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DESIGN OF OPTIMUM TMD SYSTEMS FOR SDOF AND MDOF STRUCTURES SUBJECTED TO EARTHQUAKE BASE EXCITATIONS

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ABSTRACT

Various methods have been proposed for the design of TMD systems. In these methods various assumptions have been made regarding the simulation of the acting dynamic force, its location and the criteria used in defining the optimum design parameters of the TMD system. On the other hand attention and research is increasing about generating or predicting earthquake time history records for design purposes. In the present work, a method is suggested for the optimum design of tuned mass damper (TMD) system for structures subjected to a specific or given earthquake base excitation. The method can be applied to SDOF or MDOF shear building structures. In this method, the values of the optimum stiffness and optimum damping for a given TMD mass are defined as the values that will reduce a specific response of the structure to a minimum value when subjected to an earthquake time-acceleration history. The specific response selected in the present study is the maximum relative displacement of the structure. Thus the problem can be restated as multivariable, nonlinear constrained minimization problem. Based on this formulation a MATLAB computer program is developed. The program consisted of two main subroutines. The first was a dynamic analysis subroutine for the analysis of SDOF or MDOF shear building structures with TMD system. This part is embedded in another subroutine for nonlinear constrained optimization to compute the optimum design parameters of the TMD system. The developed software is then used in selected case studies showing convergence of the present study results to optimal solutions and efficiency when compared to other methods for designing TMD systems.

Introduction

A TMD system consists of a mass, spring and a damper. If these properties are properly designed and selected, then the device can be effective in suppressing undesirable vibrations induced by earthquake or wind loads. Obtaining the optimal design for a TMD system has been the goal of many researches for decades. In these researches different assumptions have been made regarding the simulation of the acting dynamic force, its location and the criterion used in

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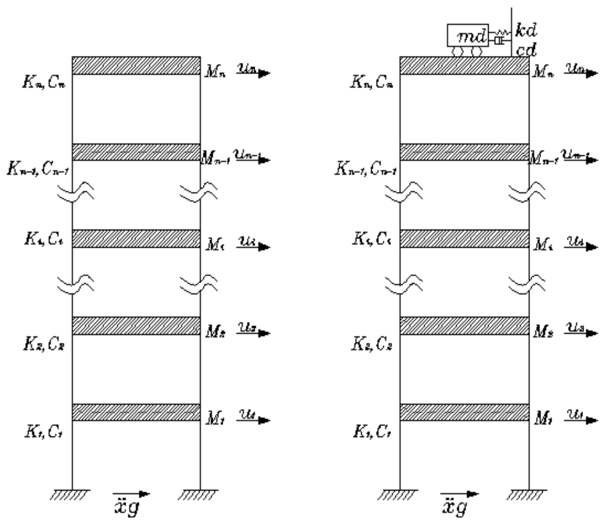
defining the optimal design parameters. (Den Hartog 1956) has derived the formula for the optimum values of the TMD parameters when attached to a SDOF structure subjected to a harmonic load. An extension to it has been made by (Warburton and Ayorinde 1980), (Tsai and Lin 1993) where the main mass damping was considered and several types of harmonic excitations were examined. Extensive research was also made by (Warburton 1982), (Rana and Soong 1998) where formulas for several types of excitations were developed. In these cases, harmonic or random excitations were applied either on the main system or at the base of the structure. (Sadek, et. al. 1997), suggested a method for estimating the design parameters of tuned mass dampers for seismic applications, the criterion used to obtain the optimum parameters was to select, for a given mass ratio, the frequency and damping ratios that would result equal and large modal damping in the first two modes of vibration. Although many of the above mentioned procedures were basically developed for TMD systems attached to SDOF structures, they were extended to be applied for MDOF structures (Rana and Soong 1998, Sadek, et.al. 1997). Hadi & Arfiadi (1998) discussed the optimum design of TMD for seismically excited building structures. In their design process the multi degree of freedom structures were considered. The genetic algorithm was used to find the optimum value of TMD parameters. The numerical examples for optimum parameters of TMD system for multi degree of freedom structures were presented to show the effectiveness of the design procedures. In their proposed procedure, the optimum value the properties of the TMD system can be determined without specifying the modes to be controlled. (Lee, et. al. 2006), proposed an optimal design theory for structures implemented with TMD. The proposed method allowed for a more extensive application. The optimal design parameters of TMD in terms of the damping coefficients and spring constants corresponding to each TMD were determined through minimizing a performance index of structural responses defined in the frequency domain. As can be noticed from the brief literature review, various assumptions have been made regarding the earthquake loading (Harmonic or Random), and about the location of the acting force (on the structure or at its base).
On the other hand, extensive research is being carried out about the prediction of possible earthquake parameters for specific site conditions. This includes acceleration-time history, response spectra and other parameters. In these works, predicted or artificial acceleration-time histories were computed using mathematical methods that take into account the geological and seismological parameters of the site at which the structure was intended to be constructed. Examples of such research work are the papers published by (Crempien-Laboriea and Oroscob 2000), (Rofooei et. al. 2001), (Varpasuo et. al. 2001), (Junn and Jo 2002), (Abdalla and Hag-Elhassam 2005), (Adnan et. al.2006), (Atkinson and Boore 2006), (Fengxin et. al. 2006).This encouraged the authors to investigate the possibility of using acceleration-time histories in designing tuned mass damper systems. In the present work, a method is suggested to obtain the optimum design parameters of a TMD system when attached to SDOF or MDOF structure. In this method, predicted or actual earthquake records are used to compute the forces acting on the structure. Then the optimal TMD properties are computed as will be described.

