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PILE FOUNDATION MODELING FOR SEISMIC ANALYSIS OF HIGHWAY BRIDGES LOCATED IN EASTERN NORTH AMERICA

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ABSTRACT

This paper presents the seismic analysis of a bridge supported on piles in Eastern North America (ENA). The influence of pile modeling on inelastic displacements at bridge bearings, the period of the structure and forces at the column bases is examined. The response of a pile group is analyzed for three different soil profiles represented by sets of characteristic p-y curves. An equivalent stiffness matrix at pile cap level is derived at different load intensities using incremental loading analysis. A 90 m long, two span prestressed concrete highway bridge is studied along its longitudinal direction. The central pier is on pile foundation Two models are used: fixed at the base of the pier, and with equivalent lumped spring elements derived from the previous analysis. The inelastic behavior of the RC columns is carefully modeled using a fiber model. Modal, elastic and inelastic time history analyses are performed. The pile foundation modeling does not have a significant effect on displacements and forces. In turn, several discrepancies were identified between modal and inelastic time-history analyses results. The effective (cracked) stiffness of reinforced concrete columns must be considered in modal analyses to better approximate forces and displacement. However, inelastic displacements at high level of ductility are largely underestimated by an elastic analysis (by 20%). Recommendations targeted at practicing engineers are formulated regarding seismic modeling of bridges supported by piles located in ENA.

Introduction

During recent earthquake events, notably Chi-Chi (Taiwan) in 1999 and Kobe in 1995, several bridges fell from their supports due to insufficient seating lengths or inadequate bearing supports. Unseating could range from impairing the post-seismic functionality of bridges to complete span and bridge deck failure. Therefore, a proper quantification of inelastic seismic displacements occurring at bridge bearings is essential in a seismic safety assessment of existing bridges and design of new bridges.

Many bridges are supported on pile foundations, especially those located on soft soils. Ground motions at the bedrock will propagate through soft soil while interacting with pile-bridge

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systems. The amplitude and frequency of ground shaking, and the corresponding wavelengths, will be altered by the soil deposit. On the other hand, when subjected to ground accelerations, the superstructure's mass transmits inertial forces to piles and induces displacements at the pile cap-level. This leads to a soil-structure interaction (SSI) problem involving both kinematic and inertial interactions that could be solved with increasing levels of modeling complexities. In practice, the problem is often simplified by considering equivalent springs at the base of the piers to approximate the lateral load (p) – displacement (y) response of the piles. Only kinematics interactions are considered with free-field seismic excitations applied to the system. Alternatively, the piles could be modelled explicitly with various arrangements of lateral and vertical supporting soil elements, ranging from simple non-linear springs to detailed finite element models. At the other end of the spectrum, the problem is often simplified to the case of fixed column bases.

This paper presents two modeling alternatives for pile foundations in seismic conditions. An efficient pile group modeling procedure, based on equivalent foundation springs, that could realistically be used in practice is applied to an existing highway bridge in Eastern North America (ENA) and compared to the fixed base case. The effects of pile foundation modeling on inelastic displacements, elastic forces and ductility demands are also quantified through modal and linear and nonlinear time-history analyses.

Soil-structure interaction model

Modeling alternatives

Several methods are available to model pile foundations. The most complete one is a detailed finite-element model (not shown herein). Both bridge structures and soils are modelled by elements with their own stiffness, damping, and mass properties. The input motion is applied at the rock level, where the far-field boundary of the model is located. It is crucial in that model to properly define boundary conditions so that seismic waves are not reflected and to properly model the semi-infinite space. Zhang (2003) presents the finite element analysis of a bridge using specialized elements implemented in the OpenSees computer program, focusing on boundary conditions and seismic input ground motions. Soil elements are to be selected carefully to model the effects of piles in soils of different stiffnesses and limit strengths. If a 2D model is used, soil elements around piles have to be adjusted to reproduce the 3D effects of piles producing reduction in individual stiffnesses and bearing capacity.

