



## **DISPLACEMENT-BASED PRELIMINARY DESIGN OF LOW-RISE BUILDINGS STIFFENED WITH BUCKLING-RESTRAINED BRACES**

Amador Teran-Gilmore<sup>1</sup> and Neftali Virto-Cambray<sup>2</sup>

### **ABSTRACT**

A displacement-based methodology for the preliminary design of a system of buckling-restrained braces is introduced. The methodology, which applies to the case of low-rise buildings, is applied to the conception and preliminary design of a bracing system for a five-story building located in the Lake Zone of Mexico City. From the evaluation of the global mechanical characteristics of the building and of its seismic performance, it is concluded that the proposed methodology yields an adequate level of seismic design.

### **Introduction**

Innovation in earthquake-resistant design has been directed towards the conception of structural systems, either traditional or innovative, that are capable of adequately limiting their level of structural and non-structural damage through the explicit control of their lateral deformation. Within this context, bracing a building with a system of buckling-restrained braces has emerged as an attractive alternative for response control.

A displacement-based approach for the preliminary design of a buckling-restrained bracing system for low-rise buildings is introduced herein. As part of the methodology, the yield stress of the steel with which the braces should be fabricated is defined. This definition takes into account the balance between the structural performance of the building for the immediate operation and life safety performance levels. Although the treatment that this paper gives to the system of buckling-restrained braces corresponds to the design of a new structure, the formulation can be readily adapted for seismic rehabilitation of existing structures.

### **Buckling-Restrained Braces**

A buckling-restrained brace is formed of: A) Ductile steel core that dissipates energy through axial deformation; B) Mortar, concrete or grout fill that restricts buckling of the core; and C) Steel jacket that confines the mortar, concrete or grout fill and provides further restriction

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<sup>1</sup>Professor, Universidad Autonoma Metropolitana, San Pablo 180, Col. Reynosa Tamaulipas, Mexico 02200, D.F.

<sup>2</sup>Project Engineer, Alonso y Asociados, Carretera Mexico-Toluca 1725, Col. Lomas de Palo Alto, Mexico 05110, D.F.

from buckling. Under severe ground motion, only the core of the brace should yield. Usually, the steel core is isolated from the mortar, concrete or grout fill in an attempt to minimize or eliminate the transfer of axial stresses between both materials. This is done so that the compression strength of the brace is similar to its tension strength (Black 2002). Further discussion regarding the concept and use of buckling-restrained braces can be found in Black (2002) and Tremblay (2006).

The methodology introduced herein is based on the explicit control of the lateral displacement of the building, in such manner that there is the need to develop design aids that explicitly relate the structural properties of the brace with its global mechanical characteristics. The expressions that are offered next have been obtained by neglecting the global flexural deformation of the bracing system produced by the axial deformation of the columns that support it; that is, they only consider the global shear deformation due to the axial deformation of the braces.

The lateral stiffness that a buckling-restrained brace contributes to a given story ( $K_L$ ) is related to its steel core area ( $A$ ) through the following expression (Tremblay 2006):

$$\frac{K_L}{(A/L)} = \frac{E \cos^2 \theta}{\gamma + \eta(1 - \gamma)} \quad (1)$$

where  $L$  is the total length of the brace,  $E$  its Young's modulus, and  $\theta$  its inclination angle.  $\gamma$  is the ratio of the length of the brace core segment to the total brace length, and  $\eta$  the ratio of the average axial stress in the brace outside the brace core to the stress in the brace core. Equation 1 can be used to estimate the required area of braces in a story as a function of the geometry of the bracing system and the lateral stiffness that it should provide to that story.

Regarding the inter-story drift at yield:

$$IDI_y = \left( \frac{\Delta_L}{h} \right)_y = \frac{f_y [\gamma + \eta(1 - \gamma)]}{E \sin \theta \cos \theta} \quad (2)$$

where  $IDI_y$  is the inter-story drift;  $\Delta_L$  and  $h$ , the inter-story displacement and story height, respectively; and  $f_y$  the yield stress of the steel core. The sub index  $y$  denotes yield. Equation 2 establishes the required yield stress for the braces as a function of the inter-story drift at which the bracing system should yield. If a specific steel grade is used for the braces, Equation 2 can be used to establish the inter-story drift at which the bracing system yields.