Dynamic Analysis of Shear Building Structures under Seismic Excitations

The present study is concerned with the dynamic behavior of plane shear building structures. A TMD damper system may be attached to the top floor as shown in Fig. 1 for linear elastic shear building with **n** degrees of freedom attached to TMD system and subjected to earthquake excitation, the dynamic equilibrium equation can be written (Chopra, 1995):



(a) without TMD system.   (b) with TMD system.

Figure 1. Shear building frame with **n** degrees of freedom.

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}=-[M][I]\ddot{x}_g \qquad (1)$$

Where [M], [C] and [K] are the mass matrix, the viscous damping matrix and the stiffness matrix, respectively given by:

$$[M]=\begin{bmatrix} M_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_i & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & M_n & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & m_d \end{bmatrix}_{(n+1)\times(n+1)}, [C]=\begin{bmatrix} (C_1+C_2) & -C_2 & 0 & 0 & 0 & 0 & 0 \\ -C_2 & (C_2+C_3) & -C_3 & 0 & 0 & 0 & 0 \\ 0 & -C_3 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & -C_i & (C_i+C_{i+1}) & -C_{i+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & -C_n & (C_n+c_d) & -c_d \\ 0 & 0 & 0 & 0 & 0 & -c_d & c_d \end{bmatrix}_{(n+1)\times(n+1)}$$

$$[K]=\begin{bmatrix} K_1+K_2 & -K_2 & 0 & 0 & 0 & 0 & 0 \\ -K_2 & K_2+K_3 & -K_3 & 0 & 0 & 0 & 0 \\ 0 & -K_3 & \ddots & 0 & 0 & 0 & 0 \\ 0 & 0 & -K_i & K_i+K_{i+1} & -K_{i+1} & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 0 & -K_n & K_n+k_d & -k_d \\ 0 & 0 & 0 & 0 & 0 & -k_d & k_d \end{bmatrix}_{(n+1)\times(n+1)}$$

It can be noticed that the size of the system matrices will be ($n+1$) degrees of freedom. $\{\ddot{u}\},\{\dot{u}\}$ & $\{u\}$ are respectively the relative acceleration, relative velocity, and the relative displacement vector. All these vectors are functions of time and relative to ground displacement. $\ddot{x}_g$ is the ground or earthquake acceleration-time history at the base of the structure in the horizontal direction. [***I***] is the identity matrix.

Formation of Damping Matrix for Dynamic Analysis:

For the dynamic analysis of multi degrees of freedom system using Newark's or other numerical methods, it is required to compute and form the mass, damping and stiffness matrices from the properties of the structure under consideration. The mass and stiffness matrices can be easily computed from the mass and stiffness properties of the structure elements following standard matrix structural analysis procedures. However, specifying and formation of the damping matrix is not straight forward. Damping as a property for multistory building structures can be expressed by two methods. In the first method, the modal damping ratios are estimated using measured data (Chopra 1995). Thus a damping ratio for each mode of vibration can be obtained. This method is ideal for dynamic analysis using mode superposition method and it is a suitable method to account for all energy dissipation in joints, non-structural elements as well as the elements damping. However, the main disadvantage in this method is that it is not suitable for numerical methods or problems with non-classical structural damping forms (MDOF Structure with TMD System). In the second method, damping matrix is computed from the damping properties of the structure components similar to mass & stiffness matrices. Although this is quite suitable for numerical methods but it is generally difficult to estimate these properties. In the present study, the two above mentioned cases are considered. In the case when the damping properties are given, the solution is direct, however if the modal damping ratios are specified for the MDOF structure, the damping matrix [***C***] can be computed as described below (Chopra 1995, Paz 1979). For ***n*** MDOF structure, let [***K***], [***C***] and [***M***] be the stiffness, damping and mass matrices. Then the modal damping ratios matrix can be defined by:

$$[\xi]=\begin{bmatrix} \xi_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \xi_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 & 0 \\ 0 & 0 & 0 & \xi_i & 0 & 0 \\ 0 & 0 & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & 0 & 0 & \xi_n \end{bmatrix}_{n*n} \tag{2}$$

where ξ_i is the modal damping ratio for the i th mode. The damping matrix [***C***] can be computed as follows:

$$[C]=[M][\phi][A][\phi]^{\mathrm{T}}[M] \tag{3}$$

where,

$$A_i = 2\xi_i\omega_i M_i \quad , \quad \phi_{ij} = \frac{a_{ij}}{\sqrt{\{a_j\}^T[M]\{a_j\}}} \text{ and } \quad [\phi]\text{: Normalized modal shape matrix}$$

This method is used to compute the damping matrix [*C*] for the main structure before the attachment of the TMD system. Using Newmark's method and the above mentioned description, a MATLAB computer program was developed for the dynamic analysis of plane shear building structures with TMD system when subjected to earthquake excitation. The program can deal with the structures when modal damping ratios are given or when damping coefficients are known.

Suggested Method for the Optimum Design of TMD System

Considering the structure shown in Fig. 1, the basic features of the method suggested in this study are:-

Excitation Force

In order to obtain the optimum design parameters of the TMD system, certain assumptions regarding the excitation force should be made. For example, (Den Hartog 1958) assumed that the structure mass is subjected to harmonic excitation whereas (Tsai & Lin 1993) assumed that the structure is subjected to harmonic base excitation. To simulate actual behavior, it is assumed in this study that the structure is subjected to base excitation. The excitation force vector is computed from acceleration-time history for a given earthquake. In a real design problem, a prediction for acceleration-time history should be made considering geological and seismological parameters of the site where the structure will be constructed. This can be achieved using one of the methods mentioned in the introduction section. However, in the present work all acceleration- time histories are taken from actual earthquake records because predicting earthquake records is out of the scope of the present study.

Optimization Criterion and Optimization Parameter

In the present work, the optimization criterion and parameter used by many authors is adopted (Den Hartog 1958, Rana & Song 1998 and Tsai & Lin 1993). In this criterion, the optimum design parameters $\boldsymbol{c_d}$ & $\boldsymbol{k_d}$ for a given $\boldsymbol{m_d}$ are defined as those values which minimize the maximum relative displacement of the structure when subjected to an excitation. The maximum relative displacement of a regular shear building frame usually occurs at the top floor.

Statement of the Problem as Constrained Non-Linear Optimization Problem

Fig. 2 shows a shear building structure provided with a tuned mass damper system at the top floor. When the structure is subjected to a given earthquake excitation (acceleration-time $\ddot{x}_g$), then $\boldsymbol{u_t}$ is the relative displacement occurring at the top of the frame. Defining $\boldsymbol{u_{tmax}}$ as the maximum $\boldsymbol{u_t}$ occurring during the earthquake duration. Then for a given structure properties ([$\boldsymbol{K}$], [$\boldsymbol{C}$], [$\boldsymbol{M}$]), TMD mass $\boldsymbol{m_d}$ and earthquake excitation $\ddot{x}_g$, the variable $\boldsymbol{u_{tmax}}$ will be a function of $\boldsymbol{c_d}$ & $\boldsymbol{k_d}$ only. This can be put in the following optimization problem:

Find $\boldsymbol{c_d}$, $\boldsymbol{k_d}$ that minimizes the following objective function:

$$\boldsymbol{u_{tmax}} = f(\boldsymbol{c_d}, \boldsymbol{k_d}) \quad \text{subjected to the inequality constrains } \boldsymbol{c_d} > 0 \text{ and } \boldsymbol{k_d} > 0. \tag{4}$$

This problem can be classified as multivariable, nonlinear constrained minimization problem. For the treatment of such problem, one of the functions available in the MATLAB optimization toolbox is used.