Penzien-type models are composed of lumped masses linked by beam elements, soil springs and dampers, assuming that ground deformation is only due to soil shear strain. In a modified Penzien-type model, an additional mass is rigidly linked to the pile and is equivalent to the mass of soil activated by the piles. The main advantage of that model is that seismic input is only applied at the base and the kinematic interaction is taken into account by soil masses and interaction springs. It is simple and its computation is not time-consuming. However, Sun and Goto (2001) pointed out that it is difficult to properly evaluate the soil spring parameters and the mass of soil around piles. They compared their results to damage to bridges during the Kobe earthquake and made several recommendations to improve the modified Penzien-type model.

Hutchison et al. (2004) present nonlinear dynamic analyses using a beam-on-nonlinear-Winkler foundation framework to model pile foundations (Fig. 1a). It consists of a series of non-linear lateral load (p) vs lateral deflection (y) or p-y elements spaced at regular intervals along the pile length. The elements model both the near-field plastic response and the far-field elastic response with a series of gap, drag, plastic and elastic springs. In OpenSees, Material PySimple1 can be used to model pile-soil interactions. That model is quite simple and its computation is not time-consuming, but the parameters of p-y elements are difficult to define and convergence may not be achieved with gap elements. As opposed to the modified Penzien-type model, there is no mass associated to the soil in a Winkler foundation model, but a different input signal has to be generated for all pile discretization levels to account for the variations in ground acceleration in various soil layers.

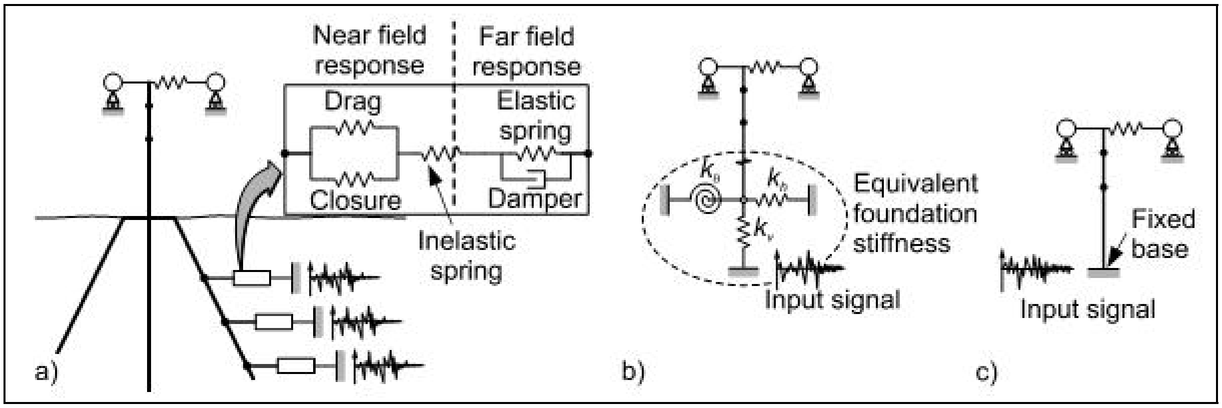

Figure 1. Some pile foundation modeling options: a) Winkler foundation model, b) Equivalent foundation spring model, and c) Fixed base model

Another approach consists of replacing the entire substructure by one or many equivalent soil springs (Fig. 1b), dividing the problem into two separate analyses. Ingham et al.(1999) present several models: i) all piles are grouped in one impedance matrix with stiffness and damping elements, ii) all piles are replaced individually by impedance matrices at mudline, iii) piles are modeled as beam-on-nonlinear-Winkler foundation (detailed model). The stiffness coefficients in the equivalent matrix were determined using linearized p-y curves along the piles. The authors also developed a procedure to determine the displacements to be applied at the base of the simplified model, taking into account SSI. All three approaches yielded similar results, but the authors recommended the use of the individual pile model because forces and plastic hinging in individual piles are easily determined while keeping the model quite simple. Kappos et al. (2004) followed a similar approach, but they studied the behaviour of piles in a detailed finite-element model before replacing them by a spring-damper-additional mass system at the pile cap level. The additional mass permits the excitation of the system with free-field motions.