## Design Scope

The methodology offered in this paper is based on the conception of a building whose gravity forces are carried by moment-resisting frames, and whose earthquake-resistance is provided by a system of buckling-restrained braces.

Under the effect of low intensity ground motion, it is assumed for illustrative purposes that the building exhibits adequate performance if it satisfies the immediate operation limit state. This implies that the gravitational and bracing systems should not exhibit significant structural damage, and that the non-structural system should remain undamaged. Regarding performance

for severe ground motion, it is assumed that the gravitational system should satisfy the immediate operation limit state while the bracing system develops significant plastic behavior that allows it to dissipate a large percentage of the input energy. An elastic gravitational system is capable of providing the braced building with significant strain hardening that stabilizes its dynamic response and reduces its residual deformations (Kiggins 2006). Once the integrated system deforms beyond its elastic limit, structural damage concentrates in the bracing system. Partial or total non-structural collapse should be avoided.

### **Performance-Based Numerical Seismic Design**

The methodology introduced herein, applicable to standard occupation buildings and schematically shown in Figure 1, considers two performance levels: immediate operation and life safety. Note that the methodology can be easily adapted to meet other design criteria once pertinent design objectives are defined for the earthquake-resistant structure. Its first step implies establishing a qualitative definition of adequate performance. The second step consists of the quantification of adequate performance through establishing response thresholds with the aid of damage indices. During the third step, the methodology establishes, through the use of displacement spectra, the value of the fundamental period of vibration of the building, which quantifies the design lateral stiffness. The sizing of the braces is established according to the value of this parameter.

Regarding the qualitative performance definitions, it is assumed for illustrative purposes that the performance levels under consideration are satisfied if:

- *Immediate Operation*: The bracing and gravitational systems can exhibit light structural damage (i.e., cracking of reinforced concrete frame elements and small plastic demands in the braces). The non-structural system should remain undamaged.
- *Life Safety*: The building should guarantee the physical integrity of its occupants and provide for easy structural rehabilitation. While the gravitational system should satisfy immediate operation requirements, the bracing system should develop significant plastic behavior. Local collapse should be avoided in the non-structural system.

For immediate operation, the gravitational system satisfies its structural performance criteria if it remains elastic. In the case of the bracing system, it may develop incipient plastic behavior. Non-structural damage is adequately controlled if the maximum inter-story drift index ( $IDI_{IO}$ ) does not exceed the threshold associated to initiation of damage ( $IDI_{NS}^{IO}$ ). Life safety is satisfied if the maximum inter-story drift index ( $IDI_{LS}$ ) is limited according to: 1) Immediate operation of the gravitational system ( $IDI_{GS}$ ), and 2) Prevention of non-structural local collapse ( $IDI_{NS}^{LS}$ ).

Numerical design starts with the conception and design of the gravitational system. This system is designed to exclusively resist the gravitational loads, and thus, standard detailing (as opposed to ductile) should be used on its structural elements. Once the gravitational system is established and designed, a nonlinear static analysis is carried out to estimate the inter-story drift

index threshold associated to immediate operation ( $IDI_{GS}$ ). For this purpose, an acceptable threshold for the plastic rotation in the structural elements of the gravitational system can be defined. The design methodology also requires an estimate of the maximum ductility demand associated to the bracing system ( $\mu_{max}$ ). A reasonable approximation for the value of  $\mu_{max}$  for a regular structure with few stories can be estimated from the ratio  $IDI_{LS}/IDI_y$ ; where  $IDI_y$  represents the inter-story drift at yield of the bracing system (Equation 2). Note that the pushover analysis would not be required in cases where the value of  $IDI_{GS}$  can be estimated from experimental evidence or practical experience.