Computer Programs for Optimum Design of TMD System

A MATLAB computer program was developed for computing the optimum design parameters of a TMD system when attached to SDOF or MDOF structures. The development of this software is based on modifying the software described in previous section to include the optimization algorithm. Details of the MATLAB software are given by (Al-Taweel 2007).

Convergence of the Proposed Method to Optimum Solution

To demonstrate the capability of the proposed method to catch the optimum design parameters for TMD system when attached to SDOF or MDOF structures, many problems were examined (Al-Taweel 2007). Two of these verification cases are discussed below.

Case Study One

A single story shear building with properties shown in Fig. 3 is considered. A TMD is attached to the top with mass $\boldsymbol{m_d}$ = 1.5 ton equal to 3% of the total mass of the structure. The objective is to determine the optimum value of TMD stiffness k_d and damping $\boldsymbol{c_d}$ that will minimize displacement $\boldsymbol{u_{tmax}}$ at the top when the structure is subjected to El-Centro earthquake excitation. To understand the variation of $\boldsymbol{u_{tmax}}$ with various values of $\boldsymbol{k_d}$ & $\boldsymbol{c_d}$ the first stage software is used to compute $\boldsymbol{u_{tmax}}$ for $\boldsymbol{k_d}$ (40 to 70 kN/m with steps of 0.2) and $\boldsymbol{c_d}$ (0 to 1.9 kN-s/m with steps of 0.1). The results are plotted as a three dimensional surface [u_{tmax} = f($\boldsymbol{c_d}$, $\boldsymbol{k_d}$)] as shown in Fig. 4 and as a contour lines as shown in Fig. 5. Next, the second stage software is used to obtain the optimum design parameters through minimization process, as described in previous sections. For the minimization process it is given that the upper bound and the lower bound value of the stiffness k_d are 0 and 1000 kN/m respectively. While the upper bound and lower bound of the damping is 0 and 100 kN-s/m respectively. After running the problem, it is

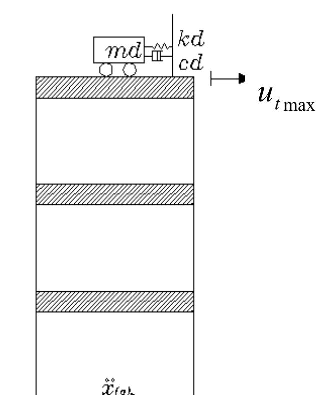
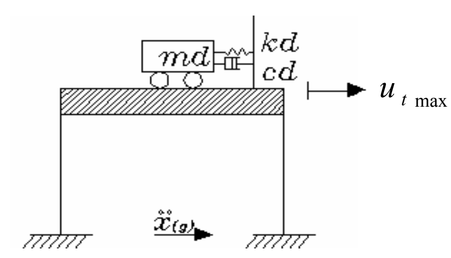


Figure 2. Shear building under earthquake excitation with single TMD system.



$C = 6.283\,kN - s/m$

$K = 1973.9\,kN/m$

$M = 50\,ton$

$m_d = 1.5\,ton$

Figure 3. Structure studied as verification problem in case study one.

found that the optimum values of stiffness and damping of the TMD are k_d = 54.08 kN/m and c_d =0.643 kN-s/m. The corresponding value of u_{tmax} is 0.1179m.When projecting these results on the contour plot in Fig. 5 it can clearly noticed that the solution given by the optimization software represents the minimum value for the surface or contour plot shown in Figs. 4 & 5 This proves the capability of the software to catch the minimum value of u_{tmax} and the corresponding optimum values of k_d & c_d.