The advantage of separating the bridge-foundation problem using substructure approach is that the bridge model and the foundation model can be developed by different persons with different tools. Both articles cited in the previous paragraph (Ingham et al. 1999, Kappos et al. 2004) included kinematic and inertial interactions because they either modified the free-field motions or added mass to the system. The approach presented herein is much simpler because it does not take into account the interaction between the soil and the piles in the definition of the input acceleration. Flexibility of the column bases is considered when subjected to inertia forces, as opposed to a fixed base (Fig. 1c) frequently used in practice. This approach is a good compromise between ease of implementation and accuracy. It can be solved by a modal superposition analysis, because time-history analyses are usually not required by design codes. Furthermore, this method is considered as a first step for inclusion of foundation effects in seismic analysis. If these effects are important, one could always use one of the aforementioned detailed methods to improve the analysis. Taking into account kinematic interaction will most likely reduce displacement and forces transmitted to the bridge, as piles will be subjected to varying, counteracting movement along their length. This may also induce forces in the piles themselves.

Bridge model

The proposed method is presented through the modal and time-history analyses of a two-span concrete highway bridge located in ENA (Fig. 2). Both spans are longitudinally fixed to the centre columns, but are free to move at the abutments. New England Bulb Tee (NEBT) prestressed concrete beams are used that are free to rotate at all supports. It is an emergency design level bridge. This level of seismic performance means that it has to be open to emergency vehicles and for security/defence purposes immediately after the design earthquake. The longitudinal response is studied herein and the analysis is simplified to a one column, one mass, 2D system. The column was designed to resist seimic forces with a force modification factor R =5, in accordance with CSA-S6-06 (CSA 2006). The column cross-section is presented in Fig. 2, along with material models used in the inelastic analyses. The concrete model by Mander et al. (1988) was used for confined and unconfined concrete materials and the Menegotto-Pinto model was specified for the rebars.

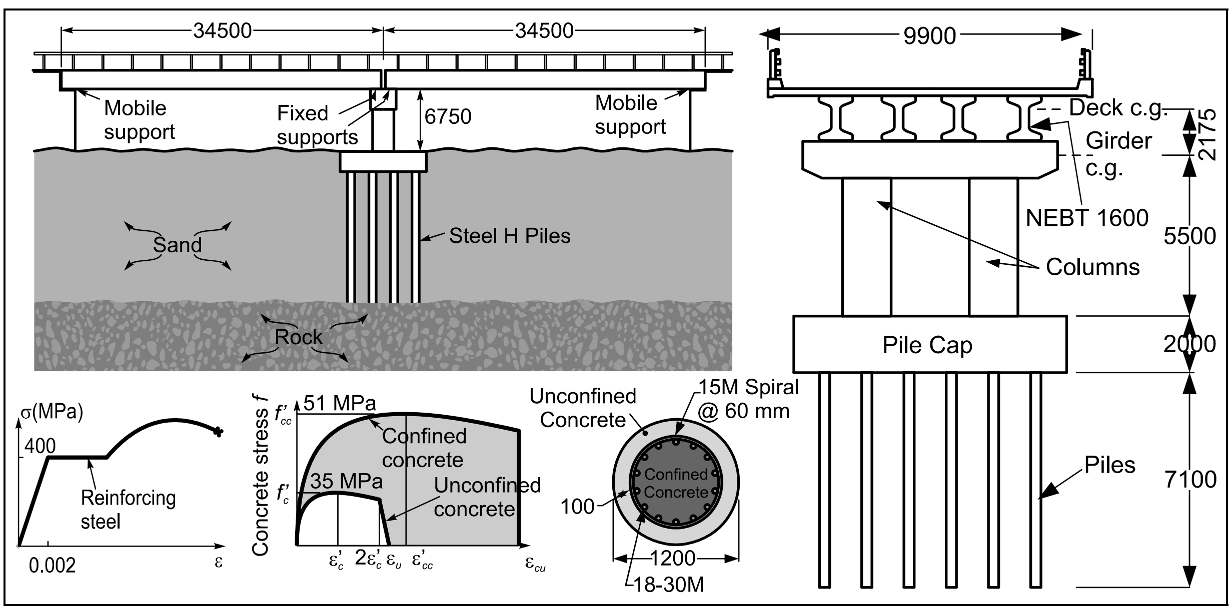

Figure 2 : Bridge under study

Three different soil profiles were studied. Soil A is a medium to dense sand, corresponding to Type I soil in CSA-S6-06. Steel H-piles are driven to rock bottom, at a depth of 9.1 m. They are designed to resist axial forces associated to the weight of the bridge and traffic loads. The second profile, Soil B, is similar but the sand is replaced by soft clay, making it a Type III soil. Pile sections remain the same. Soil C is a deep layer of clay, ranging from soft to stiff over a 40 m depth. It is considered a Type III soil as well. Tubular steel piles, 35 m long, are designed to act as friction piles. For simplification and comparison purposes, the column cross-section remains the same for all cases, even if it was designed for the first soil profile.