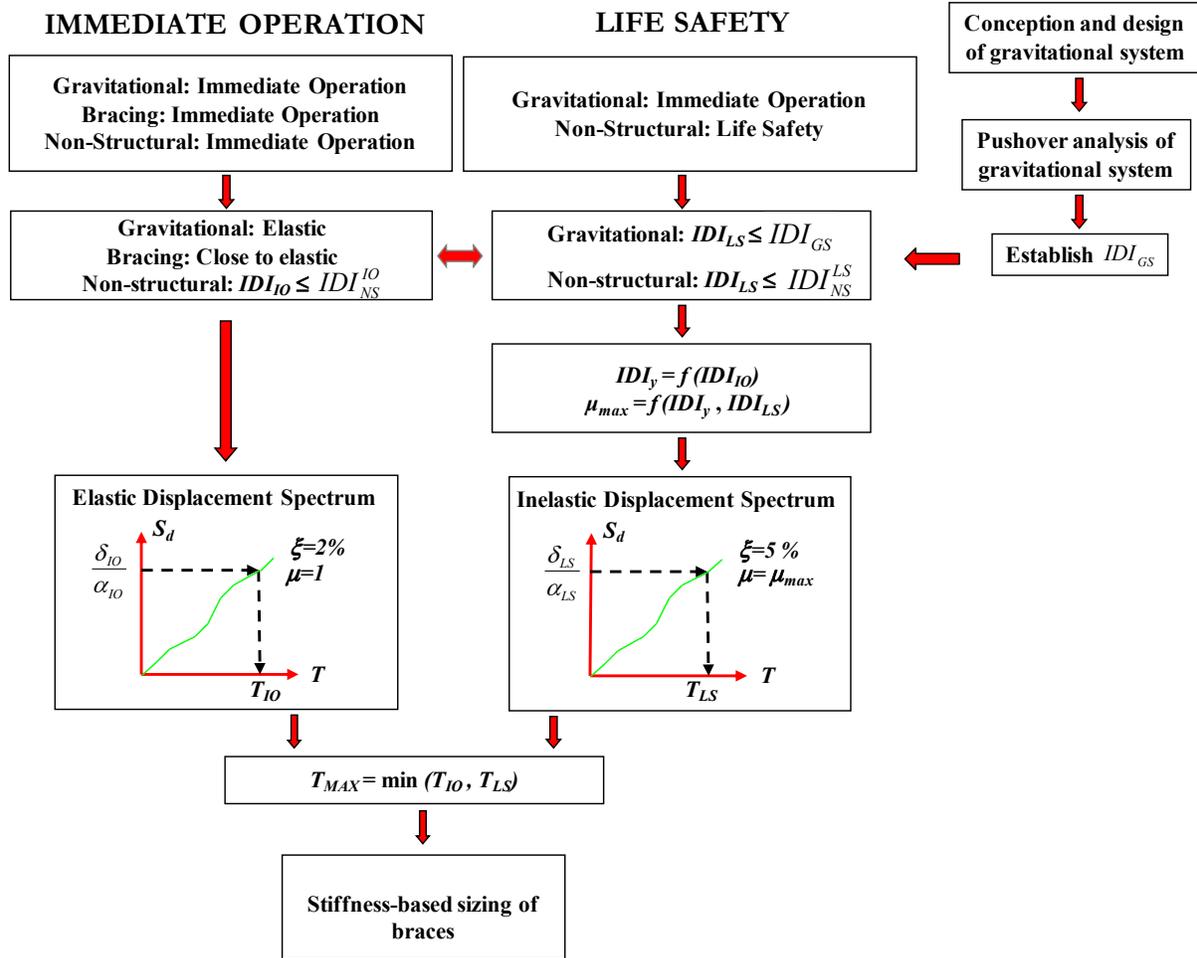


Figure 1. Preliminary seismic design methodology

The value of the fundamental period of vibration of the building is established according to Figure 1. The inter-story drift index threshold for a given limit state can be used to establish the lateral roof displacement threshold for that limit state:

$$\delta_{IO} = \frac{IDI_{IO} H}{COD_{IO}} \text{ and } \delta_{LS} = \frac{IDI_{LS} H}{COD_{LS}} \quad (3)$$

where  $H$  is the total height of the building, and  $COD$  a coefficient of distortion that considers that inter-story drift is not constant throughout the height of the building. Table 1 summarizes values

of  $COD$  for the preliminary design of fairly regular structures that exhibit shear-like global behavior. Because the deflected shapes of low-rise buckling-restrained bracing systems in which the areas of braces are varied in every story tend to exhibit a linear deformed shape even for the case of plastic behavior (Sabelli 2003), the values of  $COD$  corresponding to a global ductility of one in Table 1 can be used for this case.