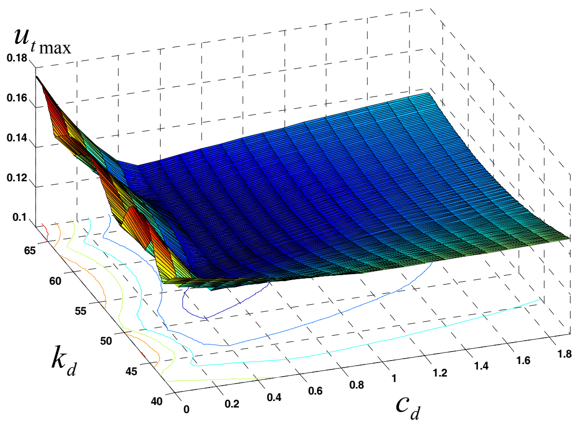


Figure 4. Variation of $u_{t\,max}$ with k_d & c_d for example in Fig. 3 as three dimensional surface.

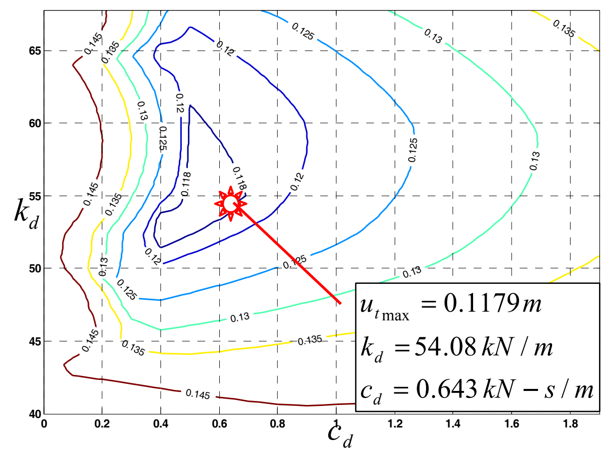
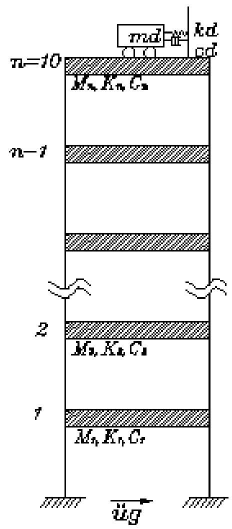


Figure 5. Variation of $u_{t\,max}$ with k_d & c_d for example in Fig. 3 as contour line.

Case Study Two

A ten story shear building with properties shown in Fig. 6 is considered. This example has been investigated by (Hadi & Arfiadi 1998). A TMD is attached to the top floor with mass $\boldsymbol{m_d}$ =108 ton equal to 3% of the total mass of the structure. The objective is to determine the optimum values of TMD stiffness $\boldsymbol{k_d}$ and damping $\boldsymbol{c_d}$ that will minimize displacement $\boldsymbol{u_{tmax}}$ at the top (tenth) floor when the structure is subjected to El-Centro 1940 earthquake excitation. To understand the variation of $\boldsymbol{u_{tmax}}$ with various values of $\boldsymbol{k_d}$ & $\boldsymbol{c_d}$, the dynamic analysis MATLAB software is used to compute $\boldsymbol{u_{tmax}}$ for values of $\boldsymbol{k_d}$ ranging from 2500 to 5000 kN/m with steps of 50 and values of $\boldsymbol{c_d}$ ranging from 0 to 400 kN-s/m with steps of 10. The results are plotted as a three dimensional surface $\boldsymbol{u_{tmax}}$ = f($\boldsymbol{c_d}$, $\boldsymbol{k_d}$) and as a contour lines as shown in Figs. 7 and 8 respectively. Next, the MATLAB optimization software is used to obtain the optimum design parameters through minimization process, as described in previous sections. After running the problem it is found that the optimum values of stiffness and damping of the TMD were $\boldsymbol{k_d}$ = 3217.3 kN/m and $\boldsymbol{c_d}$ = 53.49 kN-s/m. The corresponding value of $\boldsymbol{u_{tmax}}$ was 0.1192m. When projecting these results on the contour plot in Fig. 8, it can be clearly noticed that the solution given by the optimization software represents the minimum value for the surface or contour plot shown in Figs 7 and 8. This proves the capability of the software to catch the minimum value of $\boldsymbol{u_{tmax}}$ and the corresponding optimum values of $\boldsymbol{k_d}$ & $\boldsymbol{c_d}$. Table 1 shows a comparison between the present study results for maximum floors displacements with the results obtained by (Hadi & Arfiadi 1998) and (Lee et. al. 2006) for the structure in Fig. 6. The results show that the TMD



$$C_i = 6.20\,MN - s/m$$
$$K_i = 650\,MN/m$$
$$M_i = 360\,ton$$
$$i = 1 \text{ to } 10$$

Figure 6. Structure Studied in case study two.