Foundation modeling

The first step in the seismic analysis is to calculate the stiffness matrix of the pile foundations. This is done with the computer program GROUP (Reese and Wang 2006) which is dedicated to the calculation of the response of pile groups to pile cap loading. The program considers non-linear p-y curves to compute displacements and stresses in pile groups. The p-y curves can be input manually or automatically generated by the program using literature data. The latter option was chosen as the library is well documented and many p-y curves are recommended by the American Petroleum Institute (Po Lam et al. 2007). Scaling factors due to pile spacing and diameter are also taken into account. The program offers either static or cyclic (oceanic waves action) p-y curves. Static curves were used in the models as they were found to yield reasonable results, except for sensitive and liquefiable soils (Po Lam et al. 2007).

In GROUP, pile cap stiffness coefficients can be computed for each loading direction individually (translation, rotation). A lateral load is applied and the two stiffness coefficients of the equivalent stiffness matrix are determined at the pile cap level: lateral-lateral and lateral-rotational. Subsequently, a bending moment is applied at the pile cap and rotational-rotational and rotational-lateral coefficients are determined. In both cases, no axial load is applied on the pile group. The resulting stiffness matrix is assembled, along with the axial term, but the latter is not coupled to the other degrees of freedom. Because axial loads are not applied simultaneously with the other forces, uplift may artificially occur and significantly increase the rotation of the pile cap. The problem can be avoided by fixing the base of the piles, if non occurrence of uplifting is checked manually. Furthermore, the group matrix may not be positive-definite because of soil non-linearity. This problem can be avoided if p-y springs are linearized prior to the analysis (Po Lam et al. 2007, Ingham et al. 1999). Few iterations are then required. Kappos et al. (2004) suggest to modify the height of the piles so that the matrix is decoupled.

A different approach was adopted for the bridge studied. Since the lateral seismic forces are concentrated at the deck level, the ratio between the bending moment, M, and the lateral load, V, at the column base is constant and equal to the height from the pile cap to the center of gravity of the deck, h. Both lateral load and the bending moment increments can then be applied simultaneously, along with the full axial load. The foundation is “pushed” up to the probable capacity of the column. Lateral displacements and rotations at the pile cap level are recorded and three, uncoupled, non-linear springs are obtained for the pile cap using either tangent or secant values, as explained later. These non-linear springs, if uncoupled, can be included in widely used structural analysis programs, such as SAP2000 (CSI 2008). Figure 3a shows that using $M = Vh$ was a reasonable assumption for the bridge under study. The horizontal and rotational foundation springs computed are shown in Fig. 3b. The foundation springs lead to a longitudinal bridge stiffness between 23 kN/mm and 45 kN/mm, depending on the soil type and assuming the column is infinitely stiff. This is large compared to the lateral stiffness of the column itself (7 kN/mm assuming effective (cracked) moment of inertia).

P-Δ effects may modify the relation between moment and base shear, but the error is small compared to the uncertainties in soil stiffness and can be omitted. Because the resulting matrix is unique to the assumed load path, the method has two drawbacks, not experienced here: i) if bridge columns experience double curvature, the ratio of moment to shear changes and the load path is no longer exact; ii) if the column is rigidly linked to the deck, plastic hinge may not be located at the base of the column. The foundation springs, when measured from the top of the column, have a lateral stiffness between 23 kN/mm and 45 kN/mm. It is much stiffer that the column itself, that has an approximate effective (cracked) lateral stiffness of 7 kN/mm.