Table 1. Values of  $COD$  recommended for preliminary design of structures that exhibit shear-like global behavior

Global Ductility	$COD$	
	Minimum	Maximum
1	1.2	1.5
2+	1.5	2.0

The fundamental period of vibration of the building can be estimated through the use of the displacement thresholds  $\delta_{IO}$  and  $\delta_{LS}$  and displacement spectra corresponding to both performance levels. For this purpose,  $\delta_{IO}$  and  $\delta_{LS}$  should be modified through the use of parameter  $\alpha$  to take into consideration multi-degree-of-freedom effects. Table 2 presents values of  $\alpha$  for preliminary design of structures that exhibit shear-like global behavior.

Table 2. Values of  $\alpha$  recommended for preliminary design of structures that exhibit shear-like global behavior

Stories	$\alpha$	
	$\mu = 1$	$\mu = 2+$
1	1.0	1.0
2	1.2	1.1
3	1.3	1.2
5+	1.4	1.2

According to the acceptable level of damage for each limit state, the displacement spectrum for immediate operation contemplates elastic behavior and a percentage of critical viscous damping ( $\xi$ ) equal to 2%. In the case of life safety, the inelastic displacement spectrum corresponds to a maximum ductility equal to  $\mu_{max}$  and  $\xi$  of 5%. The values of 2 and 5% of critical viscous damping proposed herein are considered to be reasonable lower bounds for the range of values reported by Chopra (2001) for the performance levels under consideration.

Figure 1 indicates that the design value for the fundamental period of vibration ( $T_{MAX}$ ) is equal to the smaller of the values that satisfy the design requirements imposed by both performance levels. Once the value of  $T_{MAX}$  is available, the braces are sized according to it; that is, the transverse areas of the braces are deemed adequate if the actual fundamental period of the building is equal or slightly less than  $T_{MAX}$ . Once the braces have been sized, the gravitational system is adjusted using capacity design concepts to provide them with adequate support.

Once the preliminary design ends, the design process proceeds to its final stage. Final design consists of two tasks: A) The verification of the preliminary design of the bracing system through a series of nonlinear static and dynamic analyses and; B) If required, adjustment of the area of braces so that the building can meet adequately its performance levels.

## Illustrative Example

Next, the methodology is applied to the conception and preliminary design of a bracing system for a five-story reinforced concrete building.

### Gravitational System

For the five-story building under consideration, non-ductile reinforced concrete frames are used to bear the gravitational loads. Figure 2 shows the frames of the building, which is considered to be located in the Lake Zone of Mexico City. The design of the gravitational system considered live and dead loads, and standard detailing. Regarding the structural materials, a compressive strength ( $f'_c$ ) of 25 MPa was considered for concrete, and a yield stress ( $f_y$ ) of 420 MPa for the reinforcing steel. The slab has a width of 15 cm and is reinforced with #3 ( $\odot$  0.95 cm) bars @ 25 cm in both directions. Deflection and crack control was taken into consideration for the sizing of slab and beams. Two different frames were designed, one external and one internal. For construction reasons, it was considered convenient to assign the same size to all the beams (30 × 40 cm) and columns (50 × 50 cm) in the building. All beams and columns exhibit light reinforcement (in the case of columns, the area of longitudinal steel amounts for 1% of their transverse area).

A nonlinear static analysis was used to evaluate the global mechanical characteristics of the gravitational system. While a triangular load pattern through height was used, the analysis was carried out with the program *DRAIN 2DX* (Prakash 1993). According to the analysis, structural damage tends to concentrate on the beams of the frames and the bottom end of the columns located at the ground story. Figure 3 shows the capacity curve of the gravitational system. The integrated work of the frames results in a base shear close to 800 kN, which corresponds to 9% of the reactive weight of the building. Detrimental  $P-\Delta$  effects are noticeable, particularly for the internal frames. Both types of frames exhibit elastic behavior up to a roof displacement of 7 cm, and exhibit stable behavior up to a roof displacement of 10 cm. The fundamental period of the gravitational system was estimated at 1.44 seconds.

