-based and displacement-based structural response quantities. Furthermore, the consideration of higher modes of vibration to response (the first component of $\{I_{AD}\}$) as well as nonlinear effects at and around the fundamental period of vibration (the second component of $\{I_{AD}\}$) make the proposed intensity measure virtually suitable for different type of structural systems in seismic zones. The proposed intensity measure was tested on an instrumented building under a set of thirty near-fault strong ground motion records. The analyses of ranking of the earthquake ground motion records in these numerical simulations revealed the strength of the proposed *IMs* in reliably identifying the damaging potential of them. In general, the method resulted in a good correlation between the proposed *IMs* and major structural response quantities.

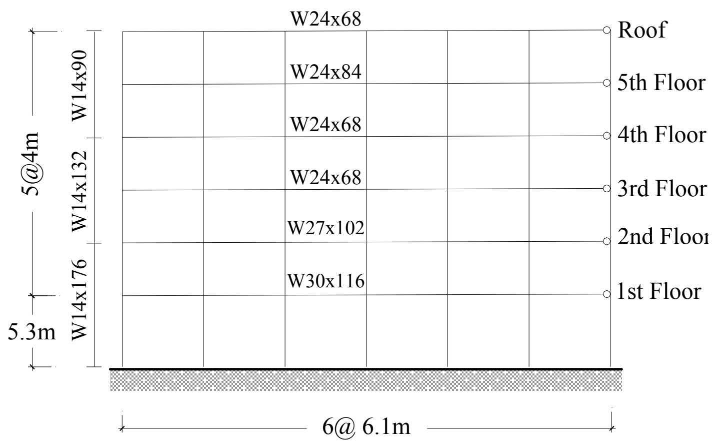


Figure 5. Elevation view of typical moment resisting perimeter frames in the building example.

Table 1. Elastic modal properties of the building example (T: modal period; Γ: modal participation factor; and α: mass participation factor).

	First Mode			Second Mode			Third Mode		
	T (sec)	Γ	α	T (sec)	Γ	α	T (sec)	Γ	α
6-story Building	1.40	2.58	0.85	0.51	0.96	0.12	0.30	0.46	0.03

Table 2. Statistics of the proposed *IMs* for the example building (*IM1*: the ordinate of the cumulative *AD* spectrum at the fundamental period of vibration, *IM2*: the slope of the cumulative *AD* spectrum around the fundamental period of vibration; and *IM3*: is Euclidean length of $\{I_{AD}\}$).

	***IM1*, g-in**	***IM2*, g-in/sec**	***IM3*, g-in**
Min	0.211	7.4	0.007
Median	4.797	269.0	0.185
Mean	6.331	419.1	0.294
Max	28.461	1773.5	1.237
Standard Deviation	5.899	451.3	0.288
COV	0.932	1.077	0.980

Table 3. Correlation coefficients of the *IMs* with demand parameters in the example building

ρ	**Roof Drift Ratio**	**Cumulative Interstory Drift Ratio**	**Peak Interstory Drift Ratio**	**Peak Floor Acceleration, g**	**Max. Base Shear, kips**
$SA(T_1)$	0.82	0.84	0.81	0.79	0.77
IM1	0.69	0.78	0.68	0.90	0.70
IM2	0.91	0.94	0.89	0.78	0.87
IM3	0.87	0.93	0.88	0.73	0.80

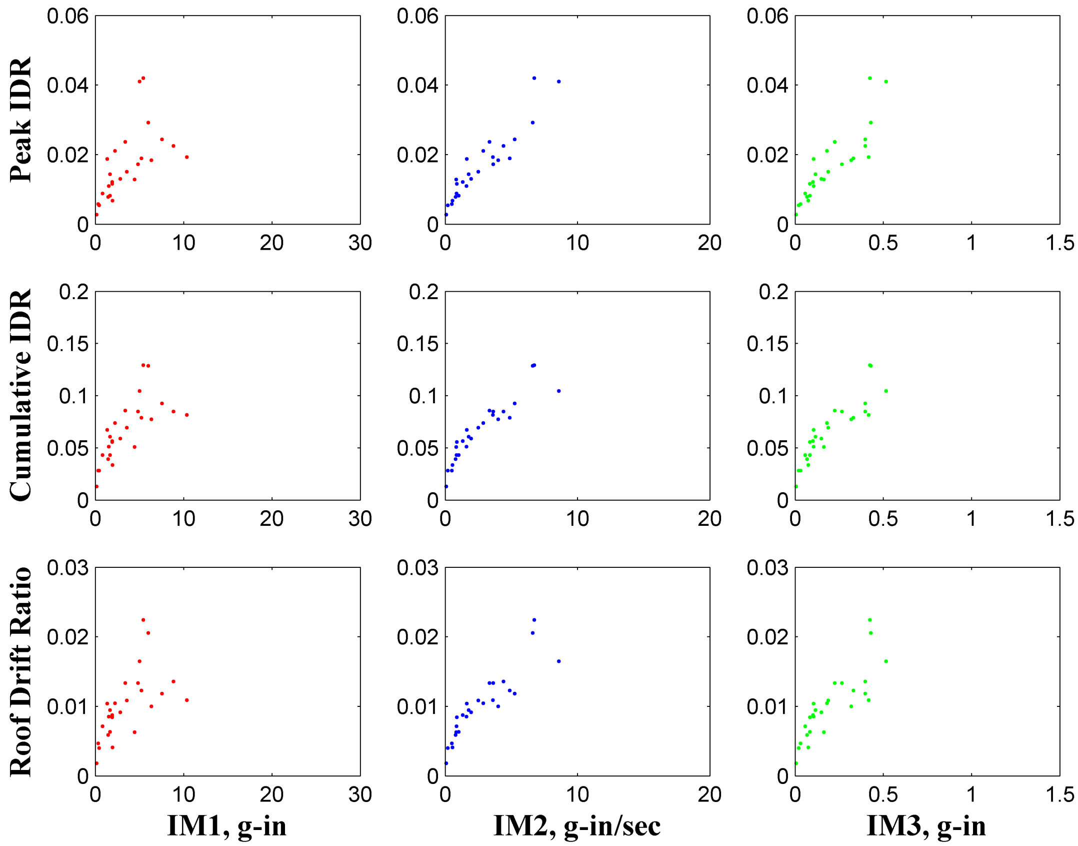


Figure 6. Correlation of proposed *IMs* with some common engineering demand parameters

The proposed *IM* is being expanded for the problem of selection and scaling of ground motion records and for probabilistic seismic hazard analysis. Work on sensitivity of $\{I_{AD}\}$ to various methods of slope calculation of cumulative *AD* spectrum is currently underway.

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