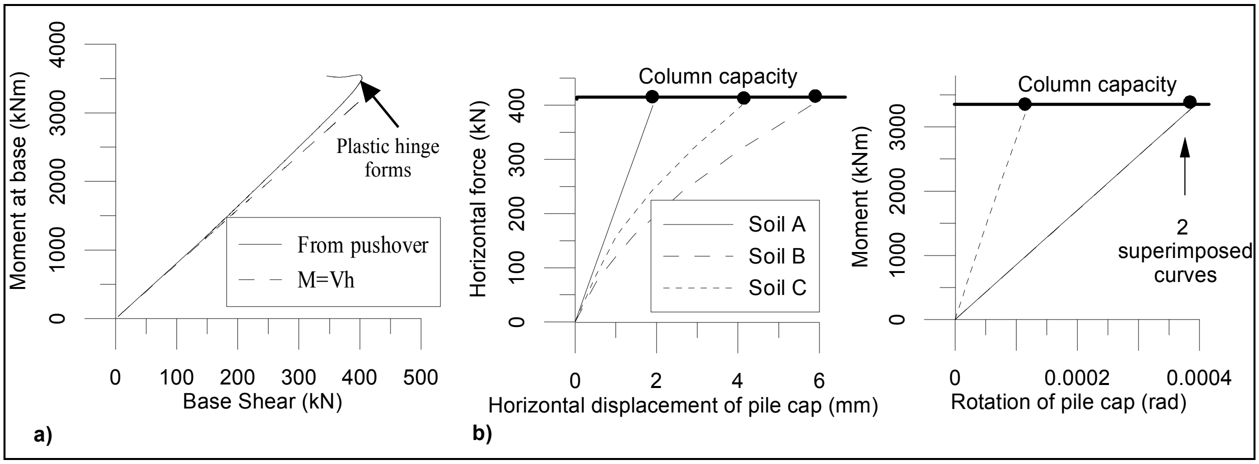

Figure 3: a) Base shear-bending moment relationships at the base of the columns; b) Lateral and rotational responses of springs modeling pile foundation as calculated with GROUP.

Analysis

Response spectrum analyses

Elastic response spectrum analyses are performed on the bridge using SAP2000. The CSA-S6-06 design spectrum used for Soil A is shown in Fig. 4a. The spectrum for Type III soils was used for Soils B and C. In previous editions of CSA-S6, it was not permitted to account for cracking in the calculation of the column stiffness when calculating the forces and two column stiffnesses are compared to examine the differences: i) the initial stiffness (un-cracked column), and ii) an effective stiffness equal to 36% of the initial stiffness to account for cracking during the seismic event (Priestley et al. 1996). A fixed base case is studied for all three soils. When the pile cap response is linear (Soil A), the foundation spring is directly added to the model. For the two other soils, a spring with initial stiffness is considered and another one with secant stiffness is also tested. The secant is traced so that the displacement of the foundation is accurate when the column reaches its flexural capacity, corresponding to a horizontal load of 400 kN in that case. Fig. 4b shows the spring responses used for Soil B.

The results of the spectral analyses are shown in Table 1, where V_e is the elastic base shear and Δ_e the elastic bridge deck displacement. The influence of the column stiffness is obvious: for all soil cases, the displacements are almost doubled while forces are reduced by 30% because the periods elongate from approximately 1.1 to 1.8 s. Results from non-linear time history analyses (discussed later) confirmed that the displacements are better predicted when the column effective stiffness is used, and only the results for this case are commented.

The effect of foundation springs on displacements and forces was the same in all cases, but the magnitude was different. For the rigid, sandy soil (Soil A), the displacement only increases by 7 mm and the forces are reduced by 19 kN, which represents an insignificant reduction of 4 kN after dividing by the R factor of 5.0! For the softer soils, the displacements increase by approximately 5% and the forces reduce by less than 5%. Notice that spectral accelerations do not change much in the period range of interest (see Fig. 4a). Foundation effects would have been much more pronounced had the bridge period been in the range of 0.5 s. For instance, in the bridge transverse direction (not examined here), the period shifts from 0.41s to 0.58 s when the effective column stiffness is used, resulting in a 25% drop in spectral accelerations.