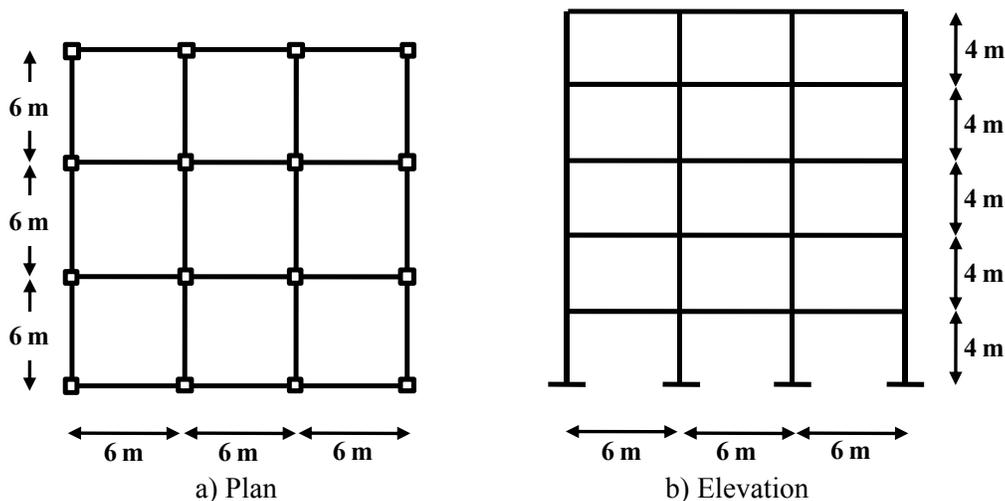


Figure 2. Structural layout of the gravitational system

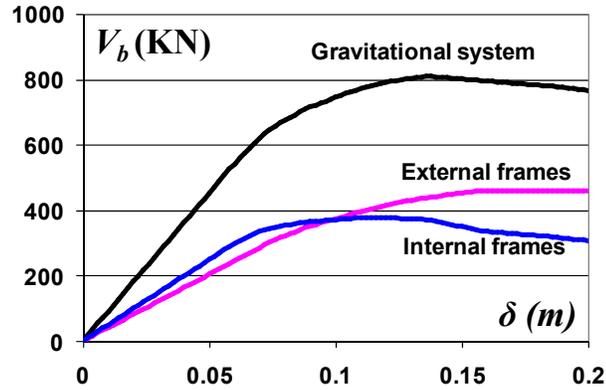


Figure 3. Capacity curve of gravitational system

The lateral deformation of the structure tends to concentrate in the second and third stories. The building starts exhibiting nonlinear behavior at a roof displacement close to 7 cm, which corresponds to an inter-story drift index close to 0.005. Under the consideration that the structural elements of a non-ductile reinforced concrete frame can accommodate plastic rotations up to 0.005 for immediate operation, the building can deform up to a roof displacement of 11.4 cm, which corresponds to an inter-story drift index of 0.0084:

$$IDI_{GS} \leq 0.0084 \quad (4)$$

It should be noted that the gravitational system of the building is formed of light reinforced concrete frames that exhibit a low longitudinal steel content and simple detailing. As a consequence, the gravitational system exhibits lateral strength and stiffness that are considerably lower than those required by an earthquake-resistant structure.

### Preliminary Design of Bracing System

The following inter-story drift index thresholds are considered to quantify the performance of the non-structural system:

$$IDI_{NS}^{IO} \leq 0.002 \quad (5)$$

$$IDI_{NS}^{LS} \leq 0.008 \quad (6)$$

where  $IDI_{NS}^{IO}$  and  $IDI_{NS}^{LS}$  are the maximum allowable inter-story drift indices corresponding to immediate operation and life safety, respectively.

Through the simultaneous consideration of the gravitational (Equation 4) and non-structural systems (Equations 5 and 6), the design inter-story drift index thresholds are:

$$IDI_{IO} \leq 0.002 \quad (7)$$

$$IDI_{LS} \leq \min(0.008, 0.0084) = 0.008 \quad (8)$$

To satisfy its performance conditions, the bracing system should yield at inter-story drifts close to 0.002, and exhibit significant plastic behavior for inter-story drifts close to 0.008.

Considering that  $\gamma = 0.5$ ,  $\eta = 0.333$ , and  $\alpha = 53.13^\circ$ , Equation 2 yields:

$$f_y = \left( \frac{\Delta_L}{h} \right)_y \frac{200,000 \sin 53.13^\circ \cos 53.13^\circ}{0.5 + 0.333(1-0.5)} = 288 \text{ MPa} \quad (9)$$

Because it has been considered acceptable for the bracing system to exhibit small plastic demands for immediate operation, the yield stress of the braces is set equal to 237.5 MPa. This results in an inter-story drift index at yield equal to (Equation 2):

$$\left( \frac{\Delta_L}{h} \right)_y = \frac{237.5 [0.5 + 0.333(1-0.5)]}{200,000 \sin 53.13^\circ \cos 53.13^\circ} = 0.00165 \quad (10)$$