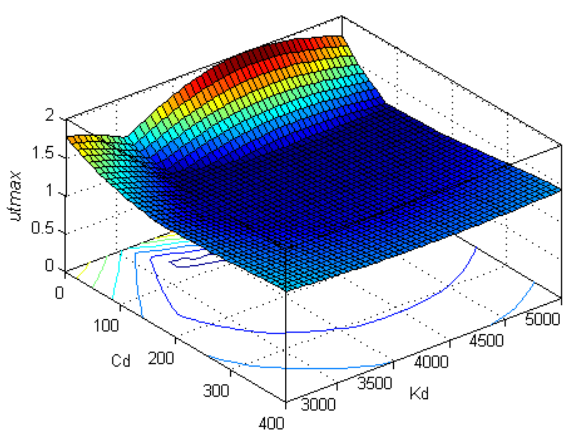


Figure 7. Variation of $\boldsymbol{u_{tmax}}$ with $\boldsymbol{k_d}$ & $\boldsymbol{c_d}$ for example in Fig. 6 as a surface.

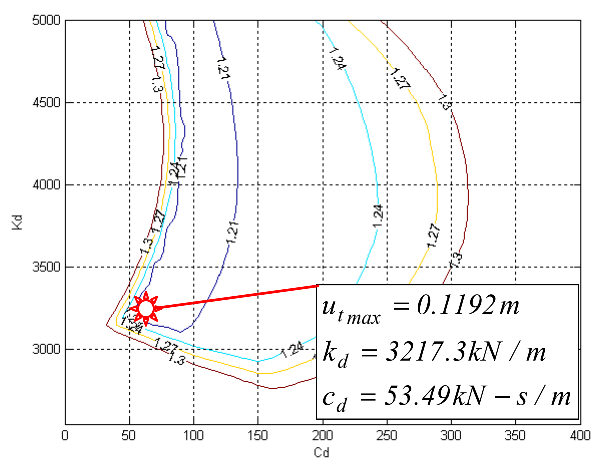


Figure 8. Variation of $\boldsymbol{u_{tmax}}$ with $\boldsymbol{k_d}$ & $\boldsymbol{c_d}$ for example in Fig. 6 as contour line.

system with properties computed according to present study is more efficient in reducing the maximum displacements in all floors than that computed by the above mentioned methods.

Conclusions

From the above discussed cases and other cases discussed by Al-Taweel (2007), the following conclusions can be drawn:

- For a given SDOF or MDOF structure, earthquake excitation and TMD mass, the present study method and software developed are capable of tracing and computing the optimum values for $\boldsymbol{k_d}$ & $\boldsymbol{c_d}$.

Table 1. Maximum floors displacements with and without TMD system under El-Centro excitation structure in case study one.

Floor No.	without TMD (m)	with TMD (m)		
		Hadi (1998)	Lee (2006)	Present St.
1	0.0292	0.0190	0.0200	0.0186
2	0.0569	0.0370	0.0390	0.0361
3	0.0890	0.0580	0.0570	0.0561
4	0.1046	0.0680	0.0730	0.0665
5	0.1261	0.0820	0.0870	0.0815
6	0.1445	0.0940	0.0990	0.0938
7	0.1600	0.1040	0.1080	0.1017
8	0.1737	0.1130	0.1170	0.1114
9	0.1829	0.1190	0.1230	0.1162
10	0.1876	0.1220	0.1260	0.1192
Prop. of TMD k_d (kN/m) c_d (kN-s/m)	-------	k_d=3750 c_d=151.5	k_d=4126 c_d=271.0	k_d=3217 c_d=53.5

- The TMD systems designed according to the present study method were more effective in reducing maximum structure displacement than other methods. This was also found true for a wide range of earthquake excitations and structure frequencies.
- The efficiency of the TMD system designed according to the present study method and other methods were generally affected by the earthquake excitation. This means that the earthquake characteristics have an important effect on the TMD behavior and should be considered in the design process.
- With the increase of research about predicting earthquake acceleration-time history, the importance of the present study will increase as an efficient method for designing TMD systems.

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