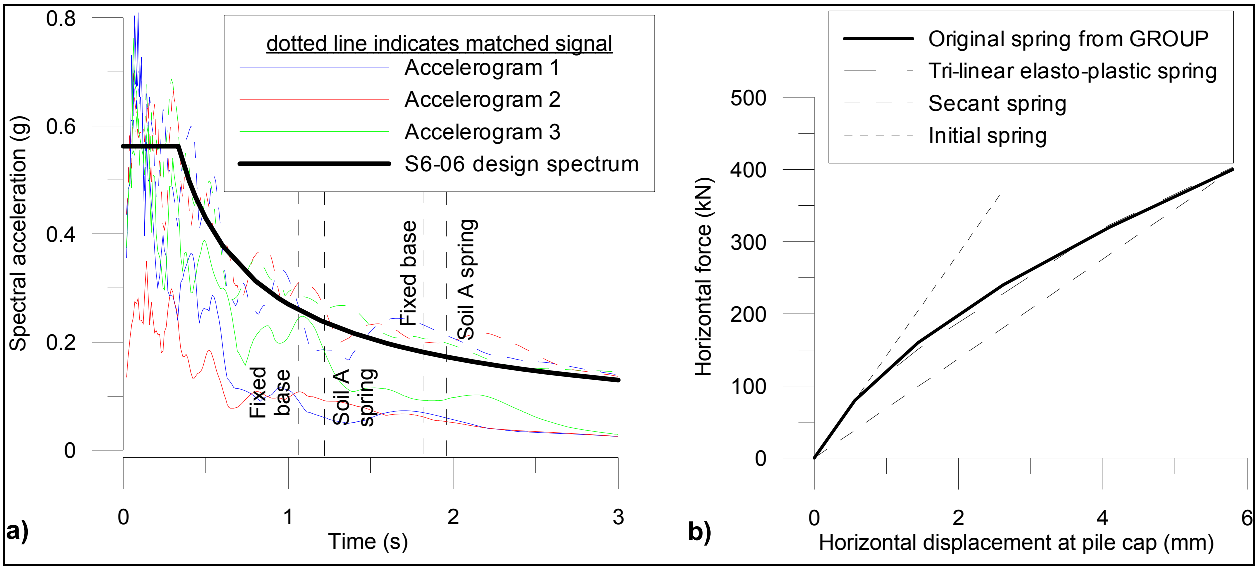

Figure 4: a) Design response spectrum and 5% damped spectral accelerations of original and matched input accelerograms for Soil A (bridge periods are also shown), b) Springs used to model the pile foundation.

Table 1: Results of modal analysis

	Base type	Column with initial stiffness			Column with effective stiffness		
		T (s)	V_e(kN)	Δ_e(mm)	T (s)	V_e(kN)	Δ_e(mm)
Soil A	Fixed	1.12	1220	78.1	1.85	867	152.9
	Elastic*	1.22	1147	87.4	1.92	848	159.9
Soil B	Fixed	1.12	1830	117.1	1.85	1300	229.3
	Initial	1.24	1705	133.8	1.93	1267	241.7
	Secant	1.29	1656	142.3	1.96	1250	247.9
Soil C	Fixed	1.12	1830	117.1	1.85	1300	229.3
	Initial	1.18	1752	126.3	1.89	1281	236.3
	Secant	1.22	1720	131.4	1.92	1272	240.0

*Springs for Soil A were found to be linear elastic within the range of the column capacity. Therefore, springs with initial and secant stiffness are identical.

Pushover and time history analyses

Nonlinear seismic analyses were performed using ZeusNL program (Elnashai et al. 2003). The program uses Bernoulli-Euler beam elements with fiber cross-sections and many material models are available, including Mander model for confined concrete. Many types of spring are also available, including a tri-linear elasto-plastic spring to model non-linear horizontal soil springs (Fig. 4b). Static incremental (pushover) analysis was first executed to determine the yield deformation, Δ_y, following the procedure proposed by Priestley et al. (1996) and shown in Fig. 5a.This value is needed to calculate the ductility demand from time-history analyses. The column effective lateral stiffness is also illustrated in the figure. The push-over analysis curve with and without concrete confinement are also shown, enforcing the importance of properly modeling confinement (and achieving it), especially for post-peak behaviour of the column.