Note that the value of  $f_y$  used in Equation 10 should be the actual yield stress, and not a reduced value. Considering that the maximum allowable inter-story drift index for life safety is equal to 0.008, the bracing system should be able to develop a maximum inter-story ductility close to  $\frac{0.008}{0.00165} = 4.8$ . Because the maximum global ductility for the building should be less than the maximum inter-story ductility (Chopra 2001), and considering that the building has only five stories, a maximum global ductility ( $\mu_{max}$ ) of 4 will be used for life safety. In most cases the yield stress of the braces is a fixed value, in such manner that Equation 2 should be used to establish  $IDI_y$  and the rest of the methodology applied as shown in Figure 1.

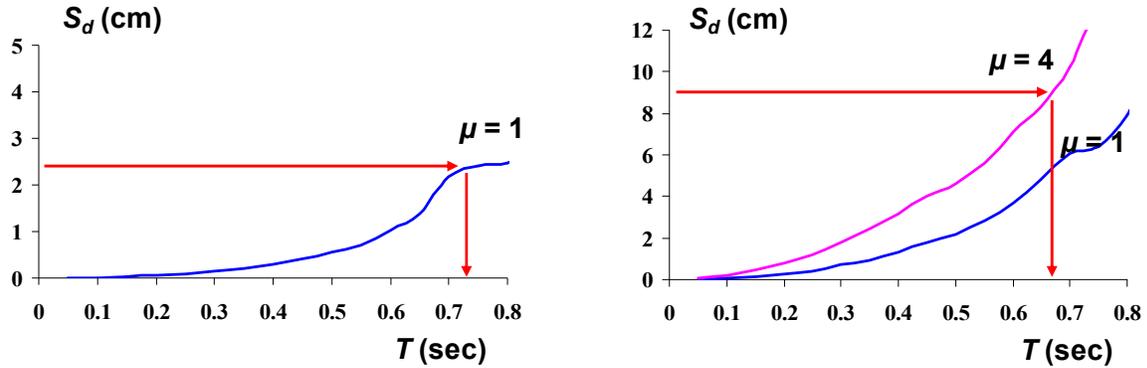
It will be considered herein that the central bays of both external frames are braced with buckling-restrained braces with a Chevron layout (four braces per story). According to the inter-story and bay dimensions, the inclination angle of the braces is equal to  $53.13^\circ$ . Figure 4 shows the design spectra for the performance levels under consideration, and illustrates how the fundamental period required to control the lateral roof displacement of the building is established. The displacement thresholds are estimated according to Equation 3:

$$\frac{\delta_{IO}}{\alpha_{IO}} = \frac{IDI_{IO} H}{\alpha_{IO} COD_{IO}} = \frac{0.002 \times 2000}{1.4 \times 1.2} = 2.4 \text{ cm} \text{ and } \frac{\delta_{LS}}{\alpha_{LS}} = \frac{IDI_{LS} H}{\alpha_{LS} COD_{LS}} = \frac{0.008 \times 2000}{1.2 \times 1.5} = 8.9 \text{ cm} \quad (11)$$

The period for which the braces should be sized corresponds to the smaller of those derived from Figure 4. This results in  $T_{MAX} = 0.66$  seconds. The stiffness-based sizing of the bracing system should be carried out in such manner that the fundamental period of vibration of the building is equal or slightly less than  $T_{MAX}$ . Under the assumption that the lateral response of the building is dominated by global shear effects, it is possible to establish that the bracing and gravitational systems work as two parallel systems, in such manner that:

$$\frac{1}{T_{BR}^2} + \frac{1}{T_{GS}^2} = \frac{1}{T_{MAX}^2} \Rightarrow \frac{1}{T_{BR}^2} = \frac{1}{T_{MAX}^2} - \frac{1}{T_{GS}^2} \quad (12)$$

where  $T_{BR}$  is the period that establishes the stiffness requirements for the braces. According to Equation 12,  $T_{BR}$  is larger than  $T_{MAX}$ , in such manner that the stiffness requirements for the braces are reduced with respect to the case in which the contribution of the gravitational system is neglected. For the five-story building, Equation 12 yields a  $T_{BR}$  equal to 0.74 seconds.