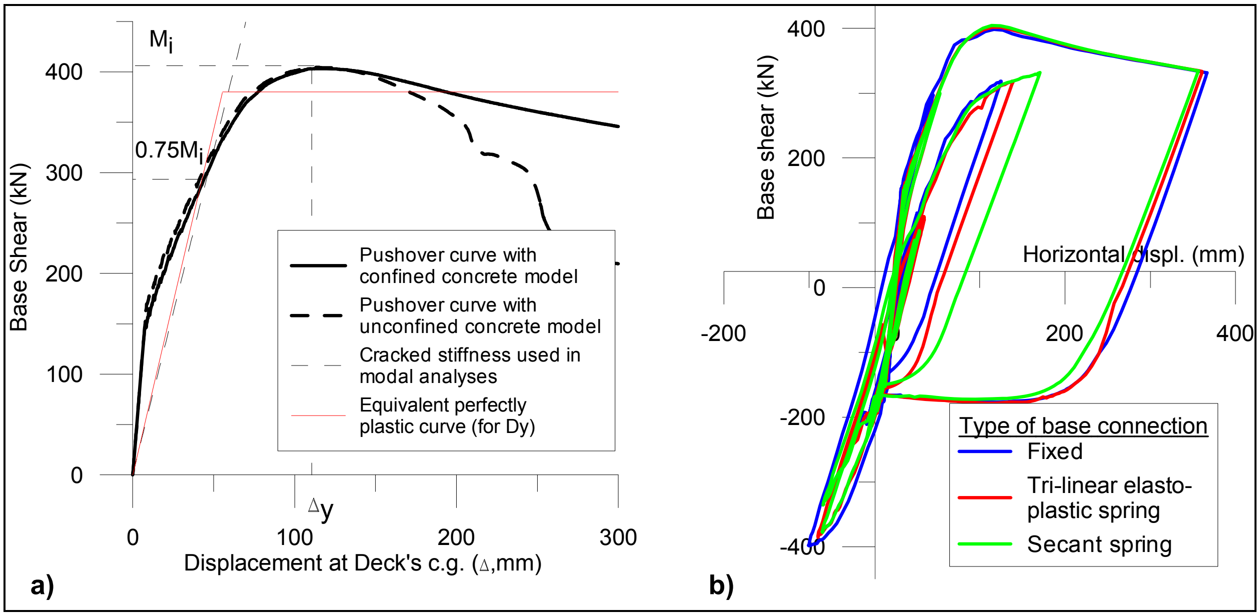

Figure 5: a) Push-over curves with confined and unconfined concrete models (equivalent perfectly plastic curve is also shown), b) Hysteretic response of the bridge to accelerogram 1 for Soil B and various base conditions.

Three input seismic accelerograms were applied to the models. De-aggregation plots of the seismic hazard at the site for a 10% in 50 years probability of exceedance, corresponding to the design level in CSA-S6-06, were used to determine dominant magnitude-distance scenarios for the selection of the motions. The accelerograms were modified by loose spectral matching in the frequency domain with respect to the CSA-S6-06 design spectrum. Response spectra of the input motions for Soil A, before and after spectral matching, are shown in Fig. 4a. Because ZeusNL applies earthquakes in the form of base displacements, rather than accelerations, baseline correction was also applied to the accelerograms to avoid high absolute velocities that could interfere with the dashpots in the structure. Rayleigh mass proportional damping equal to 5% of critical was specified, based on the effective bridge periods.

Elastic time-history analyses were performed assuming column effective stiffness and the results are presented in Table 2. In general, peak forces and displacements agree well with response spectrum analysis results. Maximum displacements reached in non-linear time history analyses are shown in Table 3. Again, the effect of pile group foundation modeling is minimal. Fig. 5b shows three hysteretic responses of the bridge to accelerogram 1 for Soil B for various spring conditions. The displacements with soil secant or trilinear springs are very close to those obtained for the fixed base case. In some cases, displacements are even reduced when accounting for foundation effects, as had been observed by Kappos et al (2004). While the structure is more flexible, inertia forces are reduced due to period elongation, but displacements can increase or decrease. In practical applications, one can compare forces and displacements with the initial stiffness of soil springs and with the secant stiffness at the capacity of the pile to obtain lower and upper bounds on structural quantities of interest. When compared to displacements from spectral analyses and elastic time-history analyses, maximal displacements from inelastic analysis are approximately 20% higher, indicating that the equal displacements principle may underpredict peak displacement demand, especially at high ductility levels. Table 3 also presents the peak displacement ductility demand, μ, in the columns. For the stiff soil (Soil A), the ductility is less than the R factor used in design (R = 5.0) but the value reaches and exceeds 5.0 for the softer soils. The ground motion intensity is smaller for the stiffer soil than for the softer soil. That contributes to the smaller ductility for Soil A.

Table 2: Results of elastic time-history analyses

	Base type	Accelerogram 1		Accelerogram 2		Accelerogram 3		Average	
		V(kN)	Δ(mm)	V(kN)	Δ(mm)	V(kN)	Δ(mm)	V(kN)	Δ(mm)
Soil A	Fixed	1080	179	924	153	986	163	997	165
	Elastic	1034	184	909	162	901	161	948	169
Soil B	Fixed	1610	267	969	160	1470	243	1350	224
	Tri-linear	1421	274	921	177	1210	233	1184	228
	Secant	1493	281	965	181	1272	239	1243	234
Soil C	Fixed	1610	267	969	160	1470	243	1350	224
	Tri-linear	1495	272	965	181	1283	233	1248	229
	Secant	1547	276	953	170	1350	240	1283	229

Table 3: Results of non-linear time-history analyses

	Base type	Accelerogram 1		Accelerogram 2		Accelerogram 3		Average	
		Δ(mm)	μ	Δ(mm)	μ	Δ(mm)	μ	Δ(mm)	μ
Soil A	Fixed	189 (0*)	3.40	164 (0)	2.95	210 (0)	3.78	187 (0)	3.37
	Elastic	202 (1.8)	3.61	167 (1.8)	2.97	217 (1.7)	3.88	195 (1.8)	3.48
Soil B	Fixed	366 (0)	6.60	195 (0)	3.52	285 (0)	5.13	282 (0)	5.08
	Tri-linear	360 (4.4)	6.40	204 (5.4)	3.58	301 (5.3)	5.33	288 (5)	5.10
	Secant	356 (4.4)	6.34	192 (5.2)	3.36	308 (5)	5.46	285 (4.9)	5.05
Soil C	Fixed	366 (0)	6.60	195 (0)	3.52	285 (0)	5.13	282 (0)	5.08
	Tri-linear	363 (3.7)	6.48	192 (5.5)	3.36	294 (3.8)	5.23	283 (4.3)	5.02
	Secant	362 (3.4)	6.46	188 (3.8)	3.32	289 (3.6)	5.15	280 (3.6)	4.98

* Displacement of pile cap is indicated in parenthesis and included in deck's displacement

Summary and conclusions

The seismic response of a simple two-span bridge was examined along its longitudinal direction. The bridge was located in ENA and subjected to high frequency ground motions. The pile foundation group was replaced by an equivalent spring for three different soil profiles. Response spectrum as well as linear and non-linear time-history analyses were performed on the bridge-foundation model and the response with tangent or secant foundation spring stiffness assumption were compared to the results obtained for a fixed base case.

The computer program GROUP was used to study the response of piles to applied forces at the pile cap level. A load path corresponding to gradually increasing the lateral load at the deck level was applied to avoid non positive-definite stiffness matrix at the pile cap level. The pile cap was found to exhibit a nonlinear lateral response and a linear rotational response for all soil cases considered. For the cases studied, the influence of the foundations on the bridge longitudinal stiffness was relatively small.

Response spectrum analysis showed that using the effective (cracked) stiffness for the columns greatly influenced the response compared to the un-cracked case: elastic displacements almost doubled while base shear forces reduced by a third. Accounting for the pile foundation group does not have such a significant effect. The natural period is lengthened, but the forces and displacements are modified by only 5% compared to the fixed base case, even with the softest of the three soil profiles. It is noted, however, that the design response spectrum does not vary much in the period range of the bridge studied range.

The ZeusNL program was used to carry out time history analyses. Inelastic time history analysis resulted in displacements larger than what was predicted from elastic time history and response spectrum analyses. In that case, the well known equal displacement principle underestimated the displacement demand, especially when the ductility demand was high. In all cases analyzed in this project, the foundation springs were not found to significantly impact on the response of bridge column and deck displacements. This conclusion may not hold true for other bridge-foundation systems exhibiting different dynamic characteristics, notably bridges with shorter natural period of vibration, say 0.5 s. Further studies are needed before the findings of this project can be generalized.

Acknowledgements

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