a) Immediate Operation,  $\zeta = 0.02$

b) Life Safety,  $\zeta = 0.05$

Figure 4. Design spectra and determination of maximum fundamental period of vibration for five-story building

Table 3 summarizes the relative value of the lateral forces ( $F_i$ ) and their corresponding story shears ( $V_i$ ) estimated from a static analysis of the five-story building. The optimal lateral stiffness distribution ( $K_{iopt}$ ) for the building is that whose variation through height is proportional to the variation through height of story shear. Practicing engineers tend to standardize the sizes of structural elements and in some cases these sizes may be restricted (pre-determined). As indicated in Table 3, in the example discussed herein it will be assumed that the actual stiffness distribution of the bracing system ( $K_{iact}$ ) does not follow exactly  $K_{iopt}$ .

Table 3. Distribution through height of lateral stiffness for five-story building

Story	$F_i$	$V_i$	$K_{iopt}$	$K_{iact}$
5	0.258 $cW$	0.258 $cW$	0.258 $K$	0.7 $K$
4	0.297 $cW$	0.555 $cW$	0.555 $K$	0.7 $K$
3	0.223 $cW$	0.778 $cW$	0.778 $K$	1.0 $K$
2	0.148 $cW$	0.926 $cW$	0.926 $K$	1.0 $K$
1	0.074 $cW$	1.000 $cW$	1.000 $K$	1.0 $K$

The total brace area was estimated at 67.2 cm<sup>2</sup> for the lower three stories, and 44.8 cm<sup>2</sup> for the upper two stories (each brace requires a fourth of the total area). The frame elements that support the bracing system were redesigned using capacity design concepts.

### Seismic Performance of the Braced Building

To establish the seismic performance of the braced building, the bracing system was added to the nonlinear model of the five-story building; and the new model subjected to the motions used to establish the design spectra. The nonlinear analyses considered a percentage of critical damping of 2% for immediate operation, and of 5% for life safety. Viscous damping was considered through a Rayleigh matrix that assigned the indicated damping to the first two modes.

Figure 5 shows the capacity curve of the braced building, and superposes the roof displacement demands for the two performance levels under consideration. Mean and mean +  $\sigma$  roof displacement demands of 2.64 and 8.24 cm were obtained for immediate operation and life safety, respectively (their respective design thresholds are estimated from Equation 11 as 3.4 and

10.7 cm, respectively). Inter-story drift index demands of 0.0017 and 0.0067 compare well with the threshold values considered for the performance levels under consideration: 0.002 and 0.008, respectively. The nonlinear model of the braced building estimates a fundamental period of vibration of 0.68 seconds, which compares well with the target value of 0.66.

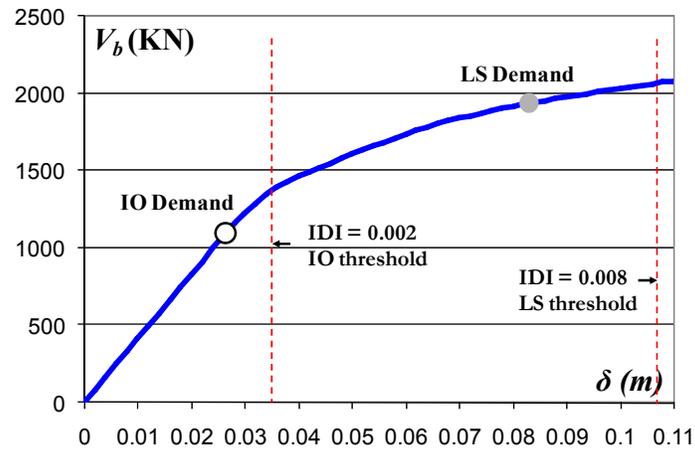


Figure 5. Structural performance of second version of braced building

## Conclusions

The application of a displacement-based methodology to a five-story building has given place to an adequate level of seismic design for immediate operation and life safety. Within the context of a displacement-based design methodology, the area of braces required for lateral stiffness should be determined as a function of the fundamental period of vibration required by the structure to control the level of damage in the gravitational and non-structural systems.

The distribution and location of braces within the building is relevant to its structural safety. In the example that has been illustrated, it was decided to concentrate the bracing system in the central bays of the external frames. The problem with this type of arrangement is the lack of redundancy